Announcements & Overview

• Administrative Stuff
  - **HW #2 grades & solutions have been posted**
  - Additional L2PL symbolization and semantics problems can be found on the website. Good additional practice.
  - **HW #3 has been posted**
    * Due date extended by one week — to March 3
  - **The mid-term is next Friday — March 3**
    * I have posted a practice mid-term
    * We will go over the practice mid-term on Tuesday
    * I have also posted a mid-term rules handout on Wednesday
  - **Special Office Hours next week: Tues 11:45–1:15 & Wed 12–2**

• Today: Natural Deduction Proofs in LSL (Chapter 4 of Forbes)
  - Quick review of rules for & and →, then onto ∨ and ~.
The Rule of Assumptions (Final Version)

- **Rule of Assumptions** (final version): At any line $j$ in a proof, any formula $p$ may be entered and labeled as an assumption (or premise, where appropriate). The number $j$ should then be written on the left. Schematically:

  $\begin{array}{c}
  j \quad (j) \quad p \\
  \end{array}$

  Assumption (or: Premise)

- We will have three (LSL) rules that will require the introduction of assumptions. **Only introduce an assumption in connection with one of these three rules** — and with an explicit strategy for discharging said assumption, in accordance with one of these three rules.
The Rule of \&-Elimination (\&E)

- **Rule of \&-Elimination**: If a conjunction \( p \& q \) occurs at line j, then at any later line k one may infer either conjunct, labeling the line ‘j \&E’ and writing on the left all the numbers which appear on the left of line j.

Schematically:

\[
\begin{align*}
& a_1, \ldots, a_n \quad (j) \quad p \& q \\
& \vdots \\
& a_1, \ldots, a_n \quad (k) \quad p \\
\end{align*}
\]

OR

\[
\begin{align*}
& a_1, \ldots, a_n \quad (j) \quad p \& q \\
& \vdots \\
& a_1, \ldots, a_n \quad (k) \quad q \\
\end{align*}
\]
The Rule of &-Introduction (&I)

- **Rule of &-Introduction**: For any formulae $p$ and $q$, if $p$ occurs at line $j$ and $q$ occurs at line $k$ then the formula ‘$p \& q$’ may be inferred at line $m$, labeling the line ‘$j, k \&I$’ and writing on the left all numbers which appear on the left of line $j$ and all which appear on the left of line $k$. [Note: we may have $j < k$, $j > k$, or $j = k$. *Why?*

\[
\begin{align*}
\begin{array}{cccccc}
a_1, \ldots, a_n & (j) & p \\
\vdots & & \\
 b_1, \ldots, b_u & (k) & q \\
\vdots & & \\
a_1, \ldots, a_n, b_1, \ldots, b_u & (m) & p \& q & j, k \&I
\end{array}
\end{align*}
\]
The Rule of →-Elimination (→E)

- **Rule of →-Elimination:** For any formulae $p$ and $q$, if $p \rightarrow q$ occurs at a line $j$ and $p$ occurs at a line $k$, then $q$ may be inferred at line $m$, labeling the line ‘$j, k \rightarrow E$’ and writing on the left all numbers which appear on the left of line $j$ and all numbers which appear on the left of line $k$.

[Note: We may have either $j < k$ or $j > k$.]

\[
\begin{align*}
  &a_1, \ldots, a_n \quad (j) \quad p \rightarrow q \\
  &\quad \vdots \\
  &b_1, \ldots, b_u \quad (k) \quad p \\
  &\quad \vdots \\
  &a_1, \ldots, a_n, b_1, \ldots, b_u \quad (m) \quad q \quad j, k \rightarrow E
\end{align*}
\]
The Rule of →-Introduction (¬I)

- Now, we need a formal Introduction Rule for the →, which captures the intuitive idea sketched above (i.e., assuming the antecedent, etc.):

