The following questions are drawn from my own readings of (and puzzlings over) chapters 1, 2, 6, and 8 of Skyrms' *Choice and Chance*. We would be well served by thinking carefully and hard about the issues they raise. Any of these questions (or sub-questions) would be appropriate short paper topics. Moreover, these are the sorts of exercises that I will be emphasizing throughout the course. As always, I urge people to come talk to me individually about these or any other questions they might have regarding the readings/lectures.

1. On pages 20–21, Skyrms argues that it would *not* be appropriate to define the inductive strength of an argument $\frac{\mathbf{P}}{\therefore C}$ as the improbability of the *conjunction* of the premises (taken jointly) and the denial of the conclusion (*i.e.*, as the improbability of the *conjunction* '$\sim C\ \&\ \mathbf{P}$'). Instead, he suggests that inductive strength should be defined as the improbability of the denial of the conclusion *given* (the conjunction of) the premises (*i.e.*, as the improbability of '$\sim C$ *given* $\mathbf{P}$').[1] His argument involves two examples which are intended to illustrate the differences between these two proposals, and to make the former reading of inductive strength seem less appropriate or correct than the latter. In essence, Skyrms' examples aim to show that the former definition of inductive strength lacks a crucial kind of 'relevance relation' between the premises and conclusion — a relation that is (presumably) captured more accurately by the latter definition. A careful analysis of this argument would be very fruitful. In particular, such an analysis should (*i*) try to pinpoint the *precise kind* of evidential 'relevance relation' that Skyrms is trying to capture, (*ii*) apply the spirit of Skyrms' criticism to the definition of *deductive validity* that Skyrms discusses with approval in chapter 1 (that is, ask why Skyrms-type reasoning wouldn't also motivate a *rejection* of the classical definition of *deductive validity* — this question has been raised forcefully by proponents of non-classical logics such as *relevance logic*), and (*iii*) assess the accuracy of the latter definition as well (*i.e.*, does the latter definition also miss some intuitive aspects of 'relevance' that might be important for inductive reasoning? We will return to this question later in our discussions of confirmation and explanation. See question 4, below.). To wit: why doesn't Skyrms have similar worries about the classical definition of deductive validity? And, can we improve upon Skyrms' definition of inductive strength (*i.e.*, should we try to make Skyrms' account even more sensitive to inductive or evidential 'relevance')? If so, how? Here, further examples would be useful (Hint: see chapter 8 for intimations from Skyrms himself — see question 4, below, for a very closely related question). The remaining questions may require some non-trivial *formal* probability reasoning:

2. When we carefully reconstruct Skyrms' first kind of 'counterexample' (page 20) to the '$\sim C\ \&\ \mathbf{P}$' definition of inductive strength, we see that it has the following sort of structure. Skyrms presents an argument $\frac{\mathbf{P}}{\therefore C}$ which he claims has the following three properties: (*i*) '$\sim C\ \&\ \mathbf{P}$' is improbable (merely) because $\mathbf{P}$ *alone* is improbable, (*ii*) '$\sim C$ *given* $\mathbf{P}$' is *not* improbable, and (*iii*) intuitively, the argument $\frac{\mathbf{P}}{\therefore C}$ is *not* inductively strong. If correct, this would indeed favor the '$\sim C$ *given* $\mathbf{P}$' account of inductive strength over the '$\sim C\ \&\ \mathbf{P}$' account (*why?*). Do you see any potential weaknesses in Skyrms' argument for (*i*)–(*iii*), or in the *particular* example Skyrms chooses for this purpose? As a hint, it will help you to perform the following *technical* exercise in probability calculus. Try to argue (using standard assumptions about probabilities over well shuffled decks of cards) that the following example has the properties (*i*)–(*iii*), above. A card is drawn at random from a standard deck. C is the proposition that the card is not a face card, and $\mathbf{P}$ is the proposition that the card is a spade. Here, you may assume that 'p is improbable' gets translated into probability calculus as '$\Pr(p) < \frac{1}{2}$' (although similar examples can

