Philosophy 57 — Day 23

- Quiz #5 to Returned Today
  - "Curve": 92–100 (A); 75–91 (B); 62–74 (C); 50–61 (D); < 50 (F)
- Quiz #6 Next Tuesday (On §6.1 of Text: Translation & Syntax of PL)
- Extra-Credit Problems to be Posted Soon on Website
  - Five questions from chapter 6
  - Will be due on (or soon after) the final exam date
- Back to Chapter 6 — Remaining Material
  - Review on the translation of conditionals
  - Truth-Functions and Truth Conditions for PL Statements
  - Truth-Tables for Arbitrary PL Sentences (§6.3)
  - Truth-Tables for Arbitrary PL Arguments (§6.4)

Chapter 6 — Propositional Logic Translations (Conditionals)

- The following are eight ways of asserting the same conditional statement (in quasi-English). All of these get translated into PL as "$p \supset q$".

Quasi-English	PL
If p, then q.	$p \supset q$
q if p.	$p \supset q$
p only if q.	$p \supset q$
q provided that p.	$p \supset q$
q on condition that p.	$p \supset q$
p implies that q.	$p \supset q$
p is a sufficient condition for q.	$p \supset q$
q is a necessary condition for p.	$p \supset q$

Chapter 6 — Propositional Logic: Truth Functions I

- Propositional Logic is truth-functional because the truth value of a compound statement is a function of the truth values of its atomic components.
- We use lower-case letters "p", "q", "r", ... to denote statement variables, which can stand for any statement in propositional logic.
- A statement form is an expression (*not* a statement of PL!) constructed out of statement variables and PL connectives which becomes a statement of PL if (simple) statements of PL are substituted for all statement variables.
  - *e.g.*, $p \bullet (q \vee r)$ is a statement form, since $A \bullet (B \vee C)$ is a statement.
  - Note: $(A \vee B) \bullet ((C \equiv D) \vee (E \supset \sim F))$ is *also* of the form $p \bullet (q \vee r)$. Why?
- With this basic terminology out of the way, we're ready to give a precise account of the truth conditions (*i.e.*, the meaning) of PL statements.
- All statement forms are defined by truth tables, which tell us how to determine the truth value of molecular statements from the truth values of their atoms.

Chapter 6 — Propositional Logic: Truth Functions II

- We begin with negations, which have the simplest truth functions. The truth table for negation is as follows (we use T and F for true and false):

p	$\sim p$
T	F
F	T

- In words, this table says that if p is true than $\sim p$ is false, and if p is false, then $\sim p$ is true. This is quite intuitive, and corresponds well to the English meaning of "not". So, truth-functional (PL) negation is like English negation.
- Examples:
  - It is not the case that Wagner wrote operas. ($\sim W$)
  - It is not the case that Picasso wrote operas. ($\sim P$)
- "$\sim W$" is false, since "W" is true, and "$\sim P$" is true, since "P" is false (like English).

Chapter 6 — Propositional Logic: Truth Functions III

p	q	$p \bullet q$
T	T	T
T	F	F
F	T	F
F	F	F

- Notice how we have four (4) rows in our truth table this time (not 2). This is because there are four possible ways of assigning truth values to p and q.
- The truth-functional definition of $\bullet$ is very close to the English “and”. A PL conjunction is true if *both* conjuncts are true; and, it is false otherwise.
  - Monet and van Gogh were painters. ($M \bullet V$)
  - Monet and Beethoven were painters. ($M \bullet B$)
  - Beethoven and Einstein were painters. ($B \bullet E$)
- “$M \bullet V$” is true, since both “M” and “V” are true. “$M \bullet B$” is false, since “B” is false. And, “$B \bullet E$” is false, since “B” and “E” are both false (like English).