- **Rule of →-Introduction**: For any formulae \( p \) and \( q \), if \( q \) has been inferred at a line \( k \) in a proof and \( p \) is an assumption or premise occurring at line \( j \), then at line \( m \) we may infer \( [p \to q] \), labeling the line ‘\( j, k \to I \)’ and writing on the left the same assumption numbers which appear on the left of line \( k \), except that we delete \( j \) if it is one of these numbers. Note: we may have \( j < k \), \( j > k \), or \( j = k \) (why?). Schematically:

\[
\begin{align*}
\text{j (j)} & \quad p & \text{Assumption (or: Premise)} \\
\vdots & & \\
\{a_1, \ldots, a_n\}/j & \quad (m) & \quad p \to q & \quad j, k \to I
\end{align*}
\]
## The Three Negation Rules

<table>
<thead>
<tr>
<th>Negation Elimination (∼E)</th>
<th>Negation Introduction (∼I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1, \ldots, a_n$ $(j)$ $\sim q$</td>
<td>$j$ $(j)$ $p$ Assumption</td>
</tr>
<tr>
<td></td>
<td>$\vdots$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>$b_1, \ldots, b_u$ $(k)$ $q$</td>
<td>$a_1, \ldots, a_n$ $(k)$ $\land$</td>
</tr>
<tr>
<td></td>
<td>$\vdots$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>$a_1, \ldots, a_n, b_1, \ldots, b_u$ $(m)$ $\land$ $j, k \sim E$</td>
<td>${a_1, \ldots, a_n}/j$ $(m)$ $\sim p$ $j, k \sim I$</td>
</tr>
</tbody>
</table>

### Double Negation (DN)

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<table>
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</thead>
<tbody>
<tr>
<td>$a_1, \ldots, a_n$ $(j)$ $\sim \sim p$</td>
</tr>
<tr>
<td>$a_1, \ldots, a_n$ $(k)$ $p$ $j$ DN</td>
</tr>
</tbody>
</table>
Another Example Requiring DN: Using MacLogic

Here is a (MacLogic generated) proof of: \( B, \neg B \vdash A \).

1  (1)  \( B \)  Premise
2  (2)  \( \neg B \)  Premise
3  (3)  \( \neg A \)  Assumption
1,2  (4)  \( \land \)  2,1 \( \neg E \)
1,2  (5)  \( \neg \neg A \)  3,4 \( \neg I \)
1,2  (6)  \( A \)  5 \( DN \)
Important Tips For Using the Negation Rules

• If you are trying to derive a formula with ‘∼’ as its main connective, use ~I to obtain it. I.e., assume the formula within the scope of the ‘∼’ and try to derive ∧ using ~E.

• When you apply ~I, the formula which you infer must be the negation of a premise or an assumption. It cannot be the negation of a formula which has been deduced.

• If one of your premises or assumptions has ‘∼’ as its main connective, it is likely that its role in the proof will be to be one of a pair of contradictory formulae in an application of ~E. You should therefore consider trying to derive the formula within the scope of the ‘∼’ to get the other member of the contradictory pair.

• If you are trying to deduce a sentence-letter and there is no obvious way to do it, consider trying to derive its double-negation and then use DN. Last Resort!

• At this point, you should only assume a formula p if you are trying to deduce its negation or trying to deduce a conditional with p as antecedent. Only make an assumption when you’ve figured how you’re going to use ~I or ~I to discharge it.
Cautionary Remarks about *Reductio* Proofs

- Once you have deduced a contradiction (\(\land\)) in the course of a proof, you can subsequently deduce *any* formula *p* via \(\sim\text{I}\) and DN.
- But, such a deduction may depend on various assumptions, which means *they won’t be proofs from the premises alone*. From last time:

