

Philosophy 57 — Day 8

- I will return quiz #2 on Tuesday
- There is no quiz on Tuesday (Quiz #3 is on the following Tuesday 3/04/03)
- Back to Chapter 4 — Categorical Statements
 - Brief Review of Terminology
 - Quality, Quantity, and Distribution of Categorical Statements
 - Venn Diagram Representations of Categorical Statements
 - Using Venn Diagram’s to Study Simple Arguments
 - The Square of Opposition
 - Conversion, Obversion, and Contraposition
 - Later: Translating from English into Categorical Logic
 - **NOTE:** Sections 4.5–4.6 *skipped* (no Aristotelian stuff)



Chapter 4: Categorical Statements — Definition & Components

- A **categorical statement** (or **proposition**) relates two classes or categories, denoted by the **subject term** (S) and the **predicate term** (P). Categorical statements assert that either all or part of S is included in (excluded from) P .
- Categorical statements come in four **standard forms**:
 - (A) All S are P . (E) No S are P .
 - (I) Some S are P . (O) Some S are not P .
- The words “all”, “no” and “some” are called **quantifiers**.
- The words “are” and “are not” are called the **copula**.
- **Example.** All members of the American Medical Association are persons holding degrees from recognized academic institutions.
 - * quantifier = “all,” S = “members of the AMA,” P = “persons holding degrees from recognized academic institutions,” copula = “are”.



Chapter 4: Categorical Statements — Quality, Quantity & Distribution I

- The **quality** of a categorical claim is either **affirmative** or **negative**, depending on whether it *affirms* or *denies* class membership.
 - * “All S are P ” and “Some S are P ” have *affirmative* quality.
 - * “No S are P ” and “Some S are not P ” have *negative* quality.
- The **quantity** of a categorical claim is either **universal** or **particular**, depending on whether it makes a claim about *every* member or just *some* member of S .
 - * “All S are P ” and “No S are P ” are *universal*.
 - * “Some S are P ” and “Some S are not P ” are *particular*.
- A term X is **distributed** in a categorical statement if the statement asserts something about *every* member of the class X (otherwise, X is *undistributed*).
 - * S is distributed in “All S are P ” and “No S are P ”.
 - * P is distributed in “No S are P ” and “Some S are not P ”.
- Remember: **Universals distribute Subjects. Negatives distribute Predicates.**



Chapter 4: Categorical Statements — Quality, Quantity & Distribution II

Proposition	Name	Quantity	Quality	S	P
All S are P .	A	Universal	Affirmative	Distributed	Undistributed
No S are P .	E	Universal	Negative	Distributed	Distributed
Some S are P .	I	Particular	Affirmative	Undistributed	Undistributed
Some S are not P .	O	Particular	Negative	Undistributed	Distributed

- It may help to simply *memorize* the cases of distribution. The text offers two mnemonic devices for remembering the above facts about distribution.
 - Mnemonic #1. Unprepared Students Never Pass.**
Universals distribute Subjects. Negatives distribute Predicates.
 - Mnemonic #2. Any Student Earning B’s Is Not On Probation.**
A distributes Subject. **E** distributes Both.
I distributes Neither. **O** distributes Predicate.
- I prefer to *deduce* these using Venn Diagrams and the *definition* of distribution. **In Logic, answers can always be deduced from basic definitions.**

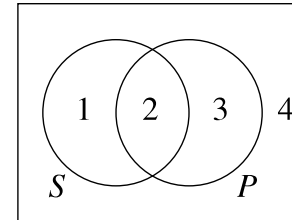


Chapter 4: Categorical Statements — Venn Diagrams & The Square of Opposition I

- Ultimately, we will use Venn Diagrams to test categorical *arguments* (*sylogisms*) for validity and invalidity. First, we need to learn how to represent categorical *statements* using Venn Diagrams.
- We will always operate from the *modern, Boolean* standpoint. You can ignore the stuff in the book about the traditional, Aristotelian standpoint.
- The standard from categorical statements can be understood as follows:
 - (A) All *S* are *P*. = No members of *S* are *outside P*.
 - (E) No *S* are *P*. = No members of *S* are *inside P*.
 - (I) Some *S* are *P*. = At least one *S* exists, and that *S* is a *P*.
 - (O) Some *S* are not *P*. = At least one *S* exists, and that *S* is not a *P*.
- **Note:** **A** and **E** do *not* imply that any *S*'s exist! This is the modern, Boolean standpoint. On the Aristotelian view, **A** and **E** *do* imply that some *S*'s exist.
- Consider "All unicorns are one-horned animals" (Boolean vs Aristotelian).

Chapter 4: Categorical Statements — Venn Diagrams & The Square of Opposition II

- To represent categorical statements using Venn Diagrams, we draw a box containing two overlapping circles. The box stands for "all things", and the two circles stand for the *S* and *P* classes in the claim being represented.