[1] Important sidebar: Is '$\sim C$ *given* $\mathbf{P}$' a *proposition* (or, if you prefer, a declarative sentence)? That is, is it accurate to say that '$\sim C$ *given* $\mathbf{P}$' even *has* a truth-value or a probability? If so, try to explain what the *truth-conditions* of '$\sim C$ *given* $\mathbf{P}$' are (*e.g.*, is '$\sim C$ *given* $\mathbf{P}$' some sort of *conditional* sentence, or what?). If not, try to show that '$\sim C$ *given* $\mathbf{P}$' is not a proposition (at least, not in the same sense that '$\sim C\ \&\ \mathbf{P}$' is a proposition). It may be useful here to perform a parallel analysis to the deductive case (as discussed above). It also may be useful to think about question 3, below, before delving into this question.

be generated for any threshold value you like). It is easy to show (i) and (ii) obtain for this example (these are just simple probability calculations). Showing that (iii) obtains will require some ingenuity. For (iii), you must argue that — *intuitively* — the argument from $\mathbf{P}$ to C is *not* inductively strong (for reasons of 'irrelevance' of some kind or other). What kind of 'irrelevance' between $\mathbf{P}$ and C can you establish, probabilistically (Hint: think about question 3, below, before making up your mind)?

An analogous reconstruction and analysis of Skyrms' second 'counterexample' would also be valuable.

3. In chapter 6, Skyrms makes several assumptions and claims about the relationship between conditional probability and stochastic (*i.e.*, probabilistic) independence. In particular, he (Definition 12 on page 119) defines conditional probability in terms of unconditional probability (Hint: with this in mind, look back at footnote 1, above). This (footnote 3 on page 119) has the consequence that whenever $\Pr(p) = 0$, the conditional probability $\Pr(q \textit{ given } p)$ is undefined. Does this make sense? In particular, do you have clear intuitions about what value $\Pr(p \textit{ given } p)$ should take (*even if* $\Pr(p) = 0$)? Related exercise: prove Skyrms' claim (footnote 4 on page 119) that stochastic independence is neither necessary nor sufficient for mutual (deductive) logical independence. First, *precisely* state Skyrms' definition of stochastic independence (keep in mind Skyrms' caveat on page 121!). Find concrete probability models which refute both directions of the biconditional "stochastic independence $\Leftrightarrow$ logical independence." The Venn diagram techniques (discussed in class) for specifying probability models may be useful here. Finally, do you think it makes sense that stochastic independence $\not\Rightarrow$ logical independence? (Hint: isn't logical dependence, intuitively, a *stronger* kind of dependence between propositions?)

4. In chapter 8, Skyrms talks about "corroboration." Can you give a *precise* (formal) definition of corroboration (using the probability calculus), in the sense Skyrms has in mind here? What is the logical relationship (if any) between the following two relations: (i) "the argument from p to q is inductively strong" and (ii) "p corroborates q." If they are not equivalent (wink-wink), then provide concrete probability models to demonstrate their non-equivalence. What might this imply about Skyrms' definition of inductive strength? That is, why *isn't* "corroboration" captured by "inductive strength" (or *vice versa*)? Moreover, does this indicate that Skyrms' definition of inductive strength may be lacking a certain kind of evidential 'relevance relation' (Hint: this relates, *directly*, back to question 1, above). Finally, note also that in chapter 8 (page 153) Skyrms talks about "varied evidence" being "*better* evidence." What, *precisely*, does Skyrms mean by "*better* evidence" here? And, what is the logical relationship (if any) between "the argument from p to q is stronger than the argument from r to s" and "p is better evidence for q than r is for s"? That is, can Skyrms' notion of inductive strength be used to fully capture his (later) notion of "*better* evidence"? This, too, relates directly back to question 1, above, and to the question concerning the relationship between inductive strength and corroboration. You should see a unifying theme in these questions (and in question 1, above). What is this theme? That is, can you be precise about what might be missing from Skyrms' definition of inductive strength? If so, can you give examples to clearly illustrate what is missing? We will return to this question later in more depth, when we discuss contemporary Bayesian confirmation theory.[2]

[2] Skyrms (page 152) not only talks about corroboration as a (qualitative) *relation* between propositions, he even talks about (quantitative) *degrees* of corroboration. Furthermore, Skyrms seems to suggest that we should measure the degree to which p corroborates q in a *particular way*. What measure of degree of corroboration does Skyrms suggest here? What alternatives might there be? Can you think of any reasons to prefer one way of measuring corroboration *versus* another? Might a change in measure of corroboration undermine the cogency of Skyrms' argument on page 152? We will return to this issue when we get to our discussion of contemporary Bayesian confirmation theory (basically, this issue was the subject of my dissertation).