VISIBILITY OF EARTH-BOUND SATELLITES:

A DEEP SPACE NETWORK STUDY

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ABSTRACT

The investigation of the visibility of Earth-bound satellites using three Deep Space Network (DSN) stations is performed in four steps, which progress from a very simplistic two-dimensional model to a general three-dimensional paradigm. These steps are as follows:

1. Two-dimensional case with line of sight (LOS) being simply the local horizontal, the satellite confined to the equatorial plane, and the three stations spaced equiangularly around the equator.

2. Two-dimensional case with an elevation angle, θ_e, constraint added to the LOS. The stations and the satellite are still equatorial in this case; however, the LOS is elevated θ_e above the local horizontal at all three stations. This constraint arises due to the degradation of electromagnetic radiation caused by atmospheric effects.

3. Three-dimensional model with DSN stations at Canberra, Goldstone, and Madrid used as observation points. The satellite is confined to the equatorial plane. A spherical coordinate system is used with the center of Earth as the origin, the North Pole as the z-axis, and the Greenwich Meridian as the x-axis. An analytical solution is not found in this case. Instead, computer-aided vector analysis is used to calculate the zenith angle at the three DSN stations for small increments of the orbital path (using a θ_e = 10-degree constraint on the LOS). Thus, numerical approximations for the visibility ratios of orbits with various values of R_o can be found.

4. Because equatorial orbits are not of particular interest here, a more general three-dimensional model is necessary to calculate visibilities of orbits inclined with respect to the equatorial plane. For this reason, linear transformations are performed to rotate the orbital vector about the x-axis (inclination) and about the z-axis (right ascension of ascending nodes). The inclination used is 28.6 degrees because the orbits of interest are those that lie in the plane of the Moon's orbit. However, DSN.FOR will calculate the visibility of any Earth-bound satellite with a given altitude (R_o), inclination angle (γ), and right ascension (μ). (Visibility data for a satellite in the Moon's orbital plane at various values of R_o and μ are graphically depicted.)

VISIBILITY OF EARTH-BOUND SATELLITES: A DEEP SPACE NETWORK STUDY

1.0 TWO-DIMENSIONAL VISIBILITY MODEL

A simple two-dimensional mathematical model can be used to simulate the visibility of an Earth-bound satellite from three ground stations. The geometry of the two-dimensional model shown in Figure 1 allows the derivation of the ratio of a given orbit that can be seen by the three ground-based stations for any given R_o. The circle represents a cross-section of Earth at the equator; the points on the circle represent three ground-based "stations." For simplicity, these points have been placed on the equator and spaced at equiangular intervals around the equator. The dotted circle represents an equatorial satellite (of altitude R_o). The derivation is as follows (see Figure 1 for explanation of variables):

$$2\theta = 2\cos^{-1}\left(\frac{R_e}{R_e + R_o}\right) \tag{1}$$

Due to symmetry, the total angle intercepted by all three stations can be written as three times this angle, specifically:

$$(3)(2)\theta = (3)(2)\cos^{-1}\left(\frac{R_e}{R_e + R_o}\right) = 6\cos^{-1}\left(\frac{R_e}{R_e + R_o}\right) \tag{2}$$

This angle, when compared to the total angle (2π radians) gives the visibility ratio of that orbit from the three ground-based stations.

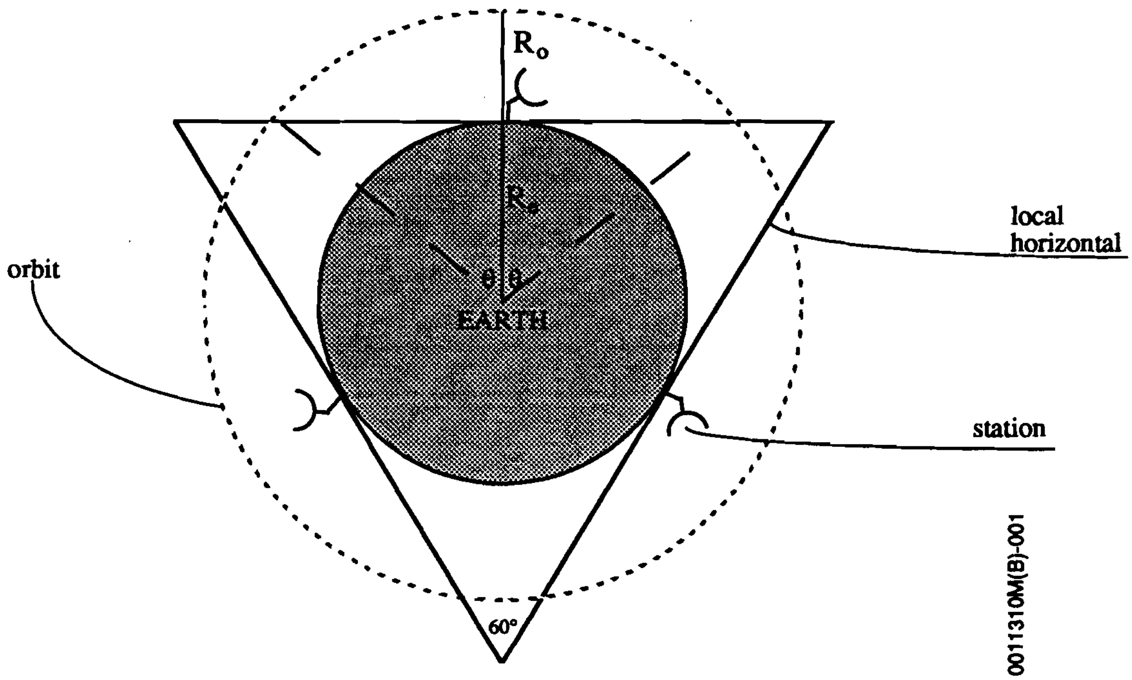
$$\text{Ratio} = \frac{6\theta}{2\pi} = \frac{6}{2\pi}\cos^{-1}\left(\frac{R_e}{R_e + R_o}\right) \tag{3}$$

Thus, the visibility can be written as follows:

$$\text{Ratio} = \frac{3}{\pi}\cos^{-1}\left(\frac{R_e}{R_e + R_o}\right) \tag{4}$$

