Let us suppose that our agent is equipped with a confidence ordering \( \preceq \) over propositions. So that \( p \preceq q \) iff \( S \) is at least as confident in the truth of \( p \) as in the truth of \( q \).

And, \( p > q \) iff \( S \) is strictly more confident in \( p \) than in \( q \).

- **Correctness Requirements** (for comparative confidence)
  - If \( p \) is false and \( q \) is true, then \( p > q \) is incorrect.
  - Heuristically, correctness requirements involve agreement with the attitudes of an omniscient agent [14, 12, 18].

- **Rational Requirements** (for comparative confidence)
  - Intransitive \( \preceq \) relations are structurally irrational. E.g., the combination of attitudes \( \{ p \preceq q, q \preceq r, r \preceq p \} \) is incoherent.
  - If one’s total evidence \( K \) supports \( p \) strictly more strongly than \( K \) supports \( q \), then \( q \preceq p \) is substantively irrational.

Bayesians also like to talk about numerical degrees of confidence (credences). Indeed, this will be our main focus today...
The Fourth Pillar of Structural Bayesianism follows from (1)–(3).

4. **Learning & Supposing.** \( \text{cr}_{t_1} (\cdot) \) should equal \( \text{cr}_{t_0} (\cdot \mid E) \).

De Finetti [3] gave Dutch Book Arguments for both (1) and (2). Accuracy-dominance arguments for (1) are now popular [18]. One can also give an accuracy-dominance argument for (2) [10].

Lewis and others have aimed to adapt de Finetti’s argument for (2) into a “diachronic Dutch Book” argument for (3) [19].

Like de Finetti, I view (1) and (2) as the *fundamental* Bayesian principles governing the (structural) rationality of credences.

I have always been more skeptical about the existence and nature of “diachronic coherence requirements.” Indeed, there seem to be *many* reasons to worry about (3) & (4) [11, 30, 24].

I will focus, primarily, on the synchronic requirements (1) & (2), and how they (allegedly) interact with Explanationism.

---

As Douven [5] explains, there have been various historical views regarding the proper formulation of Explanationism.

Rather than rehearsing Douven’s list of historical explications in detail, I will remain at a higher level of abstraction.

The basic idea behind Explanationism is that some “epistemic credit” should accrue to a hypothesis \( H \) in virtue of its being the best explanation of \( E \) (among the available alternatives \( \{H_k\} \)).

The question in which I am interested is: What is the best way to accommodate this basic Explanationist idea — of “credit” accruing to \( E \)’s best explanation — within a Bayesian framework?

Van Fraassen [25] and Douven [5] maintain that a Bayesian should incorporate Explanationism by revising some of the basic requirements of Structural Bayesianism: (1)–(3).

I will focus on Douven’s proposal, since it is more precise, and it can be couched in purely synchronic terms [as a revision of (2)].

---

There are also *substantive* Bayesian requirements. In general, these are of the following generic form: If one’s total evidence at time \( t (K_t) \) is such and so, then \( \text{cr}_t (\cdot) \) should be thus and such.

Let \( \{H_1, \ldots, H_n\} \) be some partition of alternative hypotheses (putative explanations of \( E \) entertained by an agent (at time \( t \)).

**Substantive Bayesianism** (some example requirements)

- **The Principle of Indifference** [7]. If \( K_t \) does not favor any \( H_i \) over any \( H_j \), then \( \text{cr}_t (H_i) \) should equal \( \text{cr}_t (H_j) \), \( \forall \ i, j \).

- **The Principal Principle** [17]. If \( K_t \) entails that the objective chance of \( H_i \) is \( c \) (and \( K_t \) doesn’t contain/imply any inadmissible evidence), then \( \text{cr}_t (H_i) \) should equal \( c \).

- **The Requirement of Total Evidence** [15, 1, 28]. \( \text{cr}_t (H_i) \) should be equal to the evidential probability \( \text{Pr}(H_i \mid K_t) \).


Douven recommends that Bayesians *revise the Ratio Formula* (2) in such a way that the following alternative to Bayes’s Theorem is adopted (for hypotheses \( H_i \in \{H_1, \ldots, H_n\} \) and evidence \( E \)).

\[
\text{EXPL. } \text{cr}_t (H_i \mid E) = \frac{\text{cr}_t (H_i) \cdot \text{cr}_t (E \mid H_i) + c(H_i, E)}{\sum_{k=1}^n \left[ \text{cr}_t (H_k) \cdot \text{cr}_t (E \mid H_k) + c(H_k, E) \right]}
\]

where \( c(H, E) \in [0, 1) \) is \( H \)’s “\( E \)-abductive credit score.” And, \( c(H, E) > 0 \) iff \( H \) best explains \( E \) (o.w. \( c(H, E) = 0 \)).

