

Philosophy 57 — Extra-Credit Problems

May 1, 2003 (due Tuesday, May 20, 2003)

1 Truth-Tables & PL Arguments

Use either full or “indirect” truth-tables (as described in §6.5) to determine whether the following PL-arguments are valid. Your answers should include *both* your truth-table reasoning *and* your verdict. If the argument is *invalid*, then all you need to report is a single row of the full truth-table (the one in which the premises are true and the conclusion is false). But, if the argument is *valid*, then you need to report either a *complete* truth-table, or an explanation of validity using the “indirect” truth-table method described in §6.5 of the text.

1.
$$\begin{array}{l} I \supset N \\ (\sim K \vee D) \equiv N \\ D \supset \sim I \\ \hline \therefore \sim I \supset (N \supset K) \end{array}$$
2.
$$\begin{array}{l} (\sim O \supset \sim S) \bullet (O \supset (M \bullet \sim I)) \\ \sim I \supset \sim M \\ \hline \therefore \sim S \end{array}$$

2 Knights, Knaves & Truth-Tables

The island of Knights and Knaves has two types of inhabitants, Knights who always tell the truth, and Knaves who always lie. Suppose A is the proposition person a is a knight and suppose a makes a statement S . Then, A is true if and only if S is true, since A is equivalent to S . That is, $A \approx S$. So, whenever an inhabitant x makes a claim S , we can infer that $X \equiv S$. That is, we can infer that x is a knight if and only if S is true. Here are some examples putting this to use:

If a says “I am a Knight” then we can infer from the statement that $A \equiv A$. But, since this is always true (it’s a *tautology*), we get no information from such a statement. Similarly, it cannot be the case that a native says “I am a Knave” because we could then conclude $A \equiv \sim A$, which is always false (it’s self-contradictory). If a says “I am the same type as b ,” then we can infer $A \equiv (A \equiv B)$ which is equivalent to B (that is, $B \approx A \equiv (A \equiv B)$, which you can prove using truth-tables). So, this statement allows us to infer that person b is a Knight!

Given this set-up, use truth-tables (either complete or “indirect”) to justify your answers to the following three questions about Knights and Knaves. Explain your answers as fully as possible, and use truth-tables to establish the logical relationships (*e.g.*, equivalences) you use.

3. It is rumored that there is gold buried on the island (G). You ask one of the natives, a , whether there is gold on the island. He makes the following response: “There is gold on this island if and only if I am a Knight.” Is there gold buried on the island?
4. Inhabitant a says “Either I am a Knave or b is a Knight.” What can we infer about a and b ?
5. Three of the inhabitants — a , b and c — were standing together in the garden. A stranger passed by and asked a , “Are you a Knight or a Knave?”. a answered, but rather indistinctly, so the stranger could not make out what he said. The stranger then asked b , “What did a say?”. b replied, “ a said that he is a Knave.” At this point the third man, c , said “Don’t believe b ; he’s lying!”. The question is, what are b and c ?