Philosophy 57 — Day 25

- Quiz #6 Returned Today
  - Curve: 85–100 (A); 70–80 (B); 55–65 (C); 45–50 (D); < 45 (F)
- Approximate "Curve for the course to this point" (dropping 2 quizzes)
  - 88–100 (A); 80–87 (B); 70–79 (C); 60-69 (D); < 60 (F);
- Extra-Credit Problems Posted on Website (5 problems, each worth 1 point!)
  - Extra-Credit Problems are due by Tuesday 05/20/03
  - No partial credit within problems (but you can do fewer than 5 problems)
  - You may use any tools/references you like to do these (but individually!)
  - Stay Tuned for Hints, etc. [these are all "chapter 6" problems]
- Back to Chapter 6
  - Truth-Tables for Claims (+ logical truth, *etc.*)
  - Truth-Tables for Arguments (+ validity, *etc.*)

Chapter 6 — Propositional Logic: Truth Tables I

p	$\sim p$
T	F
F	T

p	q	$p \bullet q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	q	$p \supset q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$p \equiv q$
T	T	T
T	F	F
F	T	F
F	F	T

Chapter 6 — Propositional Logic: Truth Tables II

- A statement is said to be logically true (or tautologous) if it is true regardless of the truth-values of its components. Example: $p \equiv p$ is logically true.

p	p	$\equiv$	p
T	T	T	T
F	F	T	F

- A statement is logically false (or self-contradictory) if it is false regardless of the truth-values of its components. Example: $p \bullet \sim p$ is logically false.

p	p	$\bullet$	$\sim$	p
T	T	F	F	T
F	F	F	T	F

- A statement is contingent if its truth-value varies depending on the truth-values of its components. Example: A (or *any* atom) is contingent.

A	A
T	T
F	F

Chapter 6 — Propositional Logic: Truth Tables III

- Classify the following statements as logically true (tautologous), logically false (self-contradictory), or contingent (exercise 6.3.I):

1. $N \supset (N \supset N)$
2. $(G \supset G) \supset G$
3. $(S \supset R) \bullet (S \bullet \sim R)$
4. $((E \supset F) \supset F) \supset E$
6. $(M \supset P) \vee (P \supset M)$
11. $[(Q \supset P) \bullet (\sim Q \supset R)] \bullet \sim (P \vee R)$
12. $[(H \supset N) \bullet (T \supset N)] \supset [(H \vee T) \supset N]$
15. $[(F \vee E) \bullet (G \vee H)] \equiv [(G \bullet E) \vee (F \bullet H)]$

Chapter 6 — Propositional Logic: Truth Tables IV

- Here is a completed truth-table for #11, $[(Q \supset P) \bullet (\sim Q \supset R)] \bullet \sim(P \vee R)$:

P	Q	R	$[(Q$	$\supset$	$P)$	$\bullet$	$(\sim$	Q	$\supset$	$R)]$	$\bullet$	$\sim$	$(P$	$\vee$	$R)$
T	T	T	T	T	T	T	F	T	T	T	F	F	T	T	T
T	T	F	T	T	T	T	F	T	T	F	F	F	T	T	F
T	F	T	F	T	T	T	T	F	T	T	F	F	T	T	T
T	F	F	F	T	T	F	T	F	F	F	F	F	T	T	F
F	T	T	T	F	F	F	F	T	T	T	F	F	F	T	T
F	T	F	T	F	F	F	F	T	T	F	F	T	F	F	F
F	F	T	F	T	F	T	T	F	T	T	F	F	F	T	T
F	F	F	F	T	F	F	T	F	F	F	F	T	F	F	F

- Therefore, the statement "$[(Q \supset P) \bullet (\sim Q \supset R)] \bullet \sim(P \vee R)$" is *logically false*.