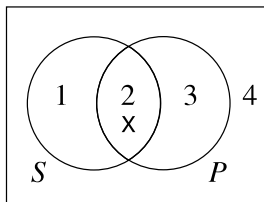


The box stands for the class of "all things".

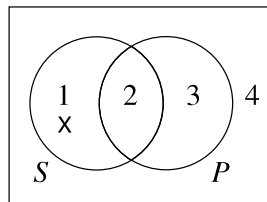
- It is helpful to think about which class of things are contained in each of 1–4.
- Region 1 = the class of things which are inside *S* but outside *P*.
Region 2 = the class of things which are inside *S* and inside *P*.
Region 3 = the class of things which are outside *S* and inside *P*.
Region 4 = the class of things which are outside *S* and outside *P*.

Chapter 4: Categorical Statements — Venn Diagrams & The Square of Opposition III

- Next, we adopt the following two Venn Diagram conventions.
 1. If a region (*i.e.*, 1–4) is *empty*, we use *shading (hashing)* to indicate this.
 2. If a region contains *at least one thing*, we use an "X" to indicate this.
- Venn Diagrams for the *particular* claims **I** and **O** involve only "X"s:



(I) Some *S* are *P*.

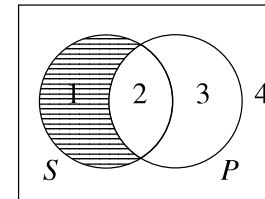


(O) Some *S* are not *P*.

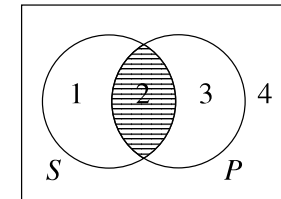
- It should be clear from these diagrams that the **I** and **O** claims *say different things*. We'll show below that *neither claim implies the other*.

Chapter 4: Categorical Statements — Venn Diagrams & The Square of Opposition IV

- The *universal* **A** and **E** claims require the *shading (hashing)* of regions.



(A) All *S* are *P*.



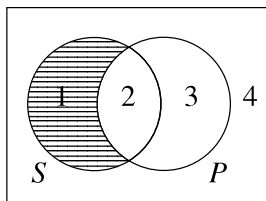
(E) No *S* are *P*.

- We can use these 2-circle Venn diagrams to investigate the *logical relationships between* the 4 standard-form categorical claims.
- For instance, we can already determine if the following four simple arguments are valid (Hurley calls these arguments "**immediate inferences**"):

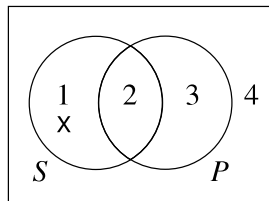
A	A	E	E
∴ O '	∴ not-O '	∴ I '	∴ not-I

Chapter 4: Categorical Statements — Venn Diagrams & The Square of Opposition V

- Three steps: (1) Draw the Venn Diagram for the premise, (2) Draw the Venn Diagram for the conclusion, (3) Does the premise-diagram contain the information in conclusion-diagram? If so, then the inference is valid.
- Example: $\frac{A}{\therefore O}$. Putting the **A** and **O** diagrams side by side, we have:



(A) All *S* are *P*.

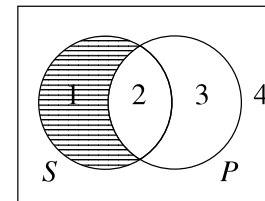


(O) Some *S* are not *P*.

- We can see that the premise-diagram does not contain the information of the conclusion diagram. So, the argument $\frac{A}{\therefore O}$ is *invalid* ($A \not\Rightarrow O$).
- What about the argument from **A** to the *denial* of **O**?

Chapter 4: Categorical Statements — Venn Diagrams & The Square of Opposition VI

- To draw the Venn diagram for the *denial* of a categorical claim, one marks the same regions as for the categorical claim itself — *but in the opposite ways*. Instead of putting an “X” in a region, one shades it (and *vice versa*).
- So, the *denial* of an **O** claim would look like this:

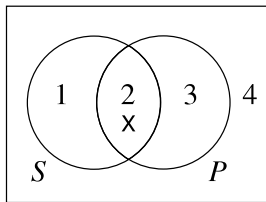


not-O: It is *not* the case that some *S* are not *P*.

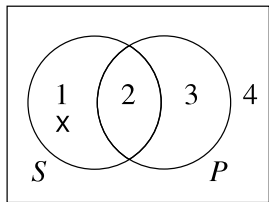
- But, this is just the **A**-diagram! That is, the **A**-diagram contains the information in the *not-O*-diagram. Hence, $\frac{A}{\therefore \text{not-O}}$ is *valid* ($A \Rightarrow \text{not-O}$).

