On Popper's Definitions of Verisimilitude
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Discussions

ON POPPER'S DEFINITIONS OF VERISIMILITUDE

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Introduction.

Sir Karl Popper's epistemological position is best characterised as an optimistic scepticism. It is a scepticism since it affirms that no non-trivial theory can be justified and that more likely than not all the theories we entertain and use are false. The position is optimistic in contending that in science we nevertheless make progress: that we have a way of improving on our false theories. Progress, however, hardly ever consists in supplanting a false theory by a true one. As a rule, the new theory is also false but somehow less so than its antecedent. Popper's epistemology thus calls for a discriminating approach to false theories: he has to assume that of two false theories, one can be preferable to the other in being 'closer to the truth' or 'more like the truth'.

In an attempt to legitimise this sort of talk Popper has proposed two rigorous definitions of verisimilitude, I shall call them logical and probabilistic. The aim of this note is to show that for simple logical reasons, both are totally inadequate.

In Section 1 are given Popper's definitions of several auxiliary notions. The logical definition of verisimilitude is considered in Section 2. It is demonstrated that on this definition a false theory can never enjoy more verisimilitude than another false theory. The probabilistic definition is dealt with in Section 3. An example of two theories $A$ and $B$ is given such that $A$ is patently closer to the truth than $B$, yet on Popper's definition $A$ has strictly less verisimilitude than $B$.

1 Preliminaries.

Consider a language having (as is usual) a finite number of primitive descriptive constants. Any finite set of (closed) sentences of the language will be called a theory. In what follows, $A$, $B$, $C$, ... are understood to be arbitrary theories. $Cn(A)$ is the set of theorems of $A$, i.e., the set of logical consequences of $A$. Furthermore, let $T$ and $F$ be the set of true and false sentences of the language respectively. Popper has proposed the following definitions.2

1 An earlier version of this paper was presented to the Philosophy Seminar of the University of Otago in March 1973. The author benefited from conversations with Sir Karl Popper, Alan Musgrave, and John Harris, and adopted a terminological suggestion made by David Miller.

2 The latest formulations of these definitions can be found in Professor Popper's [1972]. In what follows, all page references are to this book.
Definition 1.1. The truth content $A_T$ of $A$ is $\text{Cn}(A) \cap T$.

Definition 1.2. The relative content $A, B$ of $A$ given $B$ is $\text{Cn}(A \cup B) - \text{Cn}(B)$.

Definition 1.3. The falsity content $A_F$ of $A$ is the relative content of $A$ given $A_T$, i.e., $A, A_T$.

Definitions 1.1, 1.2, and 1.3 yield

**Proposition 1.4.** $A_F = \text{Cn}(A) \cap F$.

**Proof.** $A_F = A$, $A_T = \text{Cn}(A \cup A_T) - \text{Cn}(A_T)$ (by 1.3 and 1.2)

$$= \text{Cn}(A) - A_T$$

(since $A \cup A_T = A$ and $\text{Cn}(A_T) = A_T$)

$$= \text{Cn}(A) - (\text{Cn}(A) \cap T)$$

(by 1.1)

$$= \text{Cn}(A) \cap F$$

(since $\overline{T} = F$).

2 Popper's Logical Definition of Verisimilitude.

Popper never explicitly states but obviously presupposes:

**Definition 2.1.** $A_T$ and $B_T$ (or $A_F$ and $B_F$) are comparable just in case one of them is a (proper or improper) subclass of the other.

Now we can state Popper’s logical definition of verisimilitude:

**Definition 2.2.** $A$ has less verisimilitude than $B$ just in case (a) $A_T$ and $A_F$ are respectively comparable with $B_T$ and $B_F$, and (b) either $A_T \subseteq B_T$ and $A_F \not\subseteq B_F$ or $B_T \not\subseteq A_T$ and $B_F \subseteq A_F$.