<table>
<thead>
<tr>
<th>Line</th>
<th>Formula</th>
<th>Assumption/Proof Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(\sim(A &amp; B))</td>
<td>Premise</td>
</tr>
<tr>
<td>2</td>
<td>(A)</td>
<td>Assumption</td>
</tr>
<tr>
<td>3</td>
<td>(B)</td>
<td>Assumption</td>
</tr>
<tr>
<td>2, 3</td>
<td>(A &amp; B)</td>
<td>2, 3 &amp;I</td>
</tr>
<tr>
<td>1, 2, 3</td>
<td>(\land)</td>
<td>1, 4 (\sim\text{E})</td>
</tr>
<tr>
<td>1, 2</td>
<td>(\sim B)</td>
<td>3, 5 (\sim\text{I})</td>
</tr>
<tr>
<td>1</td>
<td>(A \rightarrow \sim B)</td>
<td>2, 6 (\sim\text{I})</td>
</tr>
</tbody>
</table>
You might be tempted to think that you could prove $A \rightarrow \sim B$ via $\sim I$ and DN after step (5). You can deduce it in this way, but you get:

1. $(1) \sim (A \& B)$ Premise
2. $(2) A$ Assumption
3. $(3) B$ Assumption
4. $2, 3 (4) A \& B$ 2, 3 &I
5. $1, 2, 3 (5) \&$ 1, 4 \&E
6. $(6) \sim (A \rightarrow \sim B)$ Assumption
7. $1, 2, 3 (7) \sim (A \rightarrow \sim B)$ 6, 5 \&I
8. $1, 2, 3 (8) A \rightarrow \sim B$ 7 DN

This does not help. We need to prove $A \rightarrow \sim B$ from (1) alone, not from (1), (2), and (3). [Note: (1)–(3) is an inconsistent set.]

Lesson: A strategy for proving the conclusion from the premises alone requires discharging all assumptions that are not premises.
General Strategy — Working in Both Directions

- Begin by writing the premises (if any) at the top of your scratch paper area, using the Rule of Assumptions.
- Then, write the conclusion (the main goal formula) you’re trying to derive at the bottom of your scratch area.
- Next, determine what the main connective (if any) of your conclusion is, then apply the introduction rule for that connective.
- This will yield sub-goal formula(s). Write the sub-goal formula(s) directly above your conclusion. Then try to figure-out how to prove the sub-goal formula(s) from your premises.
- This will yield sub-sub-goal formula(s). And so on …
- Repeat this process until you have worked your way all the way back up to your premises/assumptions (if a formula is resisting proof, you might try to prove its double-negation using ¬I with ¬E).
Example Proof of a *Theorem*

- Using only the rules we have learned so far, we should be able to prove the following *theorem*: \( \vdash \sim (A \& \sim A) \). Let’s do this one by hand first.

- Here’s a simple proof, generated using MacLogic (I’ll show how):

```
Problem is: \( \vdash \sim (A \& \sim A) \)
1                 (1)   A\&\sim A"""" Assumption (!)
1                 (2)   \sim A"""" 1 &E
1                 (3)   A"""" 1 &E
1                 (4)   \Lambda"""" 2,3 \sim E
(5)   \sim (A\&\sim A)"""" 1,4 \sim I
```

- This proof makes use of *no premises*, and its final line has *no numbers to its left* — indicating that we have succeeded in proving ‘\( \sim (A \& \sim A) \)’ from *nothing at all*. It’s a *theorem* (i.e., a sequent with no premises)!
The Introduction Rule for $\lor$ ($\lor$I)

**Rule of $\lor$-Introduction:** For any formula $p$, if $p$ has been inferred at line $j$, then, for any formula $q$, either \[ p \lor q \] or \[ q \lor p \] may be inferred at line $k$, labeling the line \[ j \lor I \] and writing on its left the same premise and assumption numbers as appear on the left of $j$.

