

## Basic Analytical Tools & Framework

- Administrative: new room, new students (cards)?, pictures?, please consult website (or see me) for syllabus, paper topics, grading, etc.
- Claims, propositions, and arguments (Skyrms ch. 1)
- The deductive support relation between claims (Skyrms ch. 1)
- Deductively valid arguments (Skyrms ch. 1)
- The inductive support relation between claims (Skyrms ch. 2)
- Inductively strong arguments (Skyrms ch. 2)
- The Probability Calculus (Skyrms ch. 6)
- Venn diagrams — a unified formal framework (not in readings)
- Time Permitting: Skyrms' Chapter 8 (applying of inductive logic)

## Claims (Propositions) — The Basic Building Blocks

- A claim (or proposition) is that which is expressed by a (sufficiently precise) declarative sentence (*e.g.*, 'The earth is flat.').
- Claims are either true or false, but not both.
- We will use lower-case italic letters (possibly with integer subscripts) '*p*', '*q*', '*p*<sub>1</sub>', '*p*<sub>2</sub>' ... to denote (basic) claims.
- The connectives '&', '∨', '∼' will be used to denote the *truth-functional* (Boolean) relations 'and', 'or', 'not'.
- We have the following Boolean truth-conditions for complex claims:
  - ' $p \& q$ ' is true iff both *p* and *q* are true.
  - ' $p \vee q$ ' is true iff either *p* or *q* (or both) are true.
  - ' $\sim p$ ' is true iff *p* is false.

## The Deductive Support Relation Between Claims

- The deductive support (*viz.*, entailment) relation is a qualitative (*i.e.*, yes/no – no degrees) relation between claims or propositions.
- *p* deductively supports *q* iff
  - every possible world in which *p* is true is a world in which *q* is true.
  - there is no possible world in which *p* is true and *q* is false.
  - the conjunction ' $\sim q \& p$ ' is *impossible* (in the strongest sense).
- ' $p$  deductively supports  $q$ ', ' $p$  entails  $q$ ', ' $p \models q$ ' are synonymous.
- The deductive entailment relation  $\models$  has several key properties:
  - $p \models p$  (reflexivity)
  - If  $p \models q$  and  $q \models r$ , then  $p \models r$  (transitivity)
  - If  $p \models q$ , then  $p \& r \models q$  (monotonicity)

## Deductively Valid Arguments

- An *argument*  $\mathcal{A}$  is a set of claims, one of which (*q*) is the conclusion
 
$$(\mathcal{A}) \quad \frac{p_1 \& \dots \& p_n}{\therefore q} \quad \left[ \text{abbreviated as } \frac{\mathbf{P}}{\therefore q} \right]$$
 the rest ( $p_1 \dots p_n$ ) are premises (conjunction of  $p_i$  abbreviated  $\mathbf{P}$ ).
- $\mathcal{A}$  is *deductively valid* iff  $\mathbf{P}$  deductively supports *q* (*i.e.*, iff  $\mathbf{P} \models q$ ).
- Deductive validity inherits properties of the  $\models$  relation:
  - $\frac{p}{\therefore p}$  is valid (reflexivity).
  - If  $\frac{p}{\therefore q}$  is valid and  $\frac{q}{\therefore r}$  is valid, then  $\frac{p}{\therefore r}$  is valid (transitivity).
  - If  $\frac{\mathbf{P}}{\therefore q}$  is valid, then  $\frac{\mathbf{P} \& r}{\therefore q}$  is valid (monotonicity).
- Well-known valid forms (*modus ponens*, disjunctive syllogism, *etc.*).

## The Relation of Inductive Support (informal)

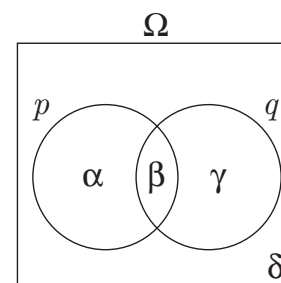
- The inductive support relation is a *quantitative* (i.e., a relation which admits of *degrees*) between claims or propositions.
- $p$  inductively supports  $q$  (on Skyrms' definition) iff
  - 'Most' possible worlds in which  $p$  is true are worlds in which  $q$  is true.
  - It is improbable that  $q$  is false, *given* that  $p$  is true.
  - **NOT** the same as: the conjunction  $\lceil \sim q \ \& \ p \rceil$  is *improbable*.
  - $\therefore$  **NOT** (strictly) analogous to definition of deductive support.<sup>a</sup>
- Skyrms presents two examples to illustrate the differences between:
  - $\lceil \sim q \ \& \ p \rceil$  is improbable, and
  - “ $\sim q$  *given*  $p$  is improbable”

[This is the subject of two paper topics — we shall return to this!]

