- We assume our agent has a credence function $b(\cdot)$, which is *probabilistic*. Probabilism for $b(\cdot)$ can *itself* be motivated *via* EUT [26]. Here, I will take this to be *common ground*.
- We assume that our agent takes exactly one of three qualitative attitudes (B, D, S) toward each member of a finite agenda $\mathcal{A}$ of (classical, possible worlds) propositions.
- We do *not* assume that these qualitative judgments can be *reduced* to $b(\cdot)$. But, we will use $b(\cdot)$ to derive a *rational coherence constraint* for qualitative judgment sets $\mathbf{B}$ (on $\mathcal{A}$).
- This derivation requires both the agent's credence function $b(\cdot)$ and their *epistemic utility function* [12, 19, 23] $u(\cdot)$.
  - ☞ Following Easwaran [4, 6], we assume our agent cares *only* about whether their qualitative judgments are *accurate*.
- Specifically, our agent attaches some *positive* utility ($\mathfrak{r}$) with making an *accurate* judgment, and some *negative* utility ($-\mathfrak{w}$) with making an *inaccurate* judgment (where $\mathfrak{w} \geq \mathfrak{r} > 0$).

- Because suspensions are neither accurate nor inaccurate (*per se*), our agent will attach *zero* epistemic utility to suspensions $S(p)$, independently of the truth-value of p.
- Thus, we have the following piecewise definition of $u(\cdot, w)$.

$$u(B(p), w) \stackrel{\text{def}}{=} \begin{cases} -\mathfrak{w} & \text{if } p \text{ is false at } w \\ \mathfrak{r} & \text{if } p \text{ is true at } w \end{cases}$$

$$u(D(p), w) \stackrel{\text{def}}{=} \begin{cases} \mathfrak{r} & \text{if } p \text{ is false at } w \\ -\mathfrak{w} & \text{if } p \text{ is true at } w \end{cases}$$

$$u(S(p), w) \stackrel{\text{def}}{=} \begin{cases} 0 & \text{if } p \text{ is false at } w \\ 0 & \text{if } p \text{ is true at } w \end{cases}$$

- With this *accuracy-centered* epistemic utility function in hand, we can derive a naïve EUT coherence requirement.

- To do so, we'll also need a *decision-theoretic principle*.
- Applications of EUT to grounding probabilism as a (synchronic) requirement for $b(\cdot)$ typically appeal to a *non-dominance* (in epistemic utility) principle [15, 27, 26].
- But, some authors apply an *expected epistemic utility maximization* (or *expected inaccuracy minimization*) principle to derive rational requirements [18, 11, 5, 25].

Coherence. An agent's belief set $\mathbf{B}$ over an agenda $\mathcal{A}$ should, from the point of view of their own credence function $b(\cdot)$, *maximize expected epistemic utility* (or *minimize expected inaccuracy*). That is, $\mathbf{B}$ should maximize

$$EEU(\mathbf{B}, b) \stackrel{\text{def}}{=} \sum_{p \in \mathcal{A}} \sum_{w \in W} b(w) \cdot u(\mathbf{B}(p), w)$$

where $\mathbf{B}(p)$ is the agent's attitude toward p, and $W \stackrel{\text{def}}{=} \bigcup \mathcal{A}$.

- For now, we assume "*act-state independence*": $\mathbf{B}(p)$ and p are *b-independent* [10, 2, 1, 16]. We'll return to this issue.

- The consequences of **Coherence** are rather simple and intuitive. It is straightforward to prove the following result.

Theorem ([4]). An agent with credence function $b(\cdot)$ and qualitative judgment set $\mathbf{B}$ over agenda $\mathcal{A}$ satisfies **Coherence** *if and only if* for all $p \in \mathcal{A}$

$$B(p) \in \mathbf{B} \textit{ iff } b(p) > \tfrac{\mathfrak{w}}{\mathfrak{r}+\mathfrak{w}},$$

$$D(p) \in \mathbf{B} \textit{ iff } b(p) < \tfrac{\mathfrak{r}}{\mathfrak{r}+\mathfrak{w}},$$

$$S(p) \in \mathbf{B} \textit{ iff } b(p) \in \left[\tfrac{\mathfrak{r}}{\mathfrak{r}+\mathfrak{w}}, \tfrac{\mathfrak{w}}{\mathfrak{r}+\mathfrak{w}}\right].$$

- ☞ In other words, **Coherence** *entails Lockean representability*, where the Lockean thresholds are determined by the way the agent (relatively) values accuracy *vs.* inaccuracy.
- This provides an elegant, EUT-based explanation of why Lockean representability is a rational requirement for agents with *both* credences *and* qualitative attitudes.
- Next, I will explain when **Coherence** entails *consistency*.

