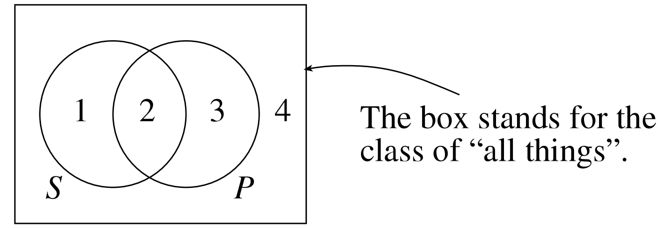
Philosophy 57 — Day 9

- Quiz #2 Returned Today (Solutions Posted on Website)
  - Curve to be announced in class ...
    - ∗ stay tuned for curve ...
- Quiz #3 is next Tuesday 03/04/03 (on chapter 4, through Thursday)
- Back to Chapter 4 — Categorical Statements (sections 4.5–4.6 *skipped*)
  - Venn Diagrams, Simple Arguments, and the Square of Opposition
  - Conversion, Obversion, and Contraposition
  - Translating from English into Categorical Logic

Chapter 4: Categorical Statements — Venn Diagrams & The Square of Opposition I



Region 1 = the class of things which are inside S but outside P.
Region 2 = the class of things which are inside S and inside P.
Region 3 = the class of things which are outside S and inside P.
Region 4 = the class of things which are outside S and outside P.

(A) All S are P. = No members of S are *outside* P (nothing in 1).

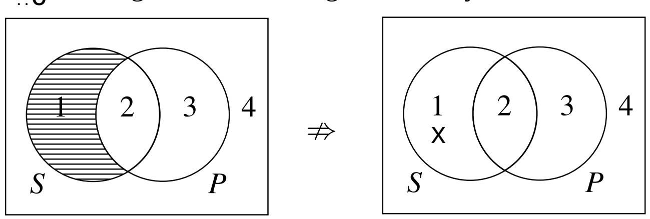
(E) No S are P. = No members of S are *inside* P (nothing in 2).

(I) Some S are P. = At least one S is inside P (something in 2).

(O) Some S are not P. = At least one S is outside P (something in 1).

Chapter 4: Categorical Statements — Venn Diagrams & The Square of Opposition II

- Three steps: (1) Draw the Venn Diagram for the premise, (2) Draw the Venn Diagram for the conclusion, (3) Does the premise-diagram contain the information in conclusion-diagram? If so, then the inference is valid.
- Example: $\frac{\mathbf{A}}{\therefore \mathbf{O}}$. Putting the **A** and **O** diagrams side by side, we have:

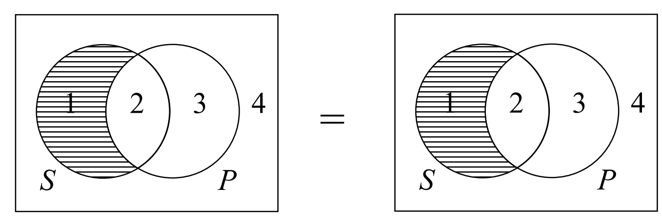


(**A**) All S are P. (**O**) Some S are not P.

- We can see that the premise-diagram does not contain the information of the conclusion diagram. So, the argument $\frac{\mathbf{A}}{\therefore \mathbf{O}}$ is *invalid* ($\mathbf{A} \not\Rightarrow \mathbf{O}$).
- What about the argument from **A** to the *denial* of **O**?