<sup>a</sup>But, deductive entailment is a “limiting case” of inductive strength. *Why?*

## Digression: The Probability Calculus I

- We can't really be precise about the relation of inductive support without using (formal) *probability calculus* (i.e., Ch. 6 of Skyrms).
- Probabilities (inductive ones) are functions from claims onto the real unit interval. These functions can be thought of (roughly!) as measures of *proportions of possible worlds in which claims are true*.
- It helps to use *Venn diagrams* to picture probabilities:



- The circles represent the sets of possible worlds in which the claims  $p$  and  $q$  are true.
- The *reference class*  $\Omega$  is the set of possible worlds with respect to which  $\text{Pr}$  is defined.
- $\text{Pr}(\Omega) = 1$  (*total area* of the r.c. 'box' is 1)
- $\text{Pr}(p \ \& \ \sim q) = \alpha$ ,  $\text{Pr}(p \ \& \ q) = \beta$ ,  $\text{Pr}(\sim p \ \& \ q) = \gamma$
- $\text{Pr}(\sim p \ \& \ \sim q) = \delta = 1 - (\alpha + \beta + \gamma)$

## The Probability Calculus II

- Thinking of probabilities as (normalized) *areas* in Venn diagrams of this kind *automatically* gives us Skyrms' first 6 rules for  $\text{Pr}(\cdot)$ :
  - If  $p$  is a logical truth, then  $\text{Pr}(p) = 1$ .
  - If  $p$  is a logical falsehood, then  $\text{Pr}(p) = 0$ .
  - If  $p$  and  $q$  are mutually exclusive (i.e., if there are no possible worlds in which  $\lceil p \ \& \ q \rceil$  is true), then  $\text{Pr}(p \vee q) = \text{Pr}(p) + \text{Pr}(q)$ .
  - If  $p$  and  $q$  are logically equivalent, then  $\text{Pr}(p) = \text{Pr}(q)$ .
  - $\text{Pr}(\sim p) = 1 - \text{Pr}(p)$
  - $\text{Pr}(p \vee q) = \text{Pr}(p) + \text{Pr}(q) - \text{Pr}(p \ \& \ q)$
- Can you see (using Venn diagrams) why the following is true?
  - If  $p \models q$ , then  $\text{Pr}(p) \leq \text{Pr}(q)$ .
- Venn d's can model *both* deductive *and* inductive relationships.

## The Probability Calculus III

- To calculate  $\text{Pr}(p \text{ given } q)$ , we treat  $q$  as if it were the “new” reference class. That is, we “conditionalize” the function  $\text{Pr}(\cdot)$  on  $q$ .
- That is, to calculate  $\text{Pr}(p \text{ given } q)$ , we ask ourselves the following question: “What is the proportion of  $q$ -worlds that are  $p$ -worlds?”
- Looking at our Venn diagram, we can see that the proportion of  $q$ -worlds that are  $p$ -worlds is given (intuitively) by:

$$\frac{\text{'area' of } p \ \& \ q\text{-worlds}}{\text{'area' of } q\text{-worlds}} = \frac{\beta}{\beta + \gamma}$$

- This leads to our definition of  $\text{Pr}(p \text{ given } q)$  (Skyrms' Def. 12):

$$\text{Pr}(p \text{ given } q) =_{df} \frac{\text{Pr}(p \ \& \ q)}{\text{Pr}(q)}$$

- NOTE: on this def.,  $\text{Pr}(p \text{ given } q)$  is *undefined* if  $\text{Pr}(q) = 0$ .

### The Probability Calculus IV

- Probabilistic (*a.k.a.*, stochastic) independence is a relation between claims or propositions. We abbreviate this relation using the symbol  $\perp$ . The relation  $p \perp q$  is defined (by Skyrms) as follows:
  - $p \perp q$  if  $\Pr(p \text{ given } q) = \Pr(p)$ .<sup>a</sup>
- With Skyrms' caveat (p. 121, see footnote), this is equivalent to:
  - $p \perp q$  if  $\Pr(p \& q) = \Pr(p) \cdot \Pr(q)$ .
- The intuition behind this definition is (roughly) that *conditionalizing on q has no effect on the probability of p*.
- In this sense, if  $p \perp q$ , then  $q$  is *irrelevant* to  $p$  (and *vice versa*, because  $\perp$  is a *symmetric* relation! Can you prove this?).
- The  $\perp$  relation captures a kind of (ir)relevance, which is *crucial* for our discussions of induction, confirmation, and explanation.

