

Philosophy 57 — Day 23

- Quiz #5 to Returned Today
 - “Curve”: 92–100 (A); 75–91 (B); 62–74 (C); 50–61 (D); < 50 (F)
- Quiz #6 Next Tuesday (On §6.1 of Text: Translation & Syntax of PL)
- Extra-Credit Problems to be Posted Soon on Website
 - Five questions from chapter 6
 - Will be due on (or soon after) the final exam date
- Back to Chapter 6 — Remaining Material
 - Review on the translation of conditionals
 - Truth-Functions and Truth Conditions for PL Statements
 - Truth-Tables for Arbitrary PL Sentences (§6.3)
 - Truth-Tables for Arbitrary PL Arguments (§6.4)

Chapter 6 — Propositional Logic Translations (Conditionals)

- The following are eight ways of asserting the same conditional statement (in quasi-English). All of these get translated into PL as “ $p \supset q$ ”.

Quasi-English	PL
If p , then q .	$p \supset q$
q if p .	$p \supset q$
p only if q .	$p \supset q$
q provided that p .	$p \supset q$
q on condition that p .	$p \supset q$
p implies that q .	$p \supset q$
p is a sufficient condition for q .	$p \supset q$
q is a necessary condition for p .	$p \supset q$

Chapter 6 — Propositional Logic: Truth Functions I

- Propositional Logic is **truth-functional** because the truth value of a compound statement is a function of the truth values of its atomic components.
- We use lower-case letters “ p ”, “ q ”, “ r ”, ... to denote **statement variables**, which can stand for any statement in propositional logic.
- A **statement form** is an expression (*not* a statement of PL!) constructed out of statement variables and PL connectives which becomes a statement of PL if (simple) statements of PL are substituted for all statement variables.
 - e.g., $p \bullet (q \vee r)$ is a statement form, since $A \bullet (B \vee C)$ is a statement.
 - Note: $(A \vee B) \bullet ((C \equiv D) \vee (E \supset \sim F))$ is *also* of the form $p \bullet (q \vee r)$. Why?
- With this basic terminology out of the way, we’re ready to give a precise account of the truth conditions (*i.e.*, the meaning) of PL statements.
- All statement forms are defined by **truth tables**, which tell us how to determine the truth value of molecular statements from the truth values of their atoms.

Chapter 6 — Propositional Logic: Truth Functions II

- We begin with negations, which have the simplest truth functions. The truth table for negation is as follows (we use T and F for true and false):

p	$\sim p$
T	F
F	T

- In words, this table says that if p is true than $\sim p$ is false, and if p is false, then $\sim p$ is true. This is quite intuitive, and corresponds well to the English meaning of “not”. So, truth-functional (PL) negation is like English negation.
- Examples:
 - It is not the case that Wagner wrote operas. ($\sim W$)
 - It is not the case that Picasso wrote operas. ($\sim P$)
- “ $\sim W$ ” is false, since “ W ” is true, and “ $\sim P$ ” is true, since “ P ” is false (like English).

Chapter 6 — Propositional Logic: Truth Functions III

p	q	$p \bullet q$
T	T	T
T	F	F
F	T	F
F	F	F

- Notice how we have four (4) rows in our truth table this time (not 2). This is because there are four possible ways of assigning truth values to p and q .
- The truth-functional definition of \bullet is very close to the English “and”. A PL conjunction is true if *both* conjuncts are true; and, it is false otherwise.
 - Monet and van Gogh were painters. ($M \bullet V$)
 - Monet and Beethoven were painters. ($M \bullet B$)
 - Beethoven and Einstein were painters. ($B \bullet E$)
- “ $M \bullet V$ ” is true, since both “ M ” and “ V ” are true. “ $M \bullet B$ ” is false, since “ B ” is false. And, “ $B \bullet E$ ” is false, since “ B ” and “ E ” are both false (like English).