Chapter 4: Categorical Statements — Venn Diagrams & The Square of Opposition III

- To draw the Venn diagram for the *denial* of a categorical claim, one marks the same regions as for the categorical claim itself — *but in the opposite ways*. Instead of putting an "X" in a region, one shades it (and *vice versa*).
- So, the *denial* of an **O** claim would look like this:

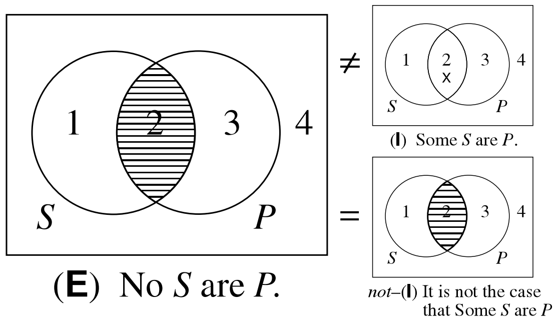


(**A**) All S are P. not–(**O**) It is not the case that Some S are not P.

- But, this *is* just the **A**-diagram! That is, the **A**-diagram contains the information in the *not*-**O**-diagram. Hence, $\frac{\mathbf{A}}{\therefore \textit{not-}\mathbf{O}}$ *is valid* ($\mathbf{A} \Rightarrow \textit{not-}\mathbf{O}$).

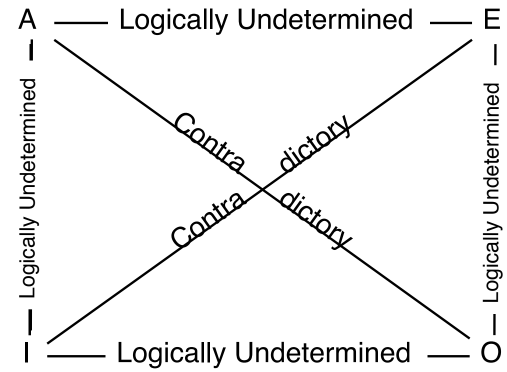
Chapter 4: Categorical Statements — Venn Diagrams & The Square of Opposition IV

- We can use the same technique to analyze $\frac{\textbf{E}}{\therefore \textbf{I}}$ and $\frac{\textbf{E}}{\therefore \textit{not-}\textbf{I}}$. Let's do $\frac{\textbf{E}}{\therefore \textit{not-}\textbf{I}}$.
- Let's draw the diagrams for **E**, **I**, and the *denial* of **I**:



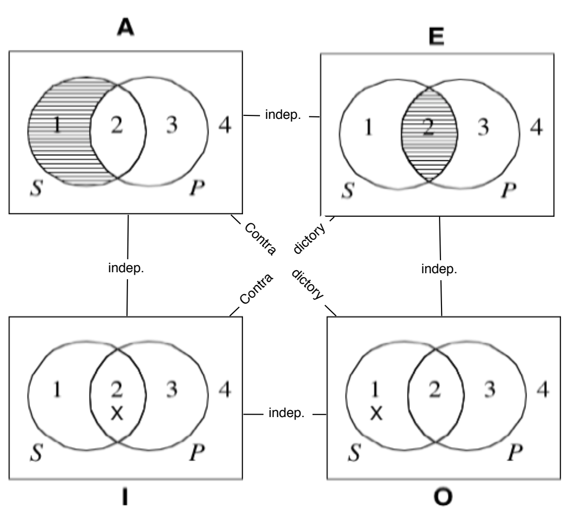
- We can see plainly that **E** ⇏ **I**, **I** ⇏ **E**, **E** ⇒ *not*-**I**, and *not*-**I** ⇒ **E**.
- Also: **I** ⇏ *not***O**, **A** ⇏ **I**, **A** ⇏ *not*-**I**, **E** ⇏ **O**, **E** ⇏ *not*-**O**. These logical relationships between **A**, **E**, **I**, **O** are summarized in the Square of Opposition.