Definitions 2.1 and 2.2 yield immediately

**Proposition 2.3.** $A$ has less verisimilitude than $B$ just in case either $A_T \subseteq B_T$ and $B_F \subseteq A_F$ or $A_T \not\subseteq B_T$ and $B_F \subset A_F$.

**Definition 2.2.** is inadequate as explication of verisimilitude in view of

**Proposition 2.4.** If $B$ is false then $A$ does not have less verisimilitude than $B$.

**Proof.** Since $B$ is false, there is a false sentence, say $f$, in $\text{Cn}(B)$. First assume

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1 This is how the concept of truth content is defined on p. 330. On p. 48 we are given a slightly different definition, whereby the truth content of $A$ is rather $(\text{Cn}(A) \cap T) - L$, where $L$ is the set of tautologies or logically valid sentences. But I take this to be a mere slip, since some statements on the same page are in conflict with this definition. At all events, the difference is marginal and does not affect our ensuing considerations.

2 This is how the concept of relative content is defined on p. 332. On p. 49 the relative content of $A$ given $B$ is characterised as the class of all sentences deducible from $A$ with the help of $B$. This might be construed as suggesting that the relative content of $A$ given $B$ is simply $\text{Cn}(A \cup B)$. However, from several subsequent remarks it transpires that this is not what is intended.

3 See pp. 49, 51 and 332.

4 The proposition shows that the definition of the falsity content of $A$ (as the class of false consequences of $A$), which is considered and rejected on p. 48 is in effect logically equivalent to the definition actually proposed at the bottom of p. 49.

5 See p. 52. The signs $\subseteq$ and $\subseteq$ stand for set inclusion and proper set inclusion respectively.
On Popper's Definitions of Verisimilitude

$A_T \subseteq B_T$. Then there is a sentence, say $b$, in $B_T - A_T$. But then $(f . b) \in B_F$. On the other hand, $(f . b) \notin A_P$, since otherwise, by i.4 and i.x, $b \in A_T$, in contradiction to the choice of $b$. Thus $B_F \nsubseteq A_P$. Now assume $B_F \subseteq A_P$. Then there is a sentence, say $a$, in $A_P - B_F$. But then $(f \supset a) \in A_T$. On the other hand, $(f \supset a) \notin A_P$, since otherwise, by i.x and i.4, $a \in A_T$, in contradiction to the choice of $a$. Thus $A_T \nsubseteq B_F$. The Proposition now follows by 2.3.

To illustrate Proposition 2.4, let $A$ consist of the sole sentence 'It is now between 9.40 and 9.48' and let $B$ consist of the sole sentence 'It is now between 9.45 and 9.48', where 'between' is understood to exclude the two bounds. Suppose that the actual time is 9.48. Then $B$ is false. Moreover, $A_T \subseteq B_T$. Yet $A$ does not have less verisimilitude than $B$ on Definition 2.2. For clearly the (only) member of $B$ is in $B_F$ but not in $A_P$, thus $B_F \nsubseteq A_P$.

3. Popper's Probabilistic Definition of Verisimilitude.

Where $A$ and $B$ are theories, let $p(A)$ be the logical probability of $A$ and $p(A, B)$ the relative logical probability of $A$ given $B$. Popper has proposed the following definitions.

\textit{Definition 3.1.} The measure $ct_T(A)$ of the truth content of $A$ is $1 - p(A_T)$.

\textit{Definition 3.2.} The measure $ct_F(A)$ of the falsity content of $A$ is $1 - p(A, A_T)$.

Popper's probabilistic explication of truthlikeness is then in terms of $ct_T$ and $ct_F$. Popper offers, in fact, two alternative explications. They will be spoken of as $\text{verisimilitude}_1$ and $\text{verisimilitude}_2$. The definitions are as follows.

\textit{Definition 3.3.} The $\text{verisimilitude}_1$ $vs_1(A)$ of $A$ is $ct_T(A) - ct_F(A)$.