$$
\begin{array}{c}
\begin{array}{c}
\vdots \\
\end{array} & \begin{array}{c}
\text{OR} \\
\end{array} & \begin{array}{c}
\vdots \\
\end{array} \\
\begin{array}{c}
\vdots \\
\end{array} & \begin{array}{c}
\text{OR} \\
\end{array} & \begin{array}{c}
\vdots \\
\end{array} \\
\end{array}
\begin{array}{c}
a_1, \ldots, a_n \ (j) \quad p \\
\end{array} & \begin{array}{c}
a_1, \ldots, a_n \ (j) \quad q \\
\end{array} & \begin{array}{c}
a_1, \ldots, a_n \ (k) \quad p \lor q \quad j \lor I \\
\end{array} & \begin{array}{c}
a_1, \ldots, a_n \ (k) \quad p \lor q \quad j \lor I \\
\end{array}
$$

- The $\lor$I rule is very simple an intuitive. Basically, it says that you may infer a disjunction from *either* of its disjuncts.

- The *elimination* rule ($\lor$E) for $\lor$, on the other hand, is considerably more complex to state and apply. It’s the hardest of our rules.
The Elimination Rule for ∨ (∨E)

- First, the idea behind the ∨-elimination rule.
- The following argument form is valid (easily verified via truth-table):
  \[
  p \lor q \\
  p \rightarrow r \\
  q \rightarrow r \\
  \therefore r
  \]
- This argument form is called the constructive dilemma. In essence, the ∨E rule reflects the constructive dilemma form of reasoning and implements it in our system of natural deduction rules.
- The ∨E rule is trickier than our other rules because it requires us to make two assumptions. This can make it rather complicated to keep track of all of our assumptions and premises during an ∨E proof.
- Now, the official definition of ∨E …
Rule of $\lor$-Elimination: If a disjunction \( p \lor q \) occurs at line g of a proof, \( p \) is assumed at line h, \( r \) is derived at line i, \( q \) is assumed at line j, and \( r \) is derived at line k, then at line m we may infer \( r \), labeling the line ‘g, h, i, j, k \lor E’ and writing on its left every number on the left at line g, and at line i (except h), and at line k (except j).

\[
\begin{align*}
\text{a}_1, \ldots, \text{a}_n & \quad (g) \quad p \lor q \\
\vdots & \\
\text{h} & \quad (h) \quad p \quad \text{Assumption} \\
\vdots & \\
\text{b}_1, \ldots, \text{b}_u & \quad (i) \quad r \\
\vdots & \\
\text{j} & \quad (j) \quad q \quad \text{Assumption} \\
\vdots & \\
\text{c}_1, \ldots, \text{c}_w & \quad (k) \quad r \\
\vdots & \\
\mathcal{A} & \quad (m) \quad r \quad \text{g, h, i, j, k \lor E}
\end{align*}
\]

where \( \mathcal{A} \) is the set: \( \{\text{a}_1, \ldots, \text{a}_n\} \cup \{\text{b}_1, \ldots, \text{b}_u\}/h \cup \{\text{c}_1, \ldots, \text{c}_w\}/j \).
An Example Involving $\vee$E and DN

Here’s a proof of the sequent: $A \vee B, \neg B \vdash A$.

Problem is: $A \vee B, \neg B \vdash A$

1. (1) $A \vee B$ Premise
2. (2) $\neg B$ Premise
3. (3) $\neg A$ Assumption (for $\neg$I)
4. (4) $A$ Assumption (for $\vee$E)
3,4 (5) $\Lambda$ 3,4 $\neg$E
6. (6) $B$ Assumption (for $\vee$E)
2,6 (7) $\Lambda$ 2,6 $\neg$E
1,2,3 (8) $\Lambda$ 1,4,5,6,7 $\vee$E
1,2 (9) $\neg \neg A$ 3,8 $\neg$I
1,2 (10) $A$ 9 DN
A Simple Example Involving $\lor$I and $\lor$E

• Here’s a proof of the sequent: $A \lor B \vdash B \lor A$.