Chapter 4: Categorical Statements — Venn Diagrams & The Square of Opposition V



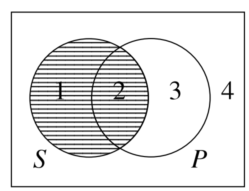
- This Square is just a handy way of summarizing the following 12 logical relationships between the four standard form categorical claims:
  - **A** ⇒ *not*-**O**, **O** ⇒ *not*-**A**, **E** ⇒ *not*-**I**, **I** ⇒ *not*-**E**, **I** ⇏ **O**, **I** ⇏ *not*-**O**, **A** ⇏ **I**, **A** ⇏ *not*-**I**, **E** ⇏ **O**, **E** ⇏ *not*-**O**, **A** ⇏ **E**, **A** ⇏ *not*-**E**.

Chapter 4: Categorical Statements — Venn Diagrams & The Square of Opposition VI

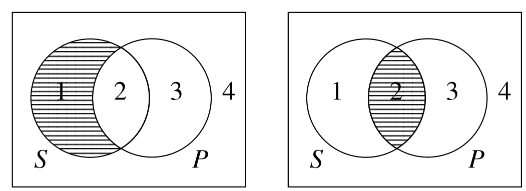


Chapter 4: Categorical Statements — Venn Diagrams & The Square of Opposition VII

- Exercise from above: prove that "Nothing is an *S*" implies *both* **A** *and* **E**.



"Nothing is an *S*"



(**A**) All *S* are *P*. (**E**) No *S* are *P*.

- The top diagram contains the information in *both* bottom diagrams.

Chapter 4: Categorical Statements — Conversion, Obversion & Contraposition I

- Conversion, Obversion, and Contraposition are three important operations or transformations that can be performed on categorical statements.
- The Converse of a categorical statement is obtained by switching its subject and predicate terms. This switching process is called Conversion.

Proposition	Name	Converse
All *A* are *B*.	**A**	All *B* are *A*.
No *A* are *B*.	**E**	No *B* are *A*.
Some *A* are *B*.	**I**	Some *B* are *A*.
Some *A* are not *B*.	**O**	Some *B* are not *A*.

- Some statements are *equivalent to* (*i.e.*, *have the same Venn Diagram as*) their converses. Some statements are *not* equivalent to their converses.
- **E** and **I** claims are equivalent to their converses, whereas **A** and **O** claims are *not* equivalent to their converses. Let's *prove* this with Venn Diagrams.

Chapter 4: Categorical Statements — Conversion, Obversion & Contraposition II

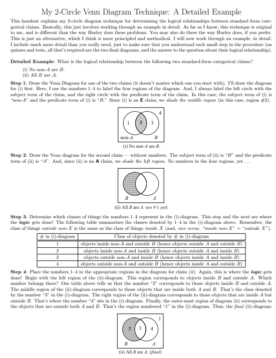
- The complement of a term "*X*" is written "non-*X*", and it denotes the class of things *not* contained in the *X*-class. Do not confuse "not" and "non-". "not" is part of the *copula* "are not", but "non-" is part of a *term* "non-*X*" ("non-*X*" can be either the subject term or the predicate term of a categorical statement).
- The Obverse of a categorical statement is obtained by: (1) switching the quality (but *not* the quantity!) of the statement, and (2) replacing the predicate term with its complement. This 2-step process is called Obversion.

Proposition	Name	Obverse
All *A* are *B*.	**A**	No *A* are non-*B*.
No *A* are *B*.	**E**	All *A* are non-*B*.
Some *A* are *B*.	**I**	Some *A* are not non-*B*.
Some *A* are not *B*.	**O**	Some *A* are non-*B*.

- *All* categorical statements are logically equivalent to their obverses. Let's *prove* this for each of the four categorical claims, using Venn Diagrams.

Chapter 4: Categorical Statements — Conversion, Obversion & Contraposition II.1

- At this point, we need to be more careful with our Venn Diagram Method! So far, we have not seen any Venn Diagrams with complemented terms in them.
- Let's do an example to see how we must handle this new case.
- Here, I will go over the handout on my 2-Circle Venn Diagram Method.