- Suppose our (naïve) agent has a belief set $\mathbf{B}_n$ on a *minimal inconsistent* agenda of size n (*e.g.*, $(n-1)$–ticket lottery).

  Theorem. For all $n \geq 2$ and any probability function $\Pr(\cdot)$, the $\Pr(\cdot)$-Lockean-representability of $\mathbf{B}_n$ (with threshold t) *entails* deductive consistency of $\mathbf{B}_n$ *iff* $t \geq \frac{n-1}{n}$.

- If we combine this with Easwaran's **Coherence** theorem, we get the following result, regarding the conditions under which the **Coherence** of $\mathbf{B}_n$ *entails* the *consistency* of $\mathbf{B}_n$.

  Theorem. For all $n \geq 2$, an agent with an accuracy-centered utility function u, a credence function $b(\cdot)$, and a belief set $\mathbf{B}_n$, the **Coherence** of $\mathbf{B}_n$ entails the consistency of $\mathbf{B}_n$ *iff*

  $$(\dagger) \qquad \mathfrak{w} \geq (n-1) \cdot \mathfrak{r}.$$

☞ Insisting that **Coherence** *implies consistency* (wrt $\mathbf{B}_n$) *requires (naïve) agents to disvalue inaccuracy at least* $(n-1)$ *times as much as they value accuracy*.

- Of course, there will be *some* agents with epistemic utility functions u, which *do* satisfy (†). But, it is very odd (from a traditional Bayesian perspective) to *require* that such an agent's epistemic utility function *must* satisfy (†).
- For example, in Lottery Paradox cases, we can make n as large as we like. And, the larger we make n, the stronger (and more implausible) the constraint (†) becomes.

☞ $\therefore$ $\mathbf{B}_n$-consistency won't be a *universal* MEEU-requirement. In other words, consistency *outstrips* the MEEU-theory of epistemic rationality. Leitgeb [17] defends an alternative.

- According to Leitgeb's Stability Theory [17], a rational agent with credence function b (over a set of possible worlds W) believes a proposition p, *viz.*, $B(p)$ iff p is entailed by some proposition B_W that is p-*stable*, where this is defined as:

  p-stability. Given a probability model $\langle W, b(\cdot)\rangle$, a proposition $x \in W$ is **p-stable** iff $b(x \mid y) > 1/2$, for all $y \in W$ such that $x \,\&\, y \neq \bot$ and $b(y) > 0$.

- Basically, Leitgeb's theory requires an agent to satisfy a *resilient* [30] version of the Lockean Thesis.
- As Leitgeb explains, his theory will require that rational agents have *consistent* (and *closed* $\therefore$ *cogent*) belief sets (even belief sets $\mathbf{B}_n$ in $(n-1)$-ticket Lottery Paradoxes).
- So, by our argument above, Stability Theory must *outstrip* MEEU-theory, which does *not* require consistency of $\mathbf{B}_n$ (at least, this is not required for *every* MEEU-rational agent).
- Leitgeb's theory has several (*prima facie*) odd consequences. I will focus on one problematic feature: the *violation* of *partition-invariance*. As we'll see, Leitgeb's theory is partition-sensitive in a particularly troubling way.
  - See Extras for some other (*prima facie*) odd consequences.
- A requirement on rational belief (or rational action) is *partition-invariant* (PI) iff its prescriptions do not depend on how the underlying space of possibilities is partitioned.

- In the case of practical rationality (*viz.*, rational action), many philosophers endorse (PI) as a *desideratum* [13, 7, 8, 20, 14].
- Savage's theory [28] and standard causal decision theories [9, 30, 21, 31] are partition-*dependent*. This has led various authors [13, 7, 14] to endorse evidential decision theories.
- We defined **Coherence** "Savage–style," and we assumed *act-state independence* (ASI) to ensure (PI). For our present examples (*e.g.*, Lotteries) this is OK. *But*, see [10, 2, 16].[1]
- Lin & Kelly [22] show: adding cogency to *any* non-trivial probabilistic acceptance rule for belief will entail *partition sensitivity* — even in Lottery cases (*i.e.*, *even if* ASI obtains).