Problem is: $A \lor B \vdash B \lor A$

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<tbody>
<tr>
<td>1</td>
<td>(1) $A \lor B$</td>
<td>Premise</td>
</tr>
<tr>
<td>2</td>
<td>(2) $A$</td>
<td>Assumption ($\lor$E)</td>
</tr>
<tr>
<td>2</td>
<td>(3) $B \lor A$</td>
<td>$2 \ \lor$I</td>
</tr>
<tr>
<td>4</td>
<td>(4) $B$</td>
<td>Assumption ($\lor$E)</td>
</tr>
<tr>
<td>4</td>
<td>(5) $B \lor A$</td>
<td>$4 \ \lor$I</td>
</tr>
<tr>
<td>1</td>
<td>(6) $B \lor A$</td>
<td>$1,2,3,4,5 \ \lor$E</td>
</tr>
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</table>
A Tricky Example Involving $\lor I$ and Negation

- Here's a proof of the theorem: $\vdash A \lor \sim A$.

Problem is: $\vdash A \lor \sim A$

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<tbody>
<tr>
<td>1</td>
<td>(1)</td>
<td>$\sim(A \lor \sim A)$</td>
<td>Assumption ($\sim I$)</td>
</tr>
<tr>
<td>2</td>
<td>(2)</td>
<td>$A$</td>
<td>Assumption ($\sim I$)</td>
</tr>
<tr>
<td>2</td>
<td>(3)</td>
<td>$A \lor \sim A$</td>
<td>2 $\lor I$</td>
</tr>
<tr>
<td>1,2</td>
<td>(4)</td>
<td>$\Delta$</td>
<td>1,3 $\sim E$</td>
</tr>
<tr>
<td>1</td>
<td>(5)</td>
<td>$\sim A$</td>
<td>2,4 $\sim I$</td>
</tr>
<tr>
<td>1</td>
<td>(6)</td>
<td>$A \lor \sim A$</td>
<td>5 $\lor I$</td>
</tr>
<tr>
<td>1</td>
<td>(7)</td>
<td>$\Delta$</td>
<td>1,6 $\sim E$</td>
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<td></td>
<td>(8)</td>
<td>$\sim \sim(A \lor \sim A)$</td>
<td>1,7 $\sim I$</td>
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<td></td>
<td>(9)</td>
<td>$A \lor \sim A$</td>
<td>8 DN</td>
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### A Third Example Involving $\lor I$ and $\lor E$

- Here's a proof of the sequent: $A \lor (B \& C) \vdash (A \lor B) \& (A \lor C)$.

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<tbody>
<tr>
<td>1</td>
<td></td>
<td>(1) $A \lor (B &amp; C)$</td>
<td>Premise</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>(2) $A$</td>
<td>Assumption ($\lor E$)</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>(3) $A \lor B$</td>
<td>2 $\lor I$</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>(4) $A \lor C$</td>
<td>2 $\lor I$</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>(5) $(A \lor B) &amp; (A \lor C)$</td>
<td>3,4 $&amp; I$</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>(6) $B &amp; C$</td>
<td>Assumption ($\lor E$)</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>(7) $B$</td>
<td>6 $&amp; E$</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>(8) $A \lor B$</td>
<td>7 $\lor I$</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>(9) $C$</td>
<td>6 $&amp; E$</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>(10) $A \lor C$</td>
<td>9 $\lor I$</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>(11) $(A \lor B) &amp; (A \lor C)$</td>
<td>8,10 $&amp; I$</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>(12) $(A \lor B) &amp; (A \lor C)$</td>
<td>1,2,5,6,11 $\lor E$</td>
</tr>
</tbody>
</table>
Another Example Involving $\lor$