My 2-Circle Venn Diagram Technique: A Detailed Example

This handout explains my 2-circle diagram technique for determining the logical relationships between standard form categorical claims. Basically, this just involves working through an example in detail. As far as I know, this technique is original to me, and is different than the way Hurley does these problems. You may also do these the way Hurley does, if you prefer. This is just an alternative, which I think is more principled and methodical. I will now work through an example, in detail. I include much more detail than you really need, just to make sure that you understand each small step in the procedure (on quizzes and tests, all that's required are the two final diagrams, and the answer to the question about their logical relationship).

Detailed Example: What is the logical relationship between the following two standard-form categorical claims?

(i) No non-*A* are *B*.
(ii) All *B* are *A*.

Step 1: Draw the Venn Diagram for one of the two claims (it doesn't matter which one you start with). I'll draw the diagram for (i) first. Here, I use the numbers 1–4 to label the four regions of the diagram. And, I always label the left circle with the subject term of the claim, and the right circle with the predicate term of the claim. In this case, the subject term of (i) is "non-*A*" and the predicate term of (i) is "*B*." Since (i) is an **E** claim, we *shade the middle region* (in this case, region #2).

(i) No non-*A* are *B*.

Step 2: Draw the Venn diagram for the second claim — without numbers. The subject term of (ii) is "*B*" and the predicate term of (ii) is "*A*". And, since (ii) is an **A** claim, we *shade the left region*. No numbers in the four regions, yet ...

(ii) All *B* are *A*. (*no #'s yet*)

Step 3: Determine which classes of things the numbers 1–4 represent in the (i)-diagram. This step and the next are where the ***logic*** gets done! The following table summarizes the classes denoted by 1–4 in the (i)-diagram above. Remember, the class of things *outside non-X* is the same as the class of things *inside X* (and, *vice versa*: "*inside non-X*" = "*outside X*").

# in (i)-diagram	Class of objects denoted by # in (i)-diagram
1	objects inside non-*A* and outside *B* (hence objects outside *A* and outside *B*)
2	objects inside non-*A* and inside *B* (hence objects outside *A* and inside *B*)
3	objects outside non-*A* and inside *B* (hence objects inside *A* and inside *B*)
4	objects outside non-*A* and outside *B* (hence objects inside *A* and outside *B*)

Step 4: Place the numbers 1–4 in the appropriate regions in the diagram for claim (ii). Again, this is where the ***logic*** gets done! Begin with the left region of the (ii)-diagram. This region corresponds to objects inside *B* and outside *A*. Which number belongs there? Our table above tells us that the number "2" corresponds to those objects inside *B* and outside *A*. The middle region of the (ii)-diagram corresponds to those objects that are inside both *A* and *B*. That's the class denoted by the number "3" in the (i)-diagram. The right region of the (ii)-diagram corresponds to those objects that are inside *A* but outside *B*. That's where the number "4" sits in the (i)-diagram. Finally, the outer-most region of diagram (ii) corresponds to the objects that are outside both *A* and *B*. That's the region numbered "1" in the (i)-diagram. Thus, the *final* (ii)-diagram:

(ii) All *B* are *A*. (*final*)

☞ *All regions with the same numbers have the same markings in the* (i) *and* (ii) *diagrams*. So, (i) ***is equivalent to*** (ii). ❑

Chapter 4: Categorical Statements — Conversion, Obversion & Contraposition III

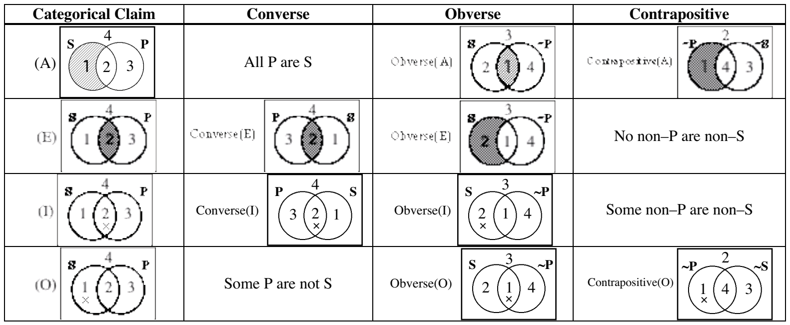
- The Contrapositive of a categorical statement is obtained by: (1) *converting* the statement, and (2) replacing both the subject term and the predicate term with their complements. This 2-step process is called Contraposition.

