

PHIL 424: Practice Mid-Term

October 16, 2014

The actual mid-term exam will have the same structure as this practice mid-term. You will have the full class period on Tuesday (10/21) to complete the actual exam. You may use a calculator.

1 Proving A Probability Theorem Algebraically

Prove the following theorem, by (a) translating it into algebra (using the stochastic truth-table below), and then (b) showing that the resulting algebraic inequality must be true (assuming, as always, that a, b, c, d are each on $[0, 1]$ and that they sum to one).

Theorem 1. $\text{cr}(X \supset Y) \geq \text{cr}(Y \mid X)$.

Please use the following stochastic truth-table to prove **Theorem 1**.

X	Y	$\text{cr}(\cdot)$
T	T	a
T	F	b
F	T	c
F	F	d

2 Finding a Probability Distribution

Find a probability distribution (*i.e.*, an assignment of numbers to a, b, c, d , which are each on $[0, 1]$ and which sum to one — as in the above stochastic truth-table) which satisfies the following three constraints. Explain how you found the solution, and why it is correct.

1. $\text{cr}(X \supset Y) = \text{cr}(Y \mid X)$.
2. $\text{cr}(X) = 1/2$.
3. $\text{cr}(Y) = 5/8$.

3 Verifying Properties of a Probability Distribution

Here is a (stochastic truth-table representation of a) probability distribution over the algebra generated by the three atomic sentences H, E, K .

H	E	K	$\text{cr}(\cdot)$
T	T	T	$a := 49/256$
T	T	F	$b := 1/16$
T	F	T	$c := 31/256$
T	F	F	$d := 1/8$
F	T	T	$e := 31/256$
F	T	F	$f := 1/8$
F	F	T	$g := 17/256$
F	F	F	$h := 3/16$

Use this table to verify the following three claims about this distribution.¹

1. $\text{cr}(H \mid E) > \text{cr}(H)$.
2. $\text{cr}(H \mid E \ \& \ K) < \text{cr}(H \mid K)$.
3. $\text{cr}(H \mid E \ \& \ \sim K) < \text{cr}(H \mid \sim K)$.

4 Proving Another Probability Theorem Algebraically

Prove the following theorem, by (a) translating it into algebra (using the stochastic truth-table below), and then (b) showing that the resulting algebraic statement must be true (assuming, as always, that a, b, c, d, e, f, g, h are each on $[0, 1]$ and that they sum to one).

Theorem 2. If $\text{cr}(X \mid Y \ \& \ Z) = 1$, then $\text{cr}(X \mid Y) \geq \text{cr}(Z \mid Y)$.

Please use the following stochastic truth-table to prove **Theorem 2**.

X	Y	Z	$\text{cr}(\cdot)$
T	T	T	a
T	T	F	b
T	F	T	c
T	F	F	d
F	T	T	e
F	T	F	f
F	F	T	g
F	F	F	h

¹This is a case in which (1) E is positively relevant to H , *unconditionally*; but, (2) E is negatively relevant to H , *conditional upon* K and (3) E is negatively relevant to H , *conditional upon* $\sim K$. What is this kind of case called? Hint: it's got "paradox" in the name.