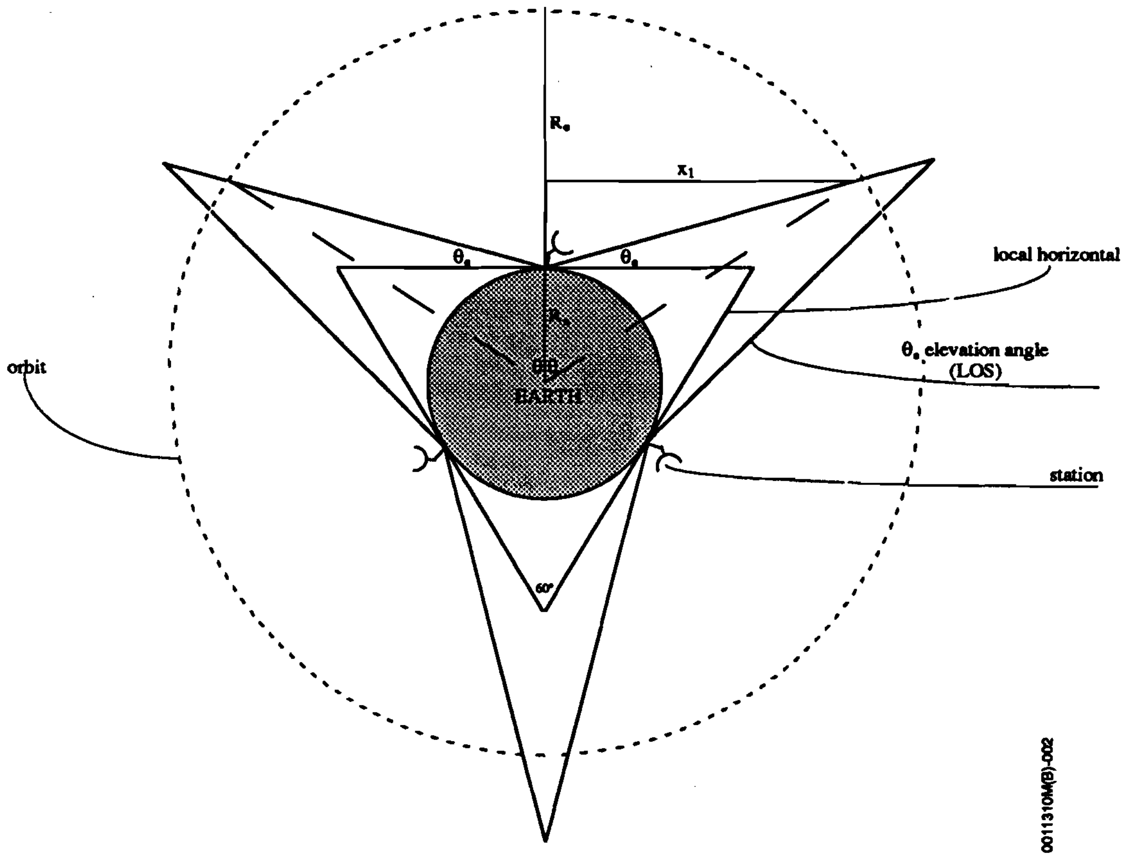
This visibility is an ideal case in which all stations lie in the plane of the orbit (i.e., the equatorial plane) and in which the observers on the surface have the ability to see straight along the horizon. (Lines of visibility are assumed to be tangent to the surface at the point of the station.)

The next step in the two-dimensional paradigm is to eliminate the last assumption and invoke an elevation angle constraint, θ_e, on the line of sight (LOS) for efficient visibility of extraterrestrial objects. The LOS elevation angle constraint arises because of the absorption of electromagnetic radiation by the atmosphere (Figure 2). Now the visibility of a given orbit is less than the ideal two-dimensional case because of the minimum elevation angle on the LOS. The derivation becomes more involved and the geometry more complex. Appendix A contains a derivation of the visibility of an equatorial satellite from three equiangularly spaced equatorial stations with a



R_e = Mean radius of the Earth (6378 km)
R_o = Altitude of orbit (variable) (km)
$(R_e + R_o)$ = Radius of orbit (dashed lines) (km)
2θ = Angle intercepted by local horizontal {out of total orbit}

Figure 1. Simple Two-Dimensional Model



R_e = Mean radius of the Earth (6378 km)
R_o = Altitude of orbit (variable)
(R_e+R_o) = Radius of orbit (dashed lines) (km)
2θ = Angle intercepted by (θ_e = minimum LOS elevation angle) LOS (out of total orbit)
x_1 is derived in Appendix A

Figure 2. Two-Dimensional Model with θ_e Elevation Angle

10-degree constraint on the LOSs of the observers; the final expression is given in Equation 5. Appendix A also contains a graphical comparison of the two two-dimensional models discussed previously.

Ratio' = (5)

$$\frac{3}{\pi}\sin^{-1}\left(\frac{-2\tan(\theta_e)R_e + \sqrt{\left(2\tan(\theta_e)R_e\right)^2 - 4\left(\tan^2(\theta_e)+1\right)\left(-R_o^2 - 2R_eR_o\right)}}{2\left(R_e + R_o\right)\left(\tan^2(\theta_e)+1\right)}\right)$$

Note that the visibility ratios can exceed unity (i.e., can be greater than 100-percent visibility) due to overlap in the visibilities of the observers. A ratio (or ratio') of greater than 1 simply means that more than one of the three observers can see the satellite at the same time.

