PHIL 424: Practice Final Exam

December 9, 2014

The actual final exam will have the same structure as this practice final. You will have 3 hours (Monday, December 22, 2014 from 12–3pm) to complete the actual exam. You are to answer all parts of all four problems — please make sure to turn in legible and complete answers to all parts. You may use a calculator.

1 Verifying a Conjunctive Fork

Consider the following probability model/stochastic truth-table (STT).

X	Y	Z	$\Pr(\cdot)$
T	T	T	$a = \frac{1}{4}$
T	T	F	$b = \frac{15}{256}$
T	F	T	$c = \frac{1}{128}$
T	F	F	$d = \frac{1}{16}$
F	T	T	$e = \frac{1}{4}$
F	T	F	$f = \frac{45}{256}$
F	F	T	$g = \frac{1}{128}$
F	F	F	$h = \frac{3}{16}$

Prove that $\{X, Y, Z\}$ form a *conjunctive fork* (in Reichenbach's sense). That is, prove that the following three facts obtain for this STT.

1. X and Z are positively correlated, *i.e.*, $\Pr(X \mid Z) > \Pr(X)$.
2. Y and Z are positively correlated, *i.e.*, $\Pr(Y \mid Z) > \Pr(Y)$.
3. Z *screens-off* X from Y, *i.e.*, $\Pr(X \mid Y \,\&\, Z) = \Pr(X \mid Z)$.

First, translate the probabilistic definitions of facts (1)–(3) into algebraic expressions in terms of the variables $a, \ldots, h$. Then, plug-in the values of $a, \ldots, h$ (in the STT above) into these algebraic expressions. Finally, thereby demonstrate that the 3 facts above obtain.

2 Proving Three Theorems From Confirmation Theory

Consider the following three Bayesian measures of the degree to which E confirms H:

$$s(H,E) \overset{\text{def}}{=} \Pr(H \mid E) - \Pr(H \mid {\sim}E)$$
$$r(H,E) \overset{\text{def}}{=} \frac{\Pr(H \mid E) - \Pr(H)}{\Pr(H \mid E) + \Pr(H)}$$
$$n(H,E) \overset{\text{def}}{=} \Pr(H \,\&\, E) - \Pr(H) \cdot \Pr(E)$$

And, consider the following two symmetry properties for relevance measures $\mathfrak{c}$:

(S_1) $\mathfrak{c}(H,E) = -\mathfrak{c}(H,{\sim}E)$

(S_2) $\mathfrak{c}(H,E) = \mathfrak{c}(E,H)$

Prove the following three theorems: (*i*) $s(H,E)$ satisfies (S_1), (*ii*) r satisfies (S_2), and (*iii*) $n(H,E)$ satisfies *both* (S_1) *and* (S_2). That is, prove (*i*) $s(H,E) = -s(H,{\sim}E)$, (*ii*) $r(H,E) = r(E,H)$, and (*iii*) both $n(H,E) = -n(H,{\sim}E)$ *and* $n(H,E) = n(E,H)$. You are to complete these proofs algebraically, using the following variables $a, \ldots, d$ to set-up your proofs.

E	H	$\Pr(s_i)$
T	T	a
T	F	b
F	T	c
F	F	d

3 Applying Bayes's Theorem & Bayesian Confirmation Theory

There are 3 coins in a box. One is a two-headed coin; another is a fair coin; and the third is a biased coin that comes up heads 75% of the time. A coin is sampled at random from the box (this means that each of the three has the same probability of being drawn, and that the draw of the coin is *independent* of its subsequent flip), and then it is flipped. The coin comes up heads. Now, answer three questions:

(a) What is the probability that the coin flipped was the two-headed coin?

(b) Does the coin's having landed heads *confirm* or *disconfirm* (in the Bayesian/relevance sense) that the coin was the 75%-biased-in-favor-of-heads coin? Explain.

(c) Does the coin's having landed heads *confirm* or *disconfirm* that the coin was the fair coin? Explain.

Hints: Bayes's Theorem for a set of n mutually exclusive and exhaustive hypotheses $\{H_i\}$ has the following form:

$$\Pr(H_i \mid E) = \frac{\Pr(E \mid H_i)\Pr(H_i)}{\Pr(E)} = \frac{\Pr(E \mid H_i)\Pr(H_i)}{\sum_{i=1}^{n}\Pr(E \mid H_i)\Pr(H_i)}$$

Here, let H_1 = the coin is two-headed, H_2 = the coin is fair, H_3 = the coin is biased 75% in favor of heads, and E = the coin landed heads on the toss. And, remember, confirmation means *positive relevance* and *dis*confirmation means *negative relevance*.

4 Explicating A Paradox/Problem

Pick **one** paradox/problem from the following list, and explain in your own words (1) what the problem/paradox is, and (2) what you take to the appropriate/best approach/response to the paradox/problem. Make your explications as complete and self-contained as possible.

- The Lottery Paradox
- The Raven Paradox
- The Problem of Old Evidence
- The Problem of the Reference Class
- The Intransitivity of Confirmation/Evidential Relevance