\textit{Definition 3.4.} The $\text{verisimilitude}_2$ $vs_2(A)$ of $A$ is

$$(ct_T(A) - ct_F(A))(2 - ct_T(A) - ct_F(A)).$$

Both concepts are drastically at variance with the intuitive notion of closeness to the truth. Preparatory to a justification of this claim I shall introduce several notational conventions and prove an auxiliary proposition.

Let $a, b, \ldots, t, \ldots$ be arbitrary sentences of the language in question. In what follows, a symbol standing for a sentence will also be used to denote the set whose only element is that sentence.

\textit{Proposition 3.5.} If $T = Cn(t)$ then $a_T = Cn(a \lor t)$.

\textit{Proof.} Assume $T = Cn(t)$ and consider an arbitrary sentence $b$. By i.I,
Proposition 3.10. If $a$ is false then $ct_r(a) = (7/8) - p(a)$ and

$$ct_r(a) = 1 - \frac{p(a)}{p(a) + 1/8}. $$

Proof. Let $a$ be false. We have:

$$ct_r(a) = 1 - p(a_T) \quad \text{(by 3.1)}$$

$$= 1 - p(a \lor t) \quad \text{(by 3.5)}$$

$$= 1 - [p(a) + p(t)] \quad \text{(by 3.7 and 3.8)},$$

$$ct_r(a) = 1 - p(a, a_T) \quad \text{(by 3.2)}$$

$$= 1 - p(a, a \lor t) \quad \text{(by 3.5)}$$

$$= 1 - [p(a) + p(a \lor t)] \quad \text{(by 3.9)}$$

$$= 1 - [p(a)/(p(a) + 1/8)] \quad \text{(by 3.7, 3.8, and propositional logic).}$$

But $p(t) = 1/8$. Which completes the proof.

From 3.10, 3.1, and 3.2 it immediately follows that the values of $v_{s_1}$ and $v_{s_2}$ at false sentences of $L$ depend solely on the logical probabilities of the sentences. A little reflection reveals that this fact alone makes $v_{s_1}$ and $v_{s_2}$ unfit to explicate the intuitive notion of proximity to the truth. Since surely we want it to be possible for one false theory to be closer to the truth than another false theory despite the two theories having the same logical probability. If Popper’s proposals were right then in order to decide which one of two false theories is closer to the truth, no factual knowledge would be required over and above the knowledge that the two theories are indeed false. Which is clearly absurd.

To illustrate this point, let us consider a couple of examples.

The following table gives the values of $ct_r, ct_f, v_{s_1},$ and $v_{s_2}$ at some false sentences of $L$:
Now imagine that Jones and Smith, two prisoners sharing a windowless and air-conditioned cell, are using $L$ to discuss the weather. Jones takes the view that it is a dry, still day, with a low temperature. In other words, Jones’s conjecture is $\sim p . \sim q . \sim r$. Smith disagrees. Although he also thinks that the temperature is low, he (rightly) insists that it is raining and windy. In other words, Smith’s theory is $p . q . \sim r$.

It seems hardly deniable that Smith is by far nearer to the truth than Jones. He is admittedly wrong on temperature, but he is right in the rain and wind. Jones, on the other hand, is wrong on three counts. He could not, in fact, be farther from the truth than he is (without contradicting himself).

Thus one would expect Smith’s theory to exceed Jones’s in measure of truth content and in verisimilitude. One would also expect Jones’s theory to exceed Smith’s in measure of falsity content. Yet, as seen in the above table, each of the functions $ct_T$, $ct_P$, $vs_1$, and $vs_2$ takes the same value at Jones’s theory as it does at Smith’s.

But Popper’s functions $vs_1$ and $vs_2$ not only fail to discriminate between theories which, like the two above, are vastly unlike in proximity to the truth. In many cases the functions accord strictly greater verisimilitude to a theory which is patentely farther from the truth than another theory.