2.0 THREE-DIMENSIONAL INVESTIGATION: THE DEEP SPACE NETWORK PROBLEM

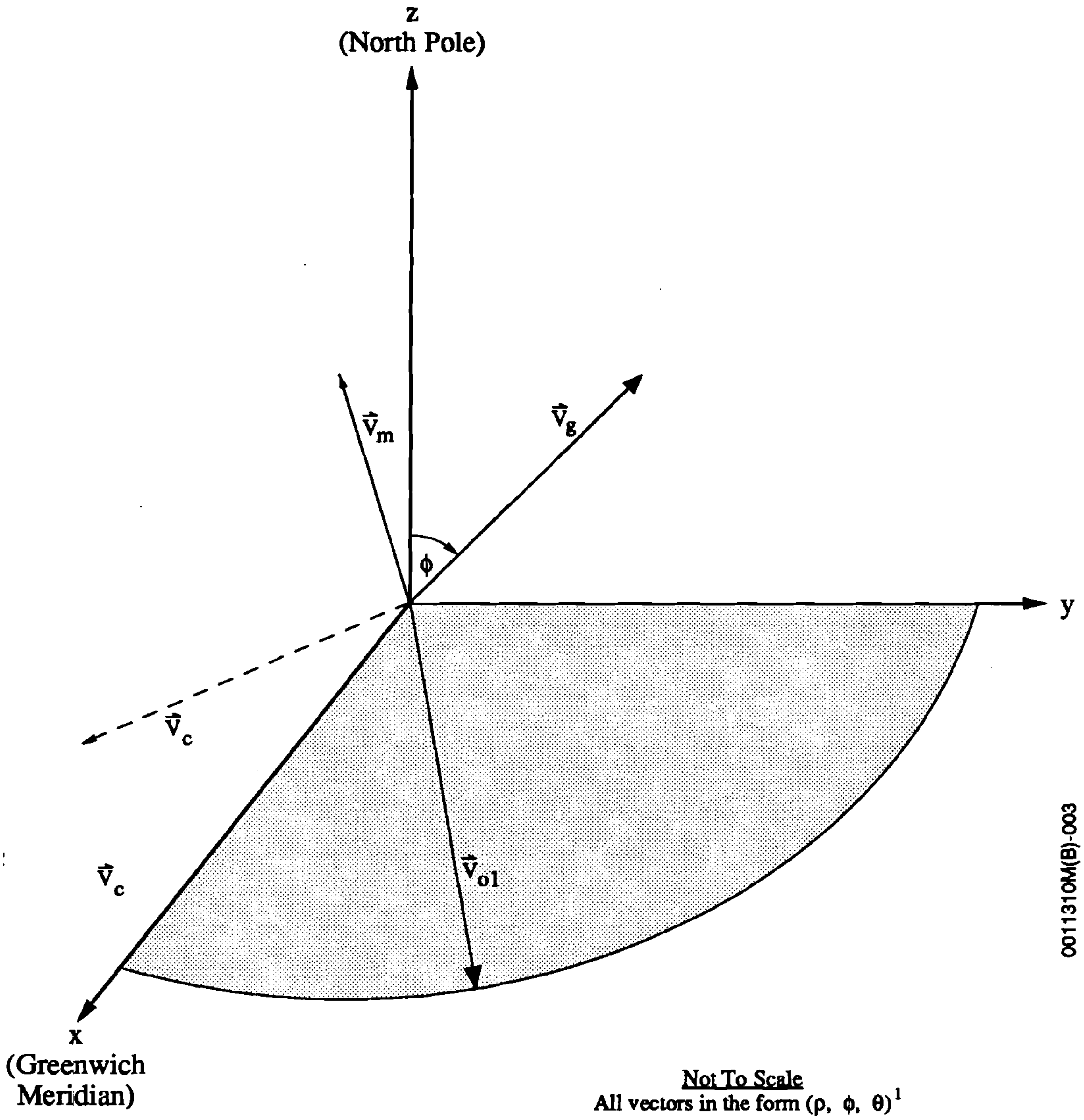
On completion of the two-dimensional analysis, the next step is to take the analysis into three dimensions. At this juncture, one must define a coordinate system in which it is possible to describe (in three dimensions) vectors that will represent the position of the spacecraft and the stations with reference to some fixed point in three-dimensional space.

The spherical coordinate system chosen is illustrated in Figure 3. The center of Earth is the origin of this coordinate system. The z-axis represents the North Pole, and the x-axis represents the Greenwich Meridian, or the line of 0 degrees longitude. The angle θ represents the angle between any vector and the positive x-axis (i.e., the longitude of any point on Earth). The angle ϕ represents the angle between any vector and the positive z-axis (i.e., the difference between 90 degrees and the latitude of any point on Earth). For example, a point that lies at 30 degrees east longitude and 50 degrees north latitude can be represented by a vector whose length is the radius of Earth, whose θ angle is 30 degrees, and whose ϕ angle is (90-50) or 40 degrees. Thus, any point in space can be described by a vector whose coordinates are r, θ, and ϕ. The relationship between (r, θ, ϕ) and (x, y, z) is as follows:

$$x=r\sin(\phi)\cos(\theta), y=r\sin(\phi)\sin(\theta), z=r\cos(\phi) \quad (6)$$

With these relationships defined, one can represent any point in space as a vector, $\vec{V}$, whose components are $\vec{x}$, $\vec{y}$, and $\vec{z}$.

Using this coordinate system, vectors describing the position of the Goldstone, Canberra, and Madrid Deep Space Network (DSN) stations are defined as $\vec{V}_g$, $\vec{V}_c$, and $\vec{V}_m$, respectively. (Figure 3 gives the definition of these vectors, as well as $\vec{V}_{o1}$, which is the vector describing the position of an equatorial satellite.)



Not To Scale

All vectors in the form $(\rho,\ \phi,\ \theta)^1$

$\vec{V}_{o1}$ = Equatorial orbit (satellite) vector = $((R_e + R_o), 90°, \theta°)$

$\vec{V}_g$ = Goldstone (DSN) vector = $(R_e, 54.7°, 243.1°)$

$\vec{V}_c$ = Canberra (DSN) vector = $(R_e, 125.4°, 149°)^1$

$\vec{V}_m$ = Madrid (DSN) vector = $(R_e, 49.5°, 355.8°)^1$

θ = Longitude angle

ϕ = Complement of latitude angle

R_e = Mean radius of Earth (6,378 km)

R_o = Altitude of orbit (km)

Figure 3. Three-Dimensional Coordinate System

It is difficult to attempt an analytic solution of the three-dimensional visibility problem. Instead, an iterative vector analysis technique can be used to arrive at numerical approximations of the visibility of such a satellite in a three-dimensional space model. This technique involves iterating the angle θ for the equatorial satellite and performing simple vector analysis to derive the values of the zenith angles [Z(c,g,m) in Figure 4] for each DSN station.

This analysis is done for every value of θ around the path of the satellite. If any one of the zenith angles is less than a critical value (which depends on the elevation angle constraint), the satellite is said to be "visible" to at least one DSN station. Counting the number of times the satellite is visible during one full orbit (i.e., 2π radians swept out by $\vec{V}_{o_1}$) and then comparing that number to the total number of iterations produces a resulting ratio that is a numerical approximation for the visibility of the satellite for that particular value of R_o. This procedure is described in the following paragraphs and is coded in the FORTRAN program called DSN.FOR (see Appendix B). Figure 4 shows the vectors and angles referred to below, except for $\vec{V}_2$ (c,g,m), which are defined in Equations 7, 8, and 9.

