

Philosophy 57 — Day 24

- Quiz #6 to be Returned Next Week
- Next Week, I will also post the “Curve for the course to this point”
 - This will tell you where you need to be on the final, etc.
- [Extra-Credit Problems Posted on Website](#) (5 problems, each worth 1 point!)
 - Extra-Credit Problems are **due by Tuesday 05/20/03**
 - No partial credit within problems (but you can do fewer than 5 problems)
 - You may use any tools/references you like to do these (but **individually!**)
 - Stay Tuned for Hints, etc. [these are all “chapter 6” problems]
- Back to Chapter 6
 - Definitions of Truth-Functional Connectives
 - Truth-Tables for Claims
 - Truth-Tables for Arguments



Chapter 6 — Propositional Logic: Truth Functions – Review

- Negation (just like English “not”), and Conjunction (just like English “and”):

p	$\sim p$	p	q	$p \bullet q$
T	F	T	T	T
T	F	T	F	F
F	T	F	T	F
F	T	F	F	F

- Disjunction is *similar* to English “or”, but *not* in the “exclusive” sense:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

- But, we can express the English exclusive “A or B, but not both”, as:

$$(A \vee B) \bullet \sim(A \bullet B)$$

- So, “ \sim ”, “ \bullet ”, and “ \vee ” do seem to match English usage for “not”, “and”, “or”.



Chapter 6 — Propositional Logic: Truth Functions – \supset

p	q	$p \supset q$
T	T	T
T	F	F
F	T	T
F	F	T

- The truth-functional definition of \supset is farther from the English “only if”. A PL conditional is false iff its antecedent is true and its consequent is false.
- In English, conditionals can be false, even if their antecedents are false. Moreover, English conditionals can be false even if their consequents are true.
 - If New York is in New Zealand, then $2 + 2 = 4$.
 - If New York is in the U.S.A., then WWII ended in 1945.
 - If WWII ended in 1941, then gold is an acid.
- So, \supset does *not* capture the English “if”. We’ll see later that $p \supset q \approx \sim p \vee q$.
- But, I will explain later why this is the *only* acceptable *truth-functional* choice.



Chapter 6 — Propositional Logic: Truth Functions – \equiv

p	q	$p \equiv q$
T	T	T
T	F	F
F	T	F
F	F	T

- The truth-functional definition of \equiv is far from the English “if and only if”. A PL biconditional is true iff both of its components have the same truth value.
- Consider these two biconditionals. [M = the moon is made of green cheese, U = there are unicorns, E = life exists on Earth, and S = the sky is blue]
 - The moon is made of green cheese if and only if there are unicorns.
 - Life exists on earth if and only if the sky is blue.
- The PL translations of these sentences are both true. $M \equiv U$ is true because M and U are false. $E \equiv S$ is true because E and S are true. This does *not* capture the English “if and only if”. We’ll see that $p \equiv q \approx (p \bullet q) \vee (\sim p \bullet \sim q)$.



Chapter 6 — Propositional Logic: Truth Tables I

- With the truth-table definitions of the five connectives in hand, we can now construct truth tables for arbitrary compound PL statements.
- The procedure for constructing the truth-table of p is as follows:
 1. Determine the number of rows in the truth-table. This is 2^n , where n is the number of atomic sentences in the compound statement p .
 2. The table will have $n + 1$ main columns: n columns for the atomic sentences in p , and one for the truth-values of p itself.
 3. The table will also have some “quasi-columns” — one for each PL statement occurring in the compound p — which needn’t be drawn explicitly, but which will go into the determination of the truth values of p .
 4. Place the atomic symbols in the left most columns, going in alphabetical order from left to right. And place p in the right most column.
 5. Write in all possible combinations of truth-values for the atomic statements. There will be 2^n of these — one for each row of the table.



6. The convention here is to start on the n th column (farthest down the alphabet) with the pattern TFTF ... repeated until the column is filled. Then, go TTFF ... in the $n - 1$ st column. And, TTTTFFFF ... in the $n - 2$ nd column, etc. ..., until the very first column has been completed.
7. Next, we need to compute the truth-values of p in each row of the table. Here, we start from the inside-out. We first copy the truth-values of the atoms, then we compute the negations, conjunctions, etc. which compose p . Finally, we will be in a position to compute the value of the main connective of p , at which point we will be done with p 's truth table.

- Example: Step-By-Step Truth-Table Construction of “ $A \equiv (B \bullet A)$.”

A	B	$A \equiv (B \bullet A)$
T	T	T
T	F	F
F	T	F
F	F	F



Chapter 6 — Propositional Logic: Truth Tables II

- A statement is said to be **logically true** (or **tautologous**) if it is true regardless of the truth-values of its components. Example: $p \equiv p$ is logically true.

p	$p \equiv p$
T	T
F	F

- A statement is **logically false** (or **self-contradictory**) if it is false regardless of the truth-values of its components. Example: $p \bullet \sim p$ is logically false.

p	$p \bullet \sim p$
T	F
F	F

- A statement is **contingent** if its truth-value varies depending on the truth-values of its components. Example: A (or *any* atom) is contingent.