Chapter 6 — Propositional Logic: Truth Tables V

- Two statements are said to be equivalent (written $p \approx q$) if they have the same truth-value in all possible worlds (*i.e.*, in all rows of a simultaneous truth-table of both statements). For instance, $A \supset B \approx \sim A \vee B$:

A	B	A	$\supset$	B	$\sim$	A	$\vee$	B
T	T	T	T	T	F	T	T	T
T	F	T	F	F	F	T	F	F
F	T	F	T	T	T	F	T	T
F	F	F	T	F	T	F	T	F

- Two statements are said to be contradictory if they have opposite truth-values in all possible worlds (*i.e.*, in all rows of a simultaneous truth-table of both statements). For instance, A and $\sim A$ are contradictory:

A	A	$\sim$	A
T	T	F	T
F	F	T	F

- Two statements are inconsistent if they are never both true in any possible world (*i.e.*, in any row of a simultaneous truth-table of both statements). For instance, $A \equiv B$ and $A \bullet \sim B$ are inconsistent (but *not* contradictory!):

A	B	A	$\equiv$	B	A	$\bullet$	$\sim$	B
T	T	T	T	T	T	F	F	T
T	F	T	F	F	T	T	T	F
F	T	F	F	T	F	F	F	T
F	F	F	T	F	F	F	T	F

- Two statements are consistent if they are both true in at least one possible world (*i.e.*, in at least one row of a simultaneous truth-table of both statements). For instance, $A \bullet B$ and $A \vee B$ are consistent:

A	B	A	$\bullet$	B	A	$\vee$	B
T	T	T	T	T	T	T	T
T	F	T	F	F	T	T	F
F	T	F	F	T	F	T	T
F	F	F	F	F	F	F	F

Chapter 6 — Propositional Logic: Truth Tables VI

- Use truth-tables to determine whether the following pairs of statements are logically equivalent, contradictory, consistent, or inconsistent (exercise 6.3.II).

2. $F \bullet M$ and $\sim(F \vee M)$
4. $R \vee \sim S$ and $S \bullet \sim R$
6. $H \equiv \sim G$ and $(G \bullet H) \vee (\sim G \bullet \sim H)$
8. $N \bullet (A \vee \sim E)$ and $\sim A \bullet (E \vee \sim N)$
10. $W \equiv (B \bullet T)$ and $W \bullet (T \supset \sim B)$
12. $R \bullet (Q \vee S)$ and $(S \vee R) \bullet (Q \vee R)$
14. $Z \bullet (C \equiv P)$ and $C \equiv (Z \bullet \sim P)$
15. $Q \supset \sim(K \vee F)$ and $(K \bullet Q) \vee (F \bullet Q)$

Chapter 6 — Propositional Logic: Truth Tables VII

- Here is a simultaneous truth-table which establishes that

$$A \equiv B \approx (A \bullet B) \vee ({\sim}A \bullet {\sim}B)$$

A	B	A	$\equiv$	B	$(A$	$\bullet$	$B)$	$\vee$	$({\sim}$	A	$\bullet$	${\sim}$	$B)$
T	T	T	T	T	T	T	T	T	F	T	F	F	T
T	F	T	F	F	T	F	F	F	F	T	F	T	F
F	T	F	F	T	F	F	T	F	T	F	F	F	T
F	F	F	T	F	F	F	F	T	T	F	T	T	F

- Can you prove the following equivalences with simultaneous truth-tables?
  - ${\sim}(A \bullet B) \approx {\sim}A \vee {\sim}B$
  - ${\sim}(A \vee B) \approx {\sim}A \bullet {\sim}B$
  - $A \approx (A \bullet B) \vee (A \bullet {\sim}B)$
  - $A \approx A \bullet (B \supset B)$
  - $A \approx A \vee (B \bullet {\sim}B)$

Chapter 6 — Propositional Logic: Truth Tables VIII

- What are the logical relationships between "p and q are equivalent", "p and q are consistent", "p and q are contradictory", and "p and q are inconsistent"?

Equivalent	Contradictory
$\Downarrow$? $\Uparrow$	$\Downarrow$? $\Uparrow$
Consistent	Inconsistent

- Answers:
  1. Equivalent $\not\Rightarrow$ Consistent (*example*?)
  2. Consistent $\not\Rightarrow$ Equivalent (*example*?)
  3. Contradictory $\Rightarrow$ Inconsistent (*why*?)
  4. Inconsistent $\not\Rightarrow$ Contradictory (*example*?)