The zenith angle at Canberra is given by

$$Z_c = \cos^{-1}\left(\frac{\vec{V}_c \cdot \vec{V}_{2c}}{\left\|\vec{V}_c\right\| \left\|\vec{V}_{2c}\right\|}\right) \text{where } \vec{V}_{2c} = \vec{V}_o - \vec{V}_c \text{ (see Figure 4)} \tag{7}$$

The zenith angle at Goldstone is given by

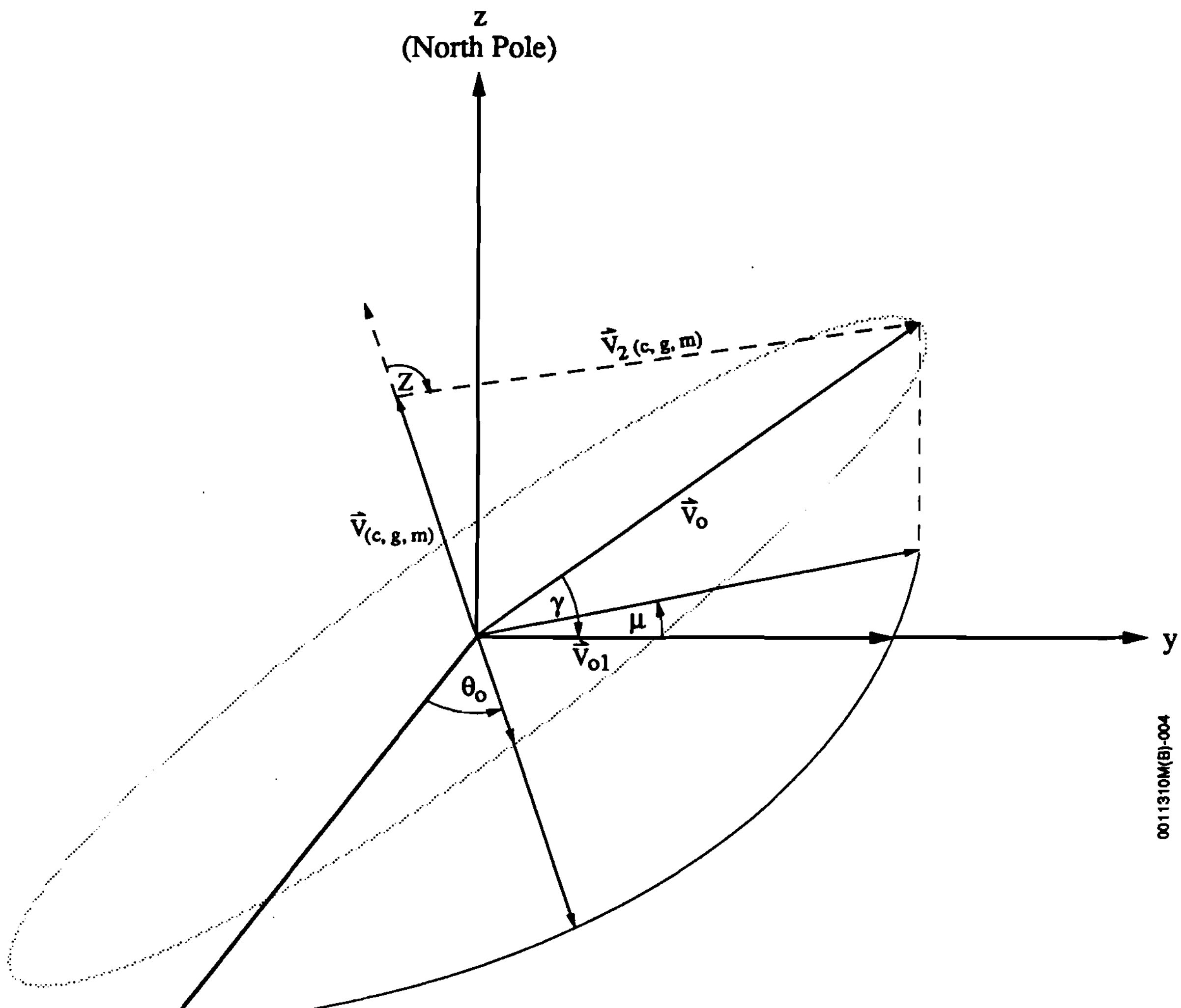
$$Z_g = \cos^{-1}\left(\frac{\vec{V}_g \cdot \vec{V}_{2g}}{\left\|\vec{V}_g\right\| \left\|\vec{V}_{2g}\right\|}\right) \text{where } \vec{V}_{2g} = \vec{V}_o - \vec{V}_g \tag{8}$$

The zenith angle at Madrid is given by

$$Z_m = \cos^{-1}\left(\frac{\vec{V}_m \cdot \vec{V}_{2m}}{\left\|\vec{V}_m\right\| \left\|\vec{V}_{2m}\right\|}\right) \text{where } \vec{V}_{2m} = \vec{V}_o - \vec{V}_m \tag{9}$$

If any one of these angles is less than 80 degrees (90-minus-10-degree elevation angle) then the satellite is "seen."

The orbits of interest here are not the equatorial orbits, but rather the orbits that lie in the Moon's orbital plane. Linear transformations are performed on the satellite vector to rotate the orbit to its proper inclination (28.6 degrees above the equatorial plane). This procedure is explained in the following paragraphs and is also coded in DSN.FOR.



Not To Scale

$\vec{V}_o$ = Transformed orbit (Equations 10 through 13)

$\vec{V}_{o1}$ = Equatorial orbit (see Figure 3 for coordinates)

γ = Angle of inclination with respect to equatorial plane (28.6° ⇛Moon's orbital plane)

μ = Right Ascension of ascending node

– – – = Transformed orbit

——— = Equatorial orbit

$\vec{V}_2$ (c, g, m) = Vector (LOS) between DSNs and Satellite

$\vec{V}$(c, g, m) = DSN Vector (Canberra, Goldstone, or Madrid)

Z = Zenith angle at DSN Station (equations, 7–9)

Figure 4. Orbit Rotation

The vector V_{o1} is simply the equatorial satellite vector. To transform this three-dimensional space vector into a vector that makes an angle γ with the equatorial plane (28.6 degrees in this study), multiply the vector by a matrix of transformation that will rotate any vector $\vec{V}_o$ about the x-axis by an angle γ. In addition to this rotation, a rotation about the z-axis is also needed to rotate the orbit around the North Pole. This angle is called the right ascension of the ascending node of the orbit and is labeled μ. Therefore, two matrix multiplications must be performed. The total transformation can be written as

$$[V_o] = [R_Z][R_X][V_{o1}] \tag{10}$$

Where $[R_Z]$,$[R_X]$ are the following 3x3 matrices, ($[R_Z]$ rotates V_{o1} by an angle μ about the z-axis; $[R_X]$ rotates V_{o1} by an angle γ about the x-axis):

$$[R_Z] = \begin{bmatrix} \cos(\mu) & -\sin(\mu) & 0 \\ \sin(\mu) & \cos(\mu) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{11}$$

$$R_y = \begin{bmatrix} \cos(\alpha) & 0 & -\sin(\alpha) \\ 0 & 1 & 0 \\ \sin(\alpha) & 0 & \cos(\alpha) \end{bmatrix}$$

and

$$[R_X] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\gamma) & -\sin(\gamma) \\ 0 & \sin(\gamma) & \cos(\gamma) \end{bmatrix} \tag{12}$$