Chapter 4: Categorical Statements — Venn Diagrams & The Square of Opposition VII

- We can use the same technique to analyze $\frac{E}{\therefore I}$ and $\frac{E}{\therefore \text{not-I}}$. Blackboard exercise.
- Let's return to the inference from **I** to **O**. Recall, I said that “Some *S* are *P*” does *not* imply “Some *S* are not *P*”. Look at the diagrams again:



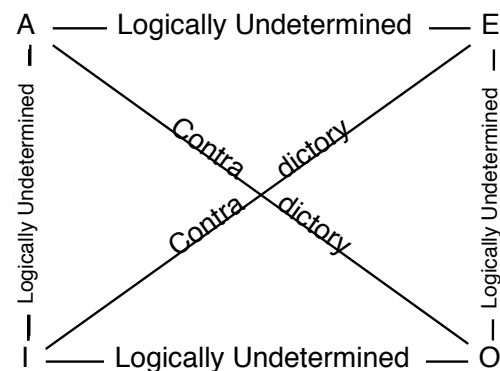
(I) Some *S* are *P*.



(O) Some *S* are not *P*.

- The **I**-diagram does *not* contain the information of the **O**-diagram. So, the argument $\frac{I}{\therefore O}$ is *invalid* ($I \not\Rightarrow O$). “Some *S* are *P*” \Rightarrow “Some *S* are not *P*”
- Also: $I \Rightarrow \text{not-O}$, $A \Rightarrow I$, $A \Rightarrow \text{not-I}$, $E \Rightarrow O$, $E \Rightarrow \text{not-O}$. These logical relationships between **A**, **E**, **I**, **O** are summarized in the **Square of Opposition**.

Chapter 4: Categorical Statements — Venn Diagrams & The Square of Opposition VIII



- This **Square** is just a handy way of summarizing the following 12 logical relationships between the four standard form categorical claims:
 * $A \Rightarrow \text{not-O}$, $O \Rightarrow \text{not-A}$, $E \Rightarrow \text{not-I}$, $I \Rightarrow \text{not-E}$, $I \Rightarrow O$, $I \Rightarrow \text{not-O}$,
 $A \Rightarrow I$, $A \Rightarrow \text{not-I}$, $E \Rightarrow O$, $E \Rightarrow \text{not-O}$, $A \Rightarrow E$, $A \Rightarrow \text{not-E}$.

Chapter 4: Categorical Statements — Conversion, Obversion & Contraposition I

- Conversion, Obversion, and Contraposition are three important operations or transformations that can be performed on categorical statements.
- The **Converse** of a categorical statement is obtained by switching its subject and predicate terms. This switching process is called **Conversion**.

Proposition	Name	Converse
All A are B.	A	All B are A.
No A are B.	E	No B are A.
Some A are B.	I	Some B are A.
Some A are not B.	O	Some B are not A.

- Some statements are *equivalent to* (i.e., have the same Venn Diagram as) their converses. Some statements are *not* equivalent to their converses.
- **E** and **I** claims are equivalent to their converses, whereas **A** and **O** claims are *not* equivalent to their converses. Let's *prove* this with Venn Diagrams.



Chapter 4: Categorical Statements — Conversion, Obversion & Contraposition II

- The **complement** of a term “X” is written “non-X”, and it denotes the class of things *not* contained in the X-class. **Do not confuse “not” and “non-”**. “not” is part of the *copula* “are not”, but “non-” is part of a *term* “non-X” (“non-X” can be either the subject term or the predicate term of a categorical statement).
- The **Obverse** of a categorical statement is obtained by: (1) switching the quality (but *not* the quantity!) of the statement, and (2) replacing the predicate term with its complement. This 2-step process is called **Obversion**.

Proposition	Name	Obverse
All A are B.	A	No A are non-B.
No A are B.	E	All A are non-B.
Some A are B.	I	Some A are not non-B.
Some A are not B.	O	Some A are non-B.

- **All categorical statements are logically equivalent to their obverses**. Let's *prove* this for each of the four categorical claims, using Venn Diagrams.



Chapter 4: Categorical Statements — Conversion, Obversion & Contraposition III

- The **Contrapositive** of a categorical statement is obtained by: (1) *converting* the statement, and (2) replacing both the subject term and the predicate term with their complements. This 2-step process is called **Contraposition**.

Proposition	Name	Contrapositive
All A are B.	A	All non-B are non-A.
No A are B.	E	No non-B are non-A.
Some A are B.	I	Some non-B are non-A.
Some A are not B.	O	Some non-B are not non-A.

- Some statements are *equivalent to* (i.e., have the same Venn Diagram as) their contrapositives. Some statements are *not* equivalent to their contrapositives.
- **A** and **O** claims are equivalent to their contrapositives, whereas **E** and **I** claims are *not* equivalent to their contrapositives. Let's *prove* this with Venn's.