☞ And, Schurz [29] shows that all cogent *Lockean* theories (*i.e.*, all *Stability* theories) must violate an even more plausible invariance constraint, which he calls *Independence*.

[1] More generally, **Coherence** will satisfy (PI) if u satisfies following, for all partitions $\{X_i\}$ of W: $(\forall X_i)\,[u(\mathbf{B}(p), X_i) = \sum_{w \in W} b(w \mid X_i) \cdot u(\mathbf{B}(p), w)]$.

- This stronger kind of partition-sensitivity can be illustrated *via* one of Leitgeb's own examples ([17, *pp*. 164–168], [3]).
- Let $\neg E \overset{\text{def}}{=}$ Adams's prediction of the secular acceleration of the moon, $T \overset{\text{def}}{=}$ Newtonian theory (the part Adams used to predict $\neg E$), $H \overset{\text{def}}{=}$ the auxiliary hypotheses (*e.g.*, negligibility of tidal friction) Adams used in his deduction of $\neg E$ from T.
- Suppose the following probability model $\langle W, b(\cdot)\rangle$ represents to the *epistemically rational degrees of belief* of a mid-19th century scientist (*e.g.*, Adams), prior to learning E.

w_i	E	H	T	$b(w_i)$
w_1	**F**	**T**	**T**	27/50
w_2	**F**	**F**	**T**	171/500
w_3	**F**	**T**	**F**	29/500
w_4	**F**	**F**	**F**	1997/50000
w_5	**T**	**F**	**T**	9/500
w_6	**T**	**F**	**F**	1/500
w_7	**T**	**T**	**F**	3/50000
w_8	**T**	**T**	**T**	0

- As Dorling explains, Bayesian confirmation theory (BCT) implies the following two verdicts regarding this example (*pace* what Quine & Duhem might have said about the case).
  (1) E *weakly* disconfirms T
  $$b(T \mid E) = 0.897308 \lessapprox 0.9 = b(T)$$
  (2) E *strongly* disconfirms H
  $$b(H \mid E) = 0.00299103 \ll 0.59806 = b(H)$$
- Leitgeb offers a qualitative analysis of Dorling's example (in ST), in which the agent starts off (prior to learning E) believing H, T, and $\neg E$. Then, after learning E, the agent comes to *disbelieve* H, while believing both E and T.

☞ Crucial to Leitgeb's analysis is his choice of $B_W = \{w_1\}$, which is the strongest p-stable set, relative to $\langle W, b(\cdot)\rangle$.

- While Leitgeb's qualitative analysis of the Dorling example is interesting, it is threatened by the non-Independence of Stability Theory. Allow me to explain, *via* a similar example.

- Add a fourth atomic sentence "C" to the language, which expresses the proposition that a fair coin-toss came up "heads". Here's the resulting probability model $\langle W', b'(\cdot)\rangle$.

w'_i	E	H	T	C	$b'(w'_i)$
w'_1	**F**	**T**	**T**	**T**	27/100
w'_2	**F**	**F**	**T**	**T**	171/1000
w'_3	**F**	**T**	**F**	**T**	29/1000
w'_4	**F**	**F**	**F**	**T**	1997/100000
w'_5	**T**	**F**	**T**	**T**	9/1000
w'_6	**T**	**F**	**F**	**T**	1/1000
w'_7	**T**	**T**	**F**	**T**	3/100000
w'_8	**T**	**T**	**T**	**T**	0
w'_9	**F**	**T**	**T**	**F**	27/100
w'_{10}	**F**	**F**	**T**	**F**	171/1000
w'_{11}	**F**	**T**	**F**	**F**	29/1000
w'_{12}	**F**	**F**	**F**	**F**	1997/100000
w'_{13}	**T**	**F**	**T**	**F**	9/1000
w'_{14}	**T**	**F**	**F**	**F**	1/1000
w'_{15}	**T**	**T**	**F**	**F**	3/100000
w'_{16}	**T**	**T**	**T**	**F**	0

- The two BCT verdicts [(1), (2)] above *remain exactly the same* in this new model $\langle W', b'(\cdot)\rangle$, since C is b'*-independent*.

☞ But, the strongest p-stable proposition relative to this new model $\langle W', b'(\cdot)\rangle$ is now $B_{W'} = \{w'_1, w'_2, w'_9, w'_{10}\}$, which *does not entail* H. So, Leitgeb's ST-analysis is *undermined*.