The transformed vector $[V_O]$ is given by

$$\left[\vec{V}_o\right] = [R_z]\,[R_x]\left[\vec{V}_{o1}\right] = \begin{bmatrix} (V_{o1_x})\cos(\mu) - (V_{o1_y})\sin(\mu)\cos(\gamma) \\ (V_{o1_x})\sin(\mu) + V_{o1_y}\cos(\mu)\cos(\gamma) \\ (V_{o1_y})\sin(\gamma) \end{bmatrix} \tag{13}$$

V_O is the transformed vector, which is used to calculate the zenith angles (see Equations 7 through 9 and program DSN.FOR in Appendix B). Note that if γ is set to zero, then the orbit reduces to an equatorial orbit; if the stations are positioned equiangularly around the equator, then the resulting data generated by DSN.FOR approaches the data for the analytically solved two-dimensional model. $\vec{V}_{o1_x}$, $\vec{V}_{o1y}$, and $\vec{V}_{o1_z}$ are the x, y, z components of the equatorial orbit satellite ($\vec{V}_{o1_z} = 0$) (see Figure 4).

3.0 DATA AND CONCLUSIONS

Table 1 shows various visibilities for various values of R_O and μ. The value of γ (inclination angle) is fixed at 28.6 degrees, the Moon's orbital plane. Values of R_O range from 0 to 50,000 kilometers. Geosynchronous orbit is at an altitude of about 35,800 kilometers. In the Moon's orbital plane, once geosynchronous orbit is reached, visibilities are well above 80 percent, indicating favorable visibility conditions. Visibility steadily increases as altitude increases, approaching 100 percent at a distance of nearly 300 megameters.

Table 1. DSN Visibility Data

R_o(km)	μ=0 deg	μ=60 deg	μ=120 deg	μ=180 deg	μ=240 deg	μ=300 deg
	Visibility (%)	Visibility (%)	Visibility (%)	Visibility (%)	Visibility (%)	Visibility (%)
0	0	0	0	0	0	0
2500	0	6.68	18.46	34.92	50.91	36.04
5000	23.87	19.33	25.22	66.43	72.87	49.88
7500	40.57	24.58	44.95	77.88	81.7	67.38
10000	48.85	27.84	52.74	83.37	86.95	80.11
12500	61.26	30.07	57.76	85.52	88.86	86.56
15000	66.91	33.25	64.52	87.11	90.14	90.06
17500	70.72	51.55	74.54	88.39	91.17	91.73
20000	73.67	58.95	79.63	89.34	92.04	92.6
22500	76.05	63.72	82.02	90.14	92.76	93.24
25000	77.88	67.3	83.45	90.85	93.32	93.87
27500	79.55	70.09	84.57	91.41	93.79	94.35
30000	80.91	72.39	85.6	91.89	94.27	94.75
32500	82.02	74.3	86.4	92.28	94.59	95.15
35000	83.05	75.97	87.11	92.68	94.91	95.47
37500	83.93	77.33	87.75	93	95.23	95.7
40000	84.65	78.60	88.23	93.32	95.47	96.02
42500	85.52	79.63	88.78	93.64	95.7	96.26
45000	86.08	80.67	89.18	93.79	95.86	96.42
47500	86.63	81.46	89.58	94.03	96.02	96.66
50000	87.19	82.26	89.98	94.19	96.26	96.82

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The right ascension of ascending nodes has a pronounced effect on the visibility of a satellite in orbit around Earth in the Moon's orbital plane. As μ increases, the visibility increases to a maximum at about $\mu = 300$ degrees. This can be explained by the presence of two DSN stations located at the equator (Goldstone and Madrid) and within 120° W of the prime meridian (243.1° east latitude, 355.8° east latitude, respectively). Thus, when the satellite is at 300 degrees "latitude" ($\mu = 300°$), it is directly "between" two DSN stations. On the other hand, when $\mu = 60$ degrees, visibility is at a minimum because the third DSN (Canberra) is below the equator at about 150° east latitude. Because the other two stations are occulted by Earth, a minimum visibility is seen at $\mu = 60$ degrees (see Figure 5 and Table 1).