Chapter 6 — Propositional Logic: Truth Tables IX

- Remember, an argument is valid if it is *impossible* for its premises to be true while its conclusion is false. Let $p_1, \ldots, p_n$ be the premises of a PL argument, and let q be the conclusion of the argument. Then, we have the following:

$$\begin{array}{l} p_1 \\ \vdots \\ p_n \\ \hline \therefore q \end{array}$$

is valid if and only if there is no row in the simultaneous truth-table of $p_1, \ldots, p_n$, and q which looks like the following:

atoms	premises			conclusion
$\cdots$	p_1	$\cdots$	p_n	q
$\cdots$	T	T	T	F

- We will use simultaneous truth-tables to prove validities and invalidities. For example, consider the following valid argument: A, $A \supset B$, therefore, B.

$$\begin{array}{l} A \\ A \supset B \\ \hline \therefore B \end{array}$$

atoms		premises				conclusion
A	B	A	A	$\supset$	B	B
T	T	T	T	T	T	T
T	F	T	T	F	F	F
F	T	F	F	T	T	T
F	F	F	F	T	F	F

VALID — since there is no row in which A and $A \supset B$ are both T, but B is F.

- In general, we'll use the following procedure for evaluating arguments:
  1. Translate and symbolize the the argument (if given in English).
  2. Write out the symbolized argument (as above).
  3. Draw a simultaneous truth-table for the symbolized argument, outlining the columns representing the premises and conclusion.
  4. Look for a row of the table in which all of the premises are true and the conclusion is false. If there is one, the argument is invalid; if not, it's valid.
- Let's practice this procedure on some examples …

Chapter 6 — Propositional Logic: Truth Tables X

- Examples (from exercise set 6.4.I) — Use LogicCoach!

2. Brazil has a huge foreign debt. Therefore, either Brazil or Argentina has a huge foreign debt.
3. If fossil fuel combustion continues at its present rate, then a greenhouse effect occurs. If a greenhouse effect occurs, then world temperatures will rise. Therefore, if fossil fuel combustion continues at its present rate, then world temperatures will rise.
7. Einstein won the Noble Prize either for explaining the photoelectric effect or for the special theory of relativity. But he did win the Noble Prize for explaining the photoelectric effect. Therefore, he did not win the Noble Prize for the special theory of relativity.
9. Either the USS Arizona or the USS Missouri was not sunk in the attack on Pearl Harbor. Therefore, it is not the case that either the USS Arizona or the USS Missouri was sunk in the attack on Pearl Harbor.

Chapter 6 — Propositional Logic: Truth Tables XI

- The corresponding conditional of an argument has as its antecedent the conjunction of the premises, and as its consequent the conclusion.

Argument	Corresponding Conditional of Argument
A $A \supset B$ $\therefore B$	$(A \bullet (A \supset B)) \supset B$
$A \supset B$ $B \supset C$ $\therefore A \supset C$	$[(A \supset B) \bullet (B \supset C)] \supset (A \supset C)$
$A \vee B$ $\sim A$ $\therefore B$	$[(A \vee B) \bullet \sim A] \supset B$

- An argument is valid if and only if its corresponding conditional is a tautology. Why? Think about the truth-functional definition of $\supset$.

Chapter 6 — Propositional Logic: Truth Tables XII

- There is a "short-cut" or "indirect" method for doing simultaneous truth-tables of arguments in propositional logic. This method can save time in some cases.
- Instead of constructing the entire truth-table, you can simply try to construct a row of the table in which the premises are true and the conclusion is false.
- If you can construct such a row, then the argument is invalid. If constructing such a row is impossible, then the argument is valid.
- Example: consider the argument from B and $A \supset B$ to A. Let's try to construct a row in which B and $A \supset B$ are both true, but A is false.

A	B	B	A	$\supset$	B	A
F	T	T	F	T	T	F

- Since we succeeded in constructing such a row, the argument is invalid.
- But, what happens when we try this on a *valid* argument? The answer is that we *cannot* consistently construct such a row. Let's try this . . .