Note that if there is no best explanation of \( E \) among the \( \{H_k\} \), then **EXPL reduces to Bayes’s Theorem** (since all of the credit scores \( c(H_k, E) \) will be equal to zero in such a case).

Because **EXPL** leads to violations of Structural Bayesianism [viz., (2)], Douven discusses various ways a defender of **EXPL** might respond to de Finetti’s [3] Dutch Book argument for (2).

---

1This definition can’t be quite right, since sometimes \( c(H, E) \) will need to be \( \geq 1 \) in order to emulate some Bayesian abductive updates. See Extras.
There is a more elegant way to capture the idea of “abductive credit,” which avoids the probabilistic incoherencies of [5, 25].

Note that there is something odd about thinking of EXPL as a structural requirement in the first place. EXPL has this form:

\[
\text{EXPL. } \text{cr}_t(H_i \mid E) \text{ should receive a boost — over and above the value prescribed by Bayes's Theorem — just in case } (A_i) H_i \text{ is the best explanation of } E \text{ (among the } \{H_k\}).
\]

Read literally, then, EXPL relates cr\(_t\)(H\(_i\) \mid E) to the fact that H\(_i\) best explains E, which makes EXPL a correctness requirement.

In order for EXPL to be a structural requirement, it would have to be stated as a relation among the agent's credences. To wit:

\[
\text{EXPL}_1. \text{ cr}_t(H_i \mid E) \text{ should receive a boost — over & above its Bayes's Thm value — iff the agent is certain at t that } A_i.\]

\[2\] We also have to be careful to allow this “boost” to occur only once — presumably, when A\(_i\) is first learned. Otherwise, H\(_i\) will be “over-boosted.”

Even EXPL\(_1\) is arguably not a (pure) structural requirement, since it only constrains agents who entertain A\(_i\), for some H\(_i\) and E.

Pure structural requirements (e.g., probabilism) do not presuppose anything about the contents of an agent’s attitudes.\(^3\)

If EXPL is going to be a (non-vacuous) rational requirement, then it must presuppose that the agent entertains \(A_i\), for some \(H_i\) and \(E\). And, in that case, I think a preferable explication exists.

**Evidential Relevance of Abduction (ERA).** Let \(A_i\) assert that \(H_j\) is the best (or perhaps the only\([4, 8]\)) explanation of \(E\) (among the available \(\{H_k\}\)). Then, it is in some cases — rationally required that an agent's credence function at \(t_0\) be such that

\[
\text{cr}_{t_0}(H_j \mid E \& A_j) > \text{cr}_{t_0}(H_j \mid E \& \neg A_j).
\]

\(^3\)One could also interpret EXPL as a relation between \(S\)'s total evidence at \(t\) \([K_t]\) and their credences at \(t\) \([\text{cr}_t(\cdot)]\). But, as with EXPL\(_1\), we would have to be careful to state such a requirement so as to allow only one “boost” to occur, per hypothesis — presumably, when \(K_t\) comes to entail \(A_j\) for the first time.

---

I’m not the first one to propose something like ERA. Climenhaga, Hartmann *et al.*, Lange, and Weisberg have all argued convincingly for similar principles [2, 16, 26, 4, 13, 8].

Roche & Sober [20] agree that ERA is the right formulation of Explanationism; but, they argue that ERA is false — viz., that \(A_j\) is never relevant to \(H_j\) (on the supposition of the evidence \(E\)).

I think Climenhaga [2] and Lange [16] do a pretty good job of responding to Roche & Sober’s skeptical argument [20].

The main thing I would add to Climenhaga’s and Lange’s trenchant responses Roche & Sober is the following point.

In order to refute R&S’s skepticism, all that is required is a single example in which \(A_j\) is relevant to \(H_j\) (given \(E\)).

I will close by discussing just such an example (involving Newton, Einstein, and the motion of Mercury), which is a well-known instance of The Problem of Old Evidence [6].