Chapter 6 — Propositional Logic: Truth Functions IV

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

- The truth-functional definition of \vee is not as close to the English “or”. A PL disjunction is true if *at least one* disjunct is true; and, it is false otherwise.
- In English, “ A or B ” often implies that “ A ” and “ B ” are *not both true*. That is called *exclusive or*. In PL, “ $A \vee B$ ” is *not exclusive*; it is *inclusive* (it is true if both disjuncts are true). But, we *can* express exclusive or in PL. How?
 - Either Jane austen or René Descartes was novelist. ($J \vee R$)
 - Either Jane Austen or Charlotte Bronte was a novelist. ($J \vee C$)
 - Either René Descartes or David Hume was a novelist. ($R \vee D$)
- The first two disjunctions are true because at least one their disjuncts is true, but the third disjunction is false, since both of its disjuncts are false.



Chapter 6 — Propositional Logic: Truth Functions V

p	q	$p \supset q$
T	T	T
T	F	F
F	T	T
F	F	T

- The truth-functional definition of \supset is farther from the English “only if”. A PL conditional is false iff its antecedent is true *and* its consequent is false.
- Consider the following English conditionals. [Let M = the moon is made of green cheese, O = life exists on other planets, and E = life exists on Earth]
 - If the moon is made of green cheese, then life exists on other planets.
 - If life exists on other planets, then life exists on earth.
- The PL translations of these sentences are both true. $M \supset O$ is true because its antecedent M is false. $O \supset E$ is true because its consequent E is true. This does *not* capture the English “if”. We’ll see later that $p \supset q \approx \sim p \vee q$.



Chapter 6 — Propositional Logic: Truth Functions VI

p	q	$p \equiv q$
T	T	T
T	F	F
F	T	F
F	F	T

- The truth-functional definition of \equiv is far from the English “if and only if”. A PL biconditional is true iff both of its components have the same truth value.
- Consider these two biconditionals. [M = the moon is made of green cheese, U = there are unicorns, E = life exists on Earth, and S = the sky is blue]
 - The moon is made of green cheese if and only if there are unicorns.
 - Life exists on earth if and only if the sky is blue.
- The PL translations of these sentences are both true. $M \equiv U$ is true because M and U are false. $E \equiv S$ is true because E and S are true. This does *not* capture the English “if and only if”. We’ll see that $p \equiv q \approx (p \bullet q) \vee (\sim p \bullet \sim q)$.



Chapter 6 — Propositional Logic: Truth Functions VII

- With the truth-table definitions of the five connectives in hand, we can now construct truth tables for arbitrary compound PL statements.
- The procedure for constructing the truth-table of p is as follows:
 1. Determine the number of rows in the truth-table. This is 2^n , where n is the number of atomic sentences in the compound statement p .
 2. The table will have $n + 1$ main columns: n columns for the atomic sentences in p , and one for the truth-values of p itself.
 3. The table will also have some “quasi-columns” — one for each PL statement occurring in the compound p — which needn’t be drawn explicitly, but which will go into the determination of the truth values of p .
 4. Place the atomic symbols in the left most columns, going in alphabetical order from left to right. And place p in the right most column.
 5. Write in all possible combinations of truth-values for the atomic statements. There will be 2^n of these — one for each row of the table.



6. The convention here is to start on the n th column (farthest down the alphabet) with the pattern TFTF ... repeated until the column is filled. Then, go TTFF ... in the $n - 1$ st column. And, TTTFFFFF ... in the $n - 2$ nd column, etc. ..., until the very first column has been completed.
7. Next, we need to compute the truth-values of p in each row of the table. Here, we start from the inside-out. We first copy the truth-values of the atoms, then we compute the negations, conjunctions, etc. which compose p . Finally, we will be in a position to compute the value of the main connective of p , at which point we will be done with p 's truth table.

- Example: Step-By-Step Truth-Table Construction of “ $A \equiv (B \bullet A)$.”

A	B	$A \equiv (B \bullet A)$
T	T	T
T	F	F
F	T	T
F	F	F