Proposition	Name	Contrapositive
All *A* are *B*.	**A**	All non-*B* are non-*A*.
No *A* are *B*.	**E**	No non-*B* are non-*A*.
Some *A* are *B*.	**I**	Some non-*B* are non-*A*.
Some *A* are not *B*.	**O**	Some non-*B* are not non-*A*.

- Some statements are *equivalent to* (*i.e.*, *have the same Venn Diagram as*) their contrapositives. Some statements are *not* equivalent to their contrapositives.
- **A** and **O** claims are equivalent to their contrapositives, whereas **E** and **I** claims are *not* equivalent to their contrapositives. Let's *prove* this with Venn's.

Chapter 4: Categorical Statements — Conversion, Obversion & Contraposition IV

Proposition	**Converse**	**Obverse**	**Contrapositive**
(**A**) All *A* are *B*.	All *B* are *A*. (≠)	No *A* are non-*B*. (=)	All non-*B* are non-*A*. (=)
(**E**) No *A* are *B*.	No *B* are *A*. (=)	All *A* are non-*B*. (=)	No non-*B* are non-*A*. (≠)
(**I**) Some *A* are *B*.	Some *B* are *A*. (=)	Some *A* are not non-*B*. (=)	Some non-*B* are non-*A*. (≠)
(**O**) Some *A* are not *B*.	Some *B* are not *A*. (≠)	Some *A* are non-*B*. (=)	Some non-*B* are not non-*A*. (=)



Chapter 4: Categorical Statements — Translation from English Overview

- Many English claims can be translated faithfully into one of the four standard form categorical claims. There are 10 things to look out for.
  - Terms Without Nouns
  - Nonstandard Verbs
  - Singular Propositions
  - Adverbs and Pronouns
  - Unexpressed Quantifiers
  - Nonstandard Quantifiers
  - Conditional Statements
  - Exclusive Propositions
  - "The Only"
  - Exceptive Pronouns
- You do not need to remember the names of these 10 watchwords, but you'll need to know how to translate English sentences which involve them.

Chapter 4: Categorical Statements — Translation from English I

- Terms Without Nouns: The subject and predicate terms of a categorical proposition must contain either a plural noun or a pronoun that serves to denote the class indicated by the term.
- Nouns and pronouns denote classes, while adjectives (and participles) connote attributes or properties. We must replace mere adjectives with noun phrases.
- Examples:
  - "Some roses are red." Here, the subject term is a noun and properly denotes a class of things (i.e., roses). But, the predicate term is a mere adjective and does not denote a class. How do we fix this?
  - "All tigers are carnivorous." Again, the subject term is a noun and properly denotes a class of things (*i.e.*, tigers). But, the predicate term is a mere adjective and does not denote a class. How do we fix this?

Chapter 4: Categorical Statements — Translation from English II

- Nonstandard Verbs: The only copulas that are allowed in standard form are "are" and "are not." Statements in English often use other forms of the verb "to be." These need to be translated into standard form.
- Examples:
  - "Some college students will become educated." How do we translate this into something of the standard form "Some college students are _______"?
  - "Some dogs would rather bark than bite." How do we translate this into something of the standard form "Some dogs are _______"?
- Sometimes the verb "to be" does not occur at all, as in:
  - "Some birds fly south for the winter." How do we translate this into something of the standard form "Some birds are _______"?
  - "All ducks swim." How do we translate this into something of the standard form "All ducks are _______"?