Let us alter slightly the above example. Imagine that while Smith sticks to his theory $p . q . \sim r$, Jones has weakened his claim to $\sim p . \sim q$. Jones’s theory is now marginally better than before: while previously Jones was positively wrong on temperature, this time he withholds judgement. But Jones’s new theory is surely not better enough to match, let alone exceed, Smith’s in closeness to the truth. Jones’s is still one of the lousiest and Smith’s one of the best false theories, as false theories go. Smith is only wrong on one count, whereas Jones is wrong on two. One would certainly expect Jones’s theory to exceed Smith’s in falsity content. Yet, the $ct_P$ of Jones’s theory is strictly less than the $ct_P$ of Smith’s. One would also expect Smith’s theory to have greater verisimilitude than Jones’s. Yet, the $vs_1$ of Smith’s theory is strictly less than the $vs_1$ of Jones’s and similarly for $vs_2$.

4 Conclusion.

To do justice to the intuitive notion of truthlikeness one must clearly make it possible for a false theory to be closer to the truth than another false theory of the same logical probability. For a simple language which, like $L$, is based on propositional logic only, this is easily done. The ‘distance’ between two constituents can be naturally defined as the number of primitive sentences negated in one of the constituents but not in the other. The verisimilitude of an arbitrary sentence $a$ can then be defined as the arithmetical mean of the distances between the true constituent $t$ and the constituents appearing in the disjunctive normal form of $a$. It is easily seen that such a definition meets all intuitive requirements.
Things, of course, get vastly more complicated when we turn to the more
typical kind of theories, i.e., theories formulated in a first-order language. But
the idea underlying the above definition of ‘distance’ can be carried over to first-
order theories if one employs, in lieu of disjunctive normal forms, Hintikka’s
distributive normal forms for first-order formulas. This, however, is a topic for a
separate article.

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REFERENCE

POPPER’S DEFINITIONS OF ‘VERISIMILITUDE’

1 One of the major problems Popper has attacked is that of finding and making
intelligible a coherent view of critical common-sense realism which agrees with
the practices of science. He sees science as progressing and finding ever better
theories. And since for him the only significant progress would be that of getting
closer to the (absolute) truth, he is obviously led to the minor problem of explain-
ing what it would mean (at least in principle) to say that one theory is closer to
the truth than another, especially in the case when both theories are false. His
attempts at explicating this particular concept usually appear in his writings under
the heading of ‘verisimilitude’.

For any two theories $A$ and $B$ let us write ‘$A \prec T B$’ if intuitively $B$ is closer to
the truth than $A$. (This is not to imply that this intuitive concept has a unique
sense and won’t someday be found to be ambiguous. But whether this is the case
is part of the problem.) Popper has given essentially two different formal definitions
of ‘$\prec_T$’, which Miller calls the qualitative and the quantitative definitions.
I will denote the first by ‘$\prec T$’. Miller and Tichy have independently shown that
neither of the formal definitions is faithful to Popper’s intuitive notion of
verisimilitude. In particular they prove that if $A$ and $B$ are false theories, then on
Popper’s qualitative definition neither can be closer to the truth than the other:

(i) $A \ll T B$ and $B \ll T A$.

The main purpose of this article is (i) to consider a mathematically more
general problem, that of comparing theories relative to an arbitrary comparison
theory, (ii) to show the philosophical relevance of this generalisation and (iii) to
shed light on the Miller-Tichy result (i) by obtaining some general results of
which (i) is a special case.

2 When we say that science finds ever better theories or that one theory is
better than another, implicit in such a statement is the assumption that there is
some criterion of comparison such as elegance, ease of calculation, degree of
falsifiability, agreement with a portion of currently accepted background know-

1 I am deeply indebted to Pavel Tichý for bringing to my attention the problem of verisi-
imilitude and his negative results. I am also indebted to P. Tichý, D. Miller and A.
Musgrave for criticism of earlier drafts of this article.

2 Miller [1974].

3 Ibid.

4 Tichý [1974].