A	A
T	T
F	F



Chapter 6 — Propositional Logic: Truth Tables III

- Classify the following statements as logically true (tautologous), logically false (self-contradictory), or contingent (exercise 6.3.I):

1. $N \supset (N \supset N)$
2. $(G \supset G) \supset G$
3. $(S \supset R) \bullet (S \bullet \sim R)$
4. $((E \supset F) \supset F) \supset E$
6. $(M \supset P) \vee (P \supset M)$
11. $[(Q \supset P) \bullet (\sim Q \supset R)] \bullet \sim(P \vee R)$
12. $[(H \supset N) \bullet (T \supset N)] \supset [(H \vee T) \supset N]$
15. $[(F \vee E) \bullet (G \vee H)] \equiv [(G \bullet E) \vee (F \bullet H)]$



Chapter 6 — Propositional Logic: Truth Tables IV

- Here is a completed truth-table for #11, $[(Q \supset P) \bullet (\sim Q \supset R)] \bullet \sim(P \vee R)$:

P	Q	R	$[(Q \supset P) \bullet (\sim Q \supset R)] \bullet \sim(P \vee R)$												
T	T	T	T	T	T	T	F	T	T	T	F	F	T	T	T
T	T	F	T	T	T	T	F	T	T	F	F	F	T	T	F
T	F	T	F	T	T	T	T	F	T	T	F	F	T	T	T
T	F	F	F	T	T	F	T	F	F	F	F	F	T	T	F
F	T	T	T	F	F	F	F	T	T	T	F	F	F	T	T
F	T	F	T	F	F	F	F	T	T	F	F	T	F	F	F
F	F	T	F	T	F	T	T	F	T	T	F	F	F	T	T
F	F	F	F	T	F	F	T	F	F	F	F	T	F	F	F

- Therefore, the statement “ $[(Q \supset P) \bullet (\sim Q \supset R)] \bullet \sim(P \vee R)$ ” is *logically false*.

Chapter 6 — Propositional Logic: Truth Tables V

- Two statements are said to be **equivalent** (written $p \approx q$) if they have the same truth-value in all possible worlds (*i.e.*, in all rows of a simultaneous truth-table of both statements). For instance, $A \supset B \approx \sim A \vee B$:

A	B	$A \supset B$	$\sim A \vee B$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

- Two statements are said to be **contradictory** if they have opposite truth-values in all possible worlds (*i.e.*, in all rows of a simultaneous truth-table of both statements). For instance, A and $\sim A$ are contradictory:

A	A	$\sim A$
T	T	F
F	F	T

- Two statements are **inconsistent** if they are never both true in any possible world (*i.e.*, in any row of a simultaneous truth-table of both statements). For instance, $A \equiv B$ and $A \bullet \sim B$ are inconsistent (but *not* contradictory!):

A	B	$A \equiv B$	$A \bullet \sim B$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	T	F

- Two statements are **consistent** if they are both true in at least one possible world (*i.e.*, in at least one row of a simultaneous truth-table of both statements). For instance, $A \bullet B$ and $A \vee B$ are consistent:

A	B	$A \bullet B$	$A \vee B$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

Chapter 6 — Propositional Logic: Truth Tables VI

- Use truth-tables to determine whether the following pairs of statements are logically equivalent, contradictory, consistent, or inconsistent (exercise 6.3.II).

- $F \bullet M$ and $\sim(F \vee M)$
- $R \vee \sim S$ and $S \bullet \sim R$
- $H \equiv \sim G$ and $(G \bullet H) \vee (\sim G \bullet \sim H)$
- $N \bullet (A \vee \sim E)$ and $\sim A \bullet (E \vee \sim N)$
- $W \equiv (B \bullet T)$ and $W \bullet (T \supset \sim B)$
- $R \bullet (Q \vee S)$ and $(S \vee R) \bullet (Q \vee R)$
- $Z \bullet (C \equiv P)$ and $C \equiv (Z \bullet \sim P)$
- $Q \supset \sim(K \vee F)$ and $(K \bullet Q) \vee (F \bullet Q)$

Chapter 6 — Propositional Logic: Truth Tables VII

- Here is a simultaneous truth-table which establishes that

$$A \equiv B \approx (A \bullet B) \vee (\sim A \bullet \sim B)$$

A	B	A	≡	B	(A	•	B)	∨	(∼	A	•	∼	B)
T	T	T	T	T	T	T	T	T	F	T	F	F	T
T	F	T	F	F	T	F	F	F	F	T	F	T	F
F	T	F	F	T	F	F	T	F	T	F	F	F	T
F	F	F	T	F	F	F	F	T	T	F	T	T	F

- Can you prove the following equivalences with simultaneous truth-tables?
 - $\sim(A \bullet B) \approx \sim A \vee \sim B$
 - $\sim(A \vee B) \approx \sim A \bullet \sim B$
 - $A \approx (A \bullet B) \vee (A \bullet \sim B)$
 - $A \approx A \bullet (B \supset B)$
 - $A \approx A \vee (B \bullet \sim B)$

