PHIL 424: Take-Home Assignment

December 9, 2014 (**revised** on 12/21/14)

The following assignment may be completed as a make-up either for (a) people who missed the mid-term exam, or (b) people who missed a homework assignment. Please turn in complete and legible answers to each of these problems *at the final exam* (Monday, December 22, 2014 at 12pm, in our usual classroom).

1 Proving Three Theorems Algebraically

Prove the following three general facts about all probability ditributions $\Pr(\cdot)$ — *algebraically*, using the following generic stochastic truth-table (STT) and variables $a, \ldots, h$.

P	Q	R	$\Pr(\cdot)$
T	T	T	a
T	T	F	b
T	F	T	c
T	F	F	d
F	T	T	e
F	T	F	f
F	F	T	g
F	F	F	h

(*i*) *If* (a) $\Pr(P \,\&\, Q) = 0$, and (b) $\Pr(P \mid P \vee Q) = \Pr(Q \mid P \vee Q)$, *then* (c) $\Pr(P) = \Pr(Q)$.

(*ii*) *If* (a) $\Pr(P \,\&\, R) = 0$, and (b) $\Pr(R \mid R \vee P) = \frac{1}{2}$, *then* (c) $\Pr(P) = \Pr(R)$.

(*iii*) *If* (a) $\Pr(P \& Q) = 0$, (b) $\Pr(P \& R) = 0$, (c) $\Pr(Q \& R) = 0$, (d) $\Pr(P \vee Q \vee R) = 1$, (e) $\Pr(P \mid P \vee Q) = \Pr(Q \mid P \vee Q)$, and (f) $\Pr(R \mid R \vee P) = \frac{1}{2}$, *then* (g) $\Pr(P) = \Pr(Q) = \Pr(R) = \frac{1}{3}$.

2 Verifying Three "Odd Properties" of Bayesian Confirmation

Consider the following probability model/STT.

A	B	C	$\Pr(\cdot)$
T	T	T	$a = \frac{14}{80}$
T	T	F	$b = \frac{8}{80}$
T	F	T	$c = \frac{10}{80}$
T	F	F	$d = \frac{8}{80}$
F	T	T	$e = \frac{1}{80}$
F	T	F	$f = \frac{17}{80}$
F	F	T	$g = \frac{15}{80}$
F	F	F	$h = \frac{7}{80}$

Prove that the following three "odd properties" of Bayesian confirmation are (simultaneously) satisfied by this (single) probability distribution (which is also an *urn model*).

(1) A confirms B and A confirms C, but A disconfirms $B \vee C$. That is:

- (1.1) $\Pr(B \mid A) > \Pr(B)$
- (1.2) $\Pr(C \mid A) > \Pr(C)$
- (1.3) $\Pr(B \vee C \mid A) < \Pr(B \vee C)$

(2) A confirms B and A confirms C, and A confirms $B \,\&\, C$. That is:

- (2.1) $\Pr(B \,\&\, C \mid A) > \Pr(B \,\&\, C)$

(3) A confirms B and A confirms C, and A confirms $B \equiv C$. That is:

- (3.1) $\Pr(B \equiv C \mid A) > \Pr(B \equiv C)$

3 Bonus (*Hard!*) Problem (worth 25% extra-credit)

Prove that *if* (a) A confirms C and (b) B confirms C, *then* (c) *either* $A \vee B$ confirms C *or* $A \,\&\, B$ confirms C. That is, prove that the following four constraints *cannot* be jointly satisfied by *any* probability distribution (using the STT and variables from the previous problem). Hint: suppose that (I)–(IV) *do* all hold, and then show this implies an *algebraic falsehood*.

- (I) $\Pr(C \mid A) > \Pr(C)$
- (II) $\Pr(C \mid B) > \Pr(C)$
- (III) $\Pr(C \mid A \vee B) \leq \Pr(C)$
- (IV) $\Pr(C \mid A \,\&\, B) \leq \Pr(C)$