Chapter 6 — Propositional Logic: Truth Functions IV

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

- The truth-functional definition of $\vee$ is not as close to the English "or". A PL disjunction is true if *at least one* disjunct is true; and, it is false otherwise.
- In English, "*A* or *B*" often implies that "*A*" and "*B*" are *not both true*. That is called *exclusive* or. In PL, "$A \vee B$" is *not* exclusive; it is *inclusive* (it is true if both disjuncts are true). But, we *can* express exclusive or in PL. How?
  - Either Jane austen or René Descartes was novelist. ($J \vee R$)
  - Either Jane Austen or Charlotte Bronte was a novelist. ($J \vee C$)
  - Either René Descartes or David Hume was a novelist. ($R \vee D$)
- The first two disjunctions are true because at least one their disjuncts is true, but the third disjunction is false, since both of its disjuncts are false.

Chapter 6 — Propositional Logic: Truth Functions V

p	q	$p \supset q$
T	T	T
T	F	F
F	T	T
F	F	T

- The truth-functional definition of $\supset$ is farther from the English "only if". A PL conditional is false iff its antecedent is true and its consequent is false.
- Consider the following English conditionals. [Let M = the moon is made of green cheese, O = life exists on other planets, and E = life exists on Earth]
  - If the moon is made of green cheese, then life exists on other planets.
  - If life exists on other planets, then life exists on earth.
- The PL translations of these sentences are both true. $M \supset O$ is true because its antecedent M is false. $O \supset E$ is true because its consequent E is true. This does *not* capture the English "if". We'll see later that $p \supset q \approx \sim p \vee q$.

Chapter 6 — Propositional Logic: Truth Functions VI

p	q	$p \equiv q$
T	T	T
T	F	F
F	T	F
F	F	T

- The truth-functional definition of $\equiv$ is far from the English "if and only if". A PL biconditional is true iff both of its components have the same truth value.
- Consider these two biconditionals. [M = the moon is made of green cheese, U = there are unicorns, E = life exists on Earth, and S = the sky is blue]
  - The moon is made of green cheese if and only if there are unicorns.
  - Life exists on earth if and only if the sky is blue.
- The PL translations of these sentences are both true. $M \equiv U$ is true because M and U are false. $E \equiv S$ is true because E and S are true. This does *not* capture the English "if and only if". We'll see that $p \equiv q \approx (p \bullet q) \vee (\sim p \bullet \sim q)$.

Chapter 6 — Propositional Logic: Truth Functions VII

- With the truth-table definitions of the five connectives in hand, we can now construct truth tables for arbitrary compound PL statements.
- The procedure for constructing the truth-table of p is as follows:
  1. Determine the number of rows in the truth-table. This is 2^n, where n is the number of atomic sentences in the compound statement p.
  2. The table will have $n + 1$ main columns: n columns for the atomic sentences in p, and one for the truth-values of p itself.
  3. The table will also have some "quasi-columns" — one for each PL statement occurring in the compound p — which needn't be drawn explicitly, but which will go into the determination of the truth values of p.
  4. Place the atomic symbols in the left most columns, going in alphabetical order from left to right. And place p in the right most column.
  5. Write in all possible combinations of truth-values for the atomic statements. There will be 2^n of these — one for each row of the table.

6. The convention here is to start on the *n*th column (farthest down the alphabet) with the pattern TFTF . . . repeated until the column is filled. Then, go TTFF . . . in the $n - 1$st column. And, TTTTFFFF . . . in the $n - 2$nd column, etc. . . , until the very first column has been completed.

7. Next, we need to compute the truth-values of p in each row of the table. Here, we start from the inside-out. We first copy the truth-values of the atoms, then we compute the negations, conjunctions, etc. which compose p. Finally, we will be in a position to compute the value of the main connective of p, at which point we will be done with p's truth table.

- Example: Step-By-Step Truth-Table Construction of "$A \equiv (B \bullet A)$."

A	B	A	$\equiv$	$(B$	$\bullet$	$A)$
T	T	T	T	T	T	T
T	F	T	F	F	F	T
F	T	F	T	T	F	F
F	F	F	T	F	F	F