4.0 REFERENCES

1. FORTRAN program VISI.FOR written by Dr. Fredric Messing, 1989

2. Frederic Messing, personal consultations (June–July 1990)

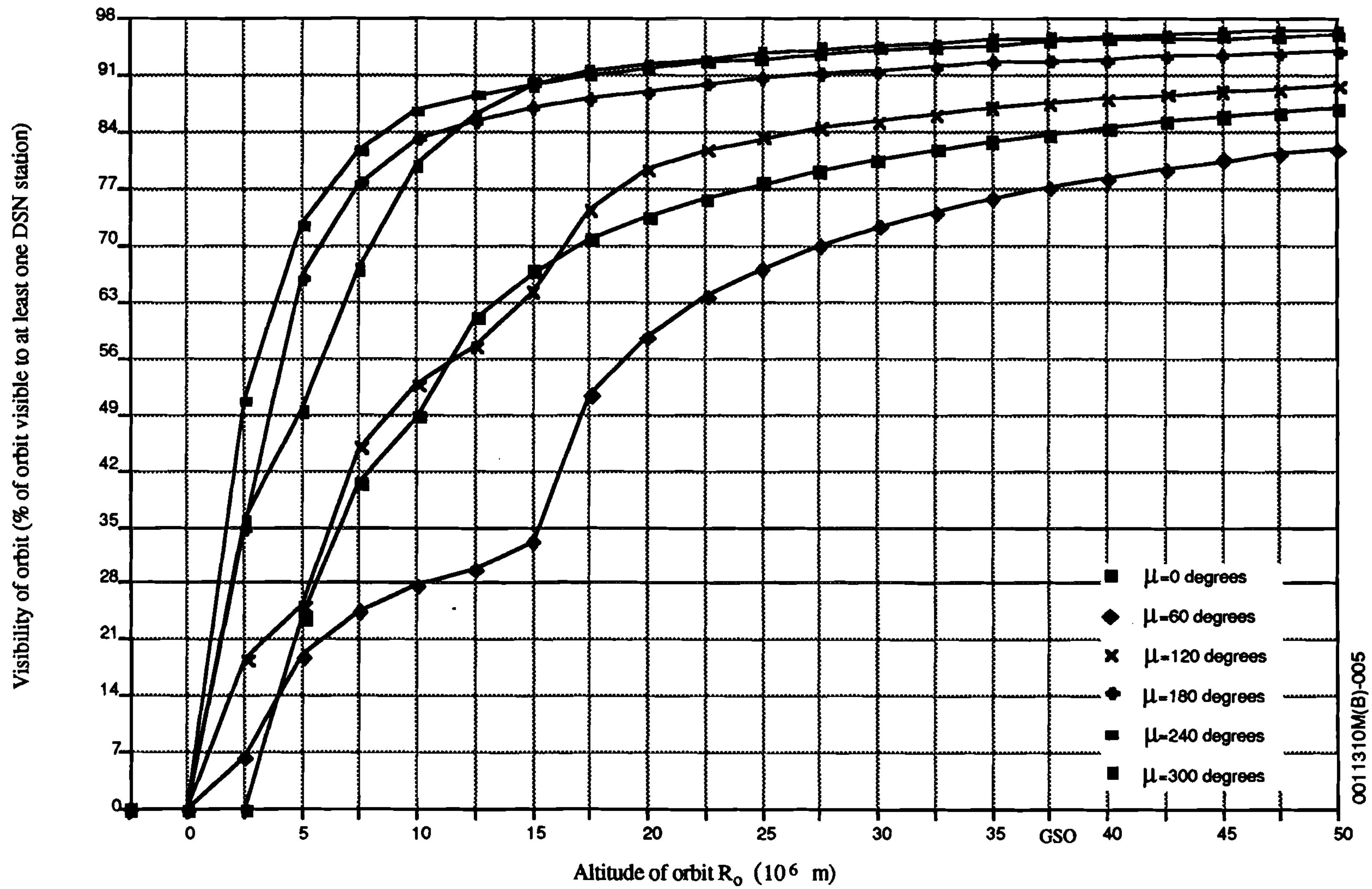


Figure 5. DSN Visibility: As a function of R_0 and μ (γ is constant at 28.6 degrees)

APPENDIX A — TWO-DIMENSIONAL VISIBILITY MODEL

The following derivation of *x* is done by setting the formula for the line of visibility (left) equal to the formula for the orbit of the satellite (right) yielding the following equation. See Figure 2 for details on variables, etc.

$R_e = 6378$ km

$$(\tan[\theta_e]x + R_e)^2 = \sqrt{(R_e + R_o)^2 - x^2}^{\,2}$$

$$R_e^2 + (\tan[\theta_e])^2 x^2 + 2\tan(\theta_e)R_e x = R_e^2 + R_o^2 + 2R_eR_o - x^2$$

$$R_e^2 + (\tan[\theta_e])^2 x^2 + x^2 + 2\tan(\theta_e)R_e x = R_e^2 + R_o^2 + 2R_eR_o$$

$$R_e^2 + \left(\left[\tan(\theta_e)\right]^2 + 1\right)x^2 + 2\tan(\theta_e)R_e x = R_e^2 + R_o^2 + 2R_eR_o$$

$$\left(\left[\tan(\theta_e)\right]^2 + 1\right)x^2 + 2\tan(\theta_e)R_e x = R_o^2 + 2R_eR_o$$

$$-R_o^2 - 2R_eR_o + \left(\left[\tan(\theta_e)\right]^2 + 1\right)x^2 + 2\tan(\theta_e)R_e x = 0$$

$$\left(\left[\tan(\theta_e)\right]^2 + 1\right)x^2 + 2\tan(\theta_e)R_e x = R_o^2 + 2R_eR_o$$

This is a quadratic equation with two solutions for x ($\theta_e = 10°$ or $\pi/18$ radians). The correct solution is x_1 (see Figure 2 for significance).

$$x_1 = \frac{-2\tan\left(\frac{\pi}{18}\right)R_e + \sqrt{\left(2\tan\left[\frac{\pi}{18}\right]R_e\right)^2 - \left(4\left[\left\{\tan\left(\frac{\pi}{18}\right)\right\}^2 + 1\right]\right)\left(-R_o^2 - 2R_eR_o\right)}}{2\left(\left[\tan\left\{\frac{\pi}{18}\right\}\right]^2 + 1\right)}$$

$$x_1 = 0.48492\left(\sqrt{-4.1244\left[-R_o^2 - 2R_eR_o\right] + \left[0.35265R_e\right]^2} - 0.35265R_e\right)$$

$$x_1 = 0.48492\left(\sqrt{-4.1244\left[-R_o^2 - 12756R_o\right] + 5.059 \times 10^6} - 2249.2\right)$$

$$x_2 = \frac{-2\tan\left(\frac{\pi}{18}\right)R_e - \sqrt{\left(2\tan\left[\frac{\pi}{18}\right]R_e\right)^2 - \left(4\left[\left\{\tan\left(\frac{\pi}{18}\right)\right\}^2 + 1\right]\right)\left(-R_o^2 - 2R_eR_o\right)}}{2\left(\left[\tan\left\{\frac{\pi}{18}\right\}\right]^2 + 1\right)}$$

$$x_2 = 0.48492\left(-\sqrt{-4.1244\left[-R_o^2 - 2R_eR_o\right] + \left[0.35265R_e\right]^2} - 0.35265R_e\right)$$

$$x_2 = 0.48492\left(-\sqrt{-4.1244\left[-R_o^2 - 12756R_o\right] + 5.059 \times 10^6} - 2249.2\right)$$

Now $ratio_1$ and $ratio_2$ can be calculated; these are the visibility ratios assuming x_1 and x_2 as solutions. See Figure 2 for explanation of variables and formulae.

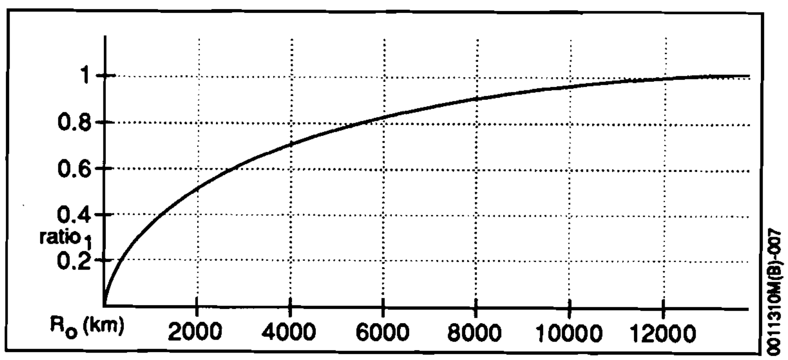
$$ratio_1 = \frac{3}{\pi}\sin^{-1}\left(\frac{x_1}{R_e + R_o}\right)$$

$$ratio_1 = 3\,\frac{\arcsin\left(0.48492\,\frac{\sqrt{-4.1244\left[-R_o^2 - 12756R_o\right] + 5.059 \times 10^6} - 2249.2}{R_e + R_o}\right)}{\pi}$$

$$ratio_1 = 0.95493 \arcsin\left(0.48492\,\frac{\sqrt{-4.1244\left[-R_o^2 - 12756R_o\right] + 5.059 \times 10^6} - 2249.2}{R_o + 6378}\right)$$

The following graph shows $ratio_1$ or the visibility as a function of satellite altitude in the elevation angle constraint problem with $\theta_e = 10°$ (see Figure 2 for significance).