---

\(^4\)Of course, \(E\) had been known for many years prior to 1914. It is widely agreed that \(t_0\) would have been no later than 1882, when Newcomb calculated the Newtonian discrepancy in Mercury’s precession precisely.
How would **EXPL₁** handle this Old Evidence Problem?

Let us suppose that our agent became certain of \( A_2 \) in \( t_2 = 1915 \).

Then, according to **EXPL₁**, \( H_2 \) receives a “positive credit score” \( c(H_2, E) > 0 \) and a boost in credence (at \( t_2 \)) — over and above what Bayes’s Thm would recommend. Hence, **EXPL₁** yields

\[
\text{cr}_{t_2}(H_2 \mid E) > \text{cr}_{t_2}(H_2).
\]

In this sense, according to **EXPL₁**, *E confirms H₂ in 1915*.

I see (at least) two problems with this application of **EXPL₁**.

- It is misleading to say that \( E \) is what confirms \( H_2 \) in 1915. It is \( A_2 \) — by ensuring \( c(H_2, E) > 0 \) — that is doing the work.

- In addition to the machinery of probability theory [modulo (2)], Douven also needs a “theory of credit scores.” In this sense, the (Bayesian) ERA approach is more parsimonious.

[See Extras for a detailed formal example, which brings this out.]

The simplest formal example of Douven v. Bayes involves a 2-element partition \( \{ H_1 = H, H_2 = \neg H \} \), evidence \( E \), and an abductive claim \( A \) asserting that \( H \) explains \( E \) better than \( \neg H \).

Our Bayesian agent will begin with a prior \( \text{cr}_{t_0}(\cdot) \) over the algebra generated by the three atoms \( H, E, A \). Between \( t_0 \) and \( t_1 \), they will learn \( E \), and between \( t_1 \) and \( t_2 \) they will learn \( A \).

I will assume \( \text{cr}_{t_0}(\cdot) \) satisfies the following six (6) constraints.

(i) \( \text{cr}_{t_0}(\cdot) \) is regular.

(ii) \( E \) (initially) confirms \( H \) \( \text{cr}_{t_0}(H \mid E) > \text{cr}_{t_0}(H \mid \neg E) \).

(iii) ERA applies \( \text{cr}_{t_0}(H \mid E \& A) > \text{cr}_{t_0}(H \mid E \& \neg A) \).

(iv) \( E \) is (a priori) irrelevant to \( A \). \( \text{cr}_{t_0}(A \mid E) = \text{cr}_{t_0}(A \mid \neg E) \).

(v) \( A \) is (a priori) irrelevant to \( H \). \( \text{cr}_{t_0}(H \mid A) = \text{cr}_{t_0}(H \mid \neg A) \).

(vi) \( \text{cr}_{t_0}(H) = \text{cr}_{t_0}(E) = \text{cr}_{t_0}(A) = \frac{1}{2} \).

There are many priors that satisfy (i)–(vi). I will choose a relatively simple one for illustrative purposes.

**Table:**

<table>
<thead>
<tr>
<th>( A )</th>
<th>( E )</th>
<th>( H )</th>
<th>( \text{cr}_{t_0}(\cdot) )</th>
<th>( \text{cr}<em>{t_1}(\cdot) = \text{cr}</em>{t_0}(\cdot \mid E) )</th>
<th>( \text{cr}<em>{t_2}(\cdot) = \text{cr}</em>{t_0}(\cdot \mid E &amp; A) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>7/32</td>
<td>7/16</td>
<td>7/8</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>3/32</td>
<td>1/16</td>
<td>1/8</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
<td>3/32</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>7/32</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>3/32</td>
<td>3/16</td>
<td>0</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>5/32</td>
<td>5/16</td>
<td>0</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>5/32</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>3/32</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Our Bayesian agent is such that \( \text{cr}_{t_2}(H) = \text{cr}_{t_0}(H \mid E \& A) = 7/8 \).

Douven’s rule for computing \( \text{cr}_{t_2}(H \mid E) \) will yield the Bayesian answer of \( 7/8 \) here only if \( H \)’s credit score is \( c(H, E) = 1 \notin \{0, 1\} \).

In fact, we’ll need to allow \( c(H, E) \in [0, \infty) \). See my PrSAT [9] notebook fitelson.org/douven.pdf (nb) for technical details.

I would argue that this is how the values of \( c(H, E) \) should be “reverse engineered.” But, then, why not just go *fully Bayesian*?