- Here's a conjecture regarding one possible way of getting to something the resembles ST, using the machinery of EUT.

  Conjecture. Let $\mathcal{Y}$ be any set of W-propositions (with nonzero b-credence). If a belief set $\mathbf{B}$ (on $\mathcal{A}$) maximizes
  $$EEU_{\mathcal{Y}}(\mathbf{B}, b) \overset{\text{def}}{=} \sum_{p \in \mathcal{A}} \sum_{w \in W} b(w \mid y) \cdot u(\mathbf{B}(p), w)$$
  for all $y \in \mathcal{Y}$, then $\mathbf{B}$ is *resiliently* Lockean representable by $b(\cdot \mid y)$, *for each* $y \in \mathcal{Y}$, with threshold $t = \frac{\mathfrak{w}}{\mathfrak{r}+\mathfrak{w}}$.
- If this conjecture is true, then "Stability Theory" emerges from "*resilient* expected epistemic utility maximization."

- S_1 and S_2 *share the same credence function* $b_1 = b_2 = b$. But, they have *very different belief states* $\mathbf{B}_1$ and $\mathbf{B}_2$ [24]. The following table depicts b, $\mathbf{B}_1$ and $\mathbf{B}_2$ (on the *contingent* p's).

w's	p	b	$\mathbf{B}_1$ (MEEU$_{1/2}$)	$\mathbf{B}_2$ (ST$_{1/2}$)
$\{w_1\}$	$\neg X \wedge \neg Y$	0.5	S	S
$\{w_2\}$	$X \wedge \neg Y$	0.25	D	S
$\{w_3\}$	$X \wedge Y$	0.125	D	S
$\{w_4\}$	$\neg X \wedge Y$	0.125	D	S
$\{w_1, w_2\}$	$\neg Y$	0.75	B	S
$\{w_1, w_3\}$	$X \equiv Y$	0.625	B	S
$\{w_1, w_4\}$	$\neg X$	0.625	B	S
$\{w_2, w_3\}$	X	0.375	D	S
$\{w_2, w_4\}$	$X \not\equiv Y$	0.375	D	S
$\{w_1, w_4\}$	Y	0.25	D	S
$\{w_1, w_2, w_3\}$	$X \vee \neg Y$	0.875	B	S
$\{w_1, w_2, w_4\}$	$\neg X \vee \neg Y$	0.875	B	S
$\{w_1, w_3, w_4\}$	$\neg X \vee Y$	0.75	B	S
$\{w_2, w_3, w_4\}$	$X \vee Y$	0.5	S	S

- There are (arbitrarily) small perturbations b' of b, which (a) do not alter the $1/2$-credence p's, (b) *lower the credence of* $\neg X \vee \neg Y$, but (c) *make it rational for* S_2 *to believe* $\neg X \vee \neg Y$.

w's	p	b	b'	$\mathbf{B}_1 = \mathbf{B}'_1$	$\mathbf{B}_2$	$\mathbf{B}'_2$
$\{w_1\}$	$\neg X \wedge \neg Y$	0.5	0.5	S	S	S
$\{w_2\}$	$X \wedge \neg Y$	0.25	0.2366	D	S	S
$\{w_3\}$	$X \wedge Y$	0.125	0.1295	D	S	D
$\{w_4\}$	$\neg X \wedge Y$	0.125	0.1339	D	S	S
$\{w_1, w_2\}$	$\neg Y$	0.75	0.7366	B	S	S
$\{w_1, w_3\}$	$X \equiv Y$	0.625	0.6295	B	S	S
$\{w_1, w_4\}$	$\neg X$	0.625	0.6339	B	S	S
$\{w_2, w_3\}$	X	0.375	0.3660	D	S	S
$\{w_2, w_4\}$	$X \not\equiv Y$	0.375	0.3705	D	S	S
$\{w_1, w_4\}$	Y	0.25	0.2634	D	S	S
$\{w_1, w_2, w_3\}$	$X \vee \neg Y$	0.875	0.8661	B	S	S
$\{w_1, w_2, w_4\}$	$\neg X \vee \neg Y$	0.875	0.8705	B	S	B
$\{w_1, w_3, w_4\}$	$\neg X \vee Y$	0.75	0.7634	B	S	S
$\{w_2, w_3, w_4\}$	$X \vee Y$	0.5	0.5	S	S	S

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