$R_o = 11{,}987$ km
$ratio_1 = 1$



Visibility is 100 percent at $R_o = (1.88)R_e$, which is 1.88 times as far as the 0 degree (local horizontal) two-dimensional model (see comparison). **Note that visibilities can exceed unity (i.e., more than 100-percent visibility) because of overlap of the three stations.

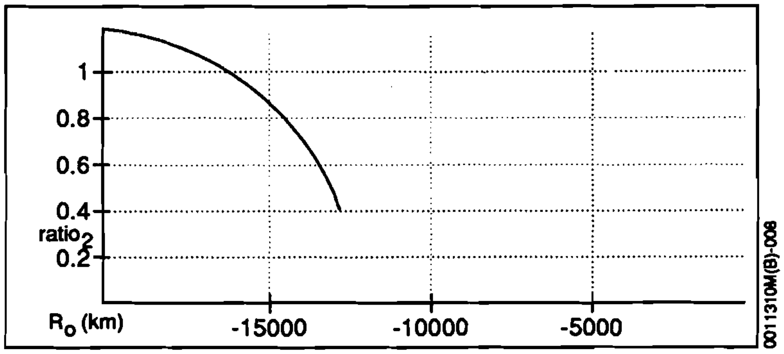
$$ratio_2 = \frac{3}{\pi}\sin^{-1}\left(\frac{x_2}{R_e + R_o}\right)$$

$$ratio_2 = 3\,\frac{\arcsin\left(0.48492\,\frac{-\sqrt{-4.1244\left[-R_o^2 - 12756R_o\right] + 5.059 \times 10^6} - 2249.2}{R_e + R_o}\right)}{\pi}$$

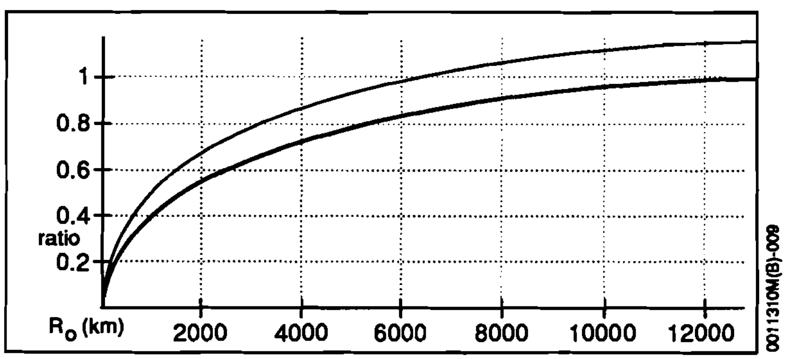
$$ratio_2 = 0.95493 \arcsin\left(0.48492\,\frac{-\sqrt{-4.1244\left[-R_o^2 - 12756R_o\right] + 5.059 \times 10^6} - 2249.2}{R_o + 6378}\right)$$

The following graph shows the visibility as a function of R_o, assuming x_2 as the proper solution to the above quadratic. However, this solution is meaningless because it requires negative values for the altitude.

R_o = -16,150 km
$ratio_2$ = 1



The following graph is a comparison between the visibilities of the simple two-dimensional model and the θ_e = 10° elevation angle constraint two-dimensional model (Figure 1 versus Figure 2). The θ_e elevation angle constraint plot (ratio') is the dark line on the bottom, indicating the decrease in visibility caused by adding the 10-degree constraint to the LOS.



$$\text{ratio}' = 3\,\frac{\arcsin\left(0.48492\,\dfrac{\sqrt{-4.1244\left[-R_o^2 - 12756R_o\right] + 5.059 \times 10^6} - 2249.2}{R_e + R_o}\right)}{\pi}$$

$$\text{ratio} = \frac{3}{\pi}\arccos\left(\frac{R_e}{R_e + R_o}\right)$$

APPENDIX B — FORTRAN PROGRAM DSN.FOR

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C   To calculate DSN visibility of any Earth-bound satellite

C   For graphical representation of coordinate system and vectors used
C   See Figures 3 and 4 in "Visibility of Earth-Bound Satellites:  A DSN Study"

C   Center of Earth is origin of spherical coordinate system

C   Phi is (90-degree latitude) (0 degree being straight up z-axis)
C   increasing in the south direction (e.g., 40 degrees north latitude
C   corresponds to 50-degree phi angle)

C   Theta is the angle of longitude with 0 degree in direction of x-axis
C   (meridian) increasing in east direction
C   (e.g., 40 deg ELONG = 40 deg theta angle)

C   c = Canberra; g = Goldstone; o = orbit; m = Madrid (subscripts indicating DSN)

C   V(c,g,o,m) = vector drawn from center of Earth to DSN(c,g,o,m)

C   V(c,g,o,m)(x,y,z) = X,Y,Z components of V(c,g,o,m)

C   V2(c,g,o,m) = vector from station to equatorial orbit {satellite}

C   V01 = vector drawn to satellite from center of Earth (origin)
C   (orbital vector) assuming equatorial orbit

C   Vo = transformed orbital vector using γ, μ (see definitions below)

C   Z = angle between V2(c,g,o,m) and V2(c,g,o,m) {zenith angle}

C   Re = radius of Earth; Ro = altitude of orbit (all lengths in km)

C   THETAo = incremented orbital angle (increment = dTHETAo)

C   GAMMA = angle of inclination of orbit with respect to equatorial plane
C   (28.6 degrees = plane of the Moon's orbit) (fixed)

C   MU = right ascension of ascending node (angle of twist about z-axis North Pole)
C   (user defined)

C   PROGRAM DSN.FOR
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```
C       DEFINING COUNTERS AND DATA
          CHARACTER*64 filename
          INTEGER*2 COUNT
          REAL*4 GAMMA1,MU1