- Let’s do a proof of: $(A \land B) \lor (A \land C) \vdash A \land (B \lor C)$

Problem is: $(A\&B)\lor(A\&C) \vdash A$

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>(1) $(A&amp;B)\lor(A&amp;C)$</th>
<th>Premise</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>(2) A&amp;B</td>
<td>Assumption ($\lor$E)</td>
<td></td>
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<tr>
<td>2</td>
<td>(3) A</td>
<td>2 &amp;E</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>(4) A&amp;C</td>
<td>Assumption ($\lor$E)</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>(5) A</td>
<td>4 &amp;E</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(6) A</td>
<td>1,2,3,4,5 $\lor$E</td>
<td></td>
</tr>
</tbody>
</table>
A Final Example Involving $\lor$ and $\neg$

Let’s do a proof of: $\neg(A \lor B) \vdash A \rightarrow B$

Problem is: $\neg(A \lor B) \vdash A \rightarrow B$

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<tbody>
<tr>
<td>1</td>
<td>(1) $\neg A \lor B$</td>
<td></td>
<td>Premise</td>
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<td></td>
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<tr>
<td>2</td>
<td>(2) $A$</td>
<td></td>
<td>Assumption ($\rightarrow I$)</td>
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<tr>
<td>3</td>
<td>(3) $\neg A$</td>
<td></td>
<td>Assumption ($\lor E$)</td>
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<td></td>
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<td></td>
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<tr>
<td>4</td>
<td>(4) $\neg B$</td>
<td></td>
<td>Assumption ($\neg I$)</td>
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<tr>
<td>2,3</td>
<td>(5) $\Lambda$</td>
<td>3,2 $\neg E$</td>
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<tr>
<td>2,3</td>
<td>(6) $\neg \neg B$</td>
<td>4,5 $\neg I$</td>
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<td>2,3</td>
<td>(7) $B$</td>
<td>6 $\text{DN}$</td>
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<td>8</td>
<td>(8) $B$</td>
<td></td>
<td>Assumption ($\lor E$)</td>
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<td>1,2</td>
<td>(9) $B$</td>
<td>1,3,7,8,8 $\lor E$</td>
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<td>1</td>
<td>(10) $A \rightarrow B$</td>
<td>2,9 $\rightarrow I$</td>
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General Tips on Proof Strategy and Planning

- As a first line of attack, always try to prove your conclusion by using the introduction rule for its main connective as the main strategy.
- This will indicate what assumptions, if any, need to be made and what other formulae will need to be derived. This is “working backward”.
- If these other formulae also contain connectives, then try to prove them by introducing their main connectives. Work backward, as far as possible.
- When this technique can no longer be applied, inspect your current stock of premises and assumptions to see if they have any obvious consequences.
- If your current premises and assumption contain a disjunction \( r \lor s \), see if you can prove your current goal formula \( p \) from each of its disjuncts \( r \) and \( s \) (using your current premises and assumptions). If you think you can, then try using \( \lor E \) to prove \( p \). If no disjunction appears anywhere in your current of premises/assumptions, then \( \lor E \) is probably not a good strategy.
- If you have tried everything you can think of to prove your current goal \( p \), try assuming \( \sim p \) and aim for \( \sim \sim p \) by \( \sim E, \sim I \); then use DN.
When to Make Assumptions, and When Not to

• In constructing a proof, any assumptions you make must eventually be discharged, so you should only make assumptions in connection with the three rules which discharge assumptions.

• In other words, if you make an assumption \( p \) in a proof, you must be able to give one of the following three reasons:

  1. \( p \) is the antecedent of a conditional \( p \rightarrow q \) you are trying to derive using the \( \rightarrow \text{I} \) rule (then, try to prove \( q \)).

  2. You are trying to derive \( \sim p \), so you assume \( p \) with an eye toward using the \( \sim \text{I} \) rule (then, try to prove \( \wedge \)).

  3. \( p \) is one of the disjuncts of a disjunction \( p \vee q \) (somewhere in your current stock of premises and assumptions!) to which you will be applying \( \vee \text{E} \) (then, try to prove some \( r \) from each).

• Remember, only the three rules \( \rightarrow \text{I} \), \( \sim \text{I} \), and \( \vee \text{E} \) involve making assumptions. No other rules can discharge assumptions.