10      Re=6378
        COUNT=3

C       CREATING A RECORD OF DATA
C               Prompt user for file name and read it:
        WRITE (*,'(A\)') ' Enter file name to store data '
                READ (*, '(A)') filename
C               OPEN FILE CALLED 'filename'
        OPEN (7, FILE=filename,ACCESS='DIRECT',
          +             FORM='FORMATTED',STATUS='NEW',RECL=40)

            PRINT *,'INPUT ANGLE INCREMENT (RADIANS)'
            READ *,dTHETAo
            PRINT *,'INPUT MINIMUM ALTITUDE (KM)'
            READ*,RMIN
            PRINT*,'INPUT MAXIMUM ALTITUDE (KM)'
            READ*,RMAX
            PRINT *,'INPUT ALTITUDE INCREMENT (KM)'
            READ*,dRo
            PRINT*,'INPUT RIGHT ASCENSION OF ASCENDING NODE (DEGREES)'
            READ*,MU1
            PRINT*,'INPUT ANGLE OF INCLINATION OF ORBIT (DEGREES)'
            READ*,GAMMA1
            DEG=.01745329252
            PI=3.141592654
            GAMMA=DEG*GAMMA1
            MU=DEG*MU1
            Q2=DEG*10
            Q=(PI/2)-Q2
            WRITE (7,'(A)',REC=1) 'DSN VISIBILITY DATA'
            WRITE (7,'(A,F9.2)',REC=2) 'MU (Right ascension)= ',MU1
            WRITE (7,'(A,F9.2)',REC=3) 'GAMMA (INCLINATION)= ',GAMMA1

C       LATITUDE & LONGITUDE OF DSN STATIONS (SEE FIGURE 3 FOR ALL
        DETAILS)
          PHIc=2.19
          PHIg=.955
          PHIm=.864
          THETAc=2.6
          THETAg=4.24
          THETAm=6.21

C       LOOPING ALTITUDE VALUES BETWEEN RMIN & RMAX
          DO Ro=RMIN,RMAX,dRo
```

```
      C=0
      COUNT1=0

C     LENGTH OF VECTORS OUT TO ORBIT AND DSNs
      RHOo=(Re+Ro)
      RHOc=Re
      RHOg=Re
      RHOm=Re

C     DEFINING COMPONENTS OF VECTOR OUT TO CANBERRA
      Vcx=RHOc*SIN(PHIc)*COS(THETAc)
      Vcy=RHOc*SIN(PHIc)*SIN(THETAc)
      Vcz=RHOc*COS(PHIc)

C     COMPONENTS OF VECTOR OUT TO GOLDSTONE
      Vgx=RHOg*SIN(PHIg)*COS(THETAg)
      Vgy=RHOg*SIN(PHIg)*SIN(THETAg)
      Vgz=RHOg*COS(PHIg)

C     COMPONENTS OF VECTOR OUT TO MADRID
      Vmx=RHOm*SIN(PHIm)*COS(THETAm)
      Vmy=RHOm*SIN(PHIm)*SIN(THETAm)
      Vmz=RHOm*COS(PHIm)

C     MOVING SATELLITE AROUND ITS ORBIT (DO LOOP FOR THETAo)

      DO THETAo=0,(2*PI),dTHETAo
      COUNT1=COUNT1+1

C     DEFINING ORBITAL VECTOR COMPONENTS ASSUMING EQUATORIAL
      PLANE
        Vo1x=RHOo*COS(THETAo)
        Vo1y=RHOo*SIN(THETAo)
        Vo1z=0

C     DEFINING ORBITAL VECTOR COMPONENTS TRANSFORMED (MU,GAMMA
      TRANSFORMS)
      Vox=(Vo1x*COS(MU))-(Vo1y*SIN(MU)*COS(GAMMA))
      Voy=(Vo1x*SIN(MU))+(Vo1y*COS(MU)*COS(GAMMA))
      Voz=(Vo1y*SIN(GAMMA))

C     Canberra to Satellite (V2c vector components)
        V2cx=Vox-Vcx
        V2cy=Voy-Vcy
        V2cz=Voz-Vcz
```

```
C       Goldstone to Satellite (V2g vector components)
          V2gx=Vox-Vgx
          V2gy=Voy-Vgy
          V2gz=Voz-Vgz

C       Madrid to Satellite (V2m vector components)
          V2mx=Vox-Vmx
          V2my=Voy-Vmy
          V2mz=Voz-Vmz

C       Dot Products between V2(c,g,m) and V(c,g,m)
          V2cdotVc=(V2cx*Vcx+V2cy*Vcy+V2cz*Vcz)
          V2gdotVg=(V2gx*Vgx+V2gy*Vgy+V2gz*Vgz)
          V2mdotVm=(V2mx*Vmx+V2my*Vmy+V2mz*Vmz)

C       Lengths of V2(c,g,m) and V(c,g,m)
          LV2c=SQRT(V2cx**2+V2cy**2+V2cz**2)
          LV2g=SQRT(V2gx**2+V2gy**2+V2gz**2)
          LV2m=SQRT(V2mx**2+V2my**2+V2mz**2)
          LVc=SQRT(Vcx**2+Vcy**2+Vcz**2)
          LVg=SQRT(Vgx**2+Vgy**2+Vgz**2)
          LVm=SQRT(Vmx**2+Vmy**2+Vmz**2)

C       DEFINING RATIOS WHICH DETERMINE COS(Zc,g,m)
          Rc=V2cdotVc/(LV2c*LVc)
          Rg=V2gdotVg/(LV2g*LVg)
          Rm=V2mdotVm/(LV2m*LVm)

C       ACCOUNTING FOR ROUNDING DISCREPANCIES (VIOLATE ACOS RANGE)
          IF (Rc .GT. 1) Rc=1
          IF (Rc .LT. -1) Rc=-1
          IF (Rg .GT. 1) Rg=1
          IF (Rg .LT. -1)  Rg=-1
          IF (Rm .GT. 1) Rm=1
          IF (Rm .LT. -1) Rm=-1

C                Angles between V2(c,g,m) and V(c,g,m) zenith angles
          Zc=ACOS(Rc)
          Zg=ACOS(Rg)
          Zm=ACOS(Rm)

C       Deciding if any one of stations (c,g,m) can see satellite
          If ((Zc .LE. Q) .OR. (Zg .LE. Q) .OR. (Zm .LE. Q)) C=C+1

          END DO
        RATIO=100*(C/COUNT1)
        PRINT*,Ro,'KM',RATIO,'%'
```

```
C       WRITING TO 'FILENAME' (DATA)
      COUNT=COUNT+1
      WRITE (7,'(F9.2,A,F9.2,A)',REC=COUNT) Ro,' Km',RATIO, '%'
      END DO
        CLOSE (7)
        GOTO 10
        END
```