<sup>a</sup>What if  $\Pr(q) = 0$ ? Skyrms, page 121, says  $p \perp q$  in this case! See paper topics.

### What *is* (Inductive) Probability? I

- Skyrms (pp. 26–28) seems skeptical about the prospects for an objective account of inductive probability and inductive logic.
- He laments that “There are no universally accepted rules for constructing inductively strong arguments; no general agreement on a way of measuring the inductive strength of arguments; no precise, uncontroversial definition of inductive probability.”
- Naively, we might try thinking of inductive probability as a quantitative generalization (or measure) of deductive (logical) necessity (or modality). But, this leads to the following problem(s):
- Can we discover (*a priori*?) what the “*logical probabilities*” are? If  $\Omega$  is the set of logical truths, then it is not clear what the values of  $\Pr(\cdot)$  should be (except for the logical truths and logical falsehoods, the probabilities of which are ‘given’ by pure deductive intuition).

### What *is* (Inductive) Probability? II

- We do seem to have pretty strong (*a priori*?) intuitions about what kinds of propositions are logically *impossible* (or *necessary*).
- But, when we move to *quantitative* judgments of “logical probability,” our intuitions seem to be much more shaky.
- There are further subtleties. Claims that are *impossible* are impossible *given any other claim(s)*. That is: if  $p$  is impossible, then  $p$  is impossible *given*  $q$  — for *any*  $q$ . Not so for *improbability*!
- For, no matter low  $\Pr(p \text{ given } \Omega)$  is,  $\Pr(p \text{ given } \Omega \& q)$  can be arbitrarily high, for appropriate choice of  $q$  (*e.g.*,  $q = p$ ).
- That is, judgments about (im)probabilities will depend very sensitively on what we take to be part of the “background” (or the “reference class”). (Im)probability seems indexical or contextual in a way that (im)possibility is not. This makes things more difficult.

### Back to Skyrms on Inductive Strength

- With  $\Pr(p \text{ given } q)$  and  $p \perp q$  under our belts, we can now return (intelligently) to Skyrms' discussion of inductive strength.
- Now, we can state Skyrms' definition more precisely:
  - An argument  $\frac{\mathbf{P}}{\therefore q}$  is inductively strong if  $\Pr(\sim q \text{ given } \mathbf{P})$  is low.
- It should be clear why this is *not* equivalent to “ $\Pr(\sim q \& \mathbf{P})$  is low”.
- Can you give a *formal reconstruction* the examples Skyrms uses (pages 19–20) to illustrate the difference between these accounts?
- Such a reconstruction is *crucial* for tackling the first paper topics.
- Hint: Skyrms gives an example in which  $\sim q \& \mathbf{P}$  is improbable *merely because*  $\mathbf{P}$  is improbable. He claims that  $\mathbf{P}$  need not be ‘evidentially relevant’ in such cases. Thus, he argues, the argument from  $\mathbf{P}$  to  $q$  need not be strong. Does  $\mathbf{P} \perp q$  hold in his example?

### 'Relevance' in the *Deductive* Support Relation

- Skyrms' complaint about the " $\sim q \ \& \ \mathbf{P}$  is improbable" account of inductive strength is (roughly) that  $\sim q \ \& \ \mathbf{P}$  can be improbable even if (intuitively)  $\mathbf{P}$  has "nothing to do with"  $q$ .
- Put another way, Skyrms' complaint seems to be that  $\sim q \ \& \ \mathbf{P}$  can be improbable merely because  $\mathbf{P}$  (or  $q$ ) by itself is improbable — regardless of the *relationship* (or lack thereof) between  $\mathbf{P}$  and  $q$ .
- Some philosophers (but not Skyrms!) have had similar complaints about the " $\sim q \ \& \ \mathbf{P}$  is impossible" account of *deductive* support.
- Such philosophers point out the (intuitive) "irrelevance" of the premises and conclusions in the following *valid* argument forms:

$$\frac{p \ \& \ \sim p}{\therefore q} \quad \frac{p}{\therefore q \vee \sim q}$$

- Why not move to something like " $\sim q$  given  $\mathbf{P}$  is impossible"?

### Various Reflections, Problems, and Issues

- Using our Venn diagram technique to prove *Bayes' Theorem*:

$$\begin{aligned} \Pr(p \text{ given } q) &= \frac{\beta}{\beta + \gamma} \\ &= \frac{\beta}{\alpha + \beta} \cdot (\alpha + \beta) \\ &= \frac{\Pr(q \text{ given } p) \cdot \Pr(p)}{\Pr(q)} \end{aligned}$$

- What is " $p$  given  $q$ "? Is it a claim? If so, can "*given*" be thought of as a sentential connective? If so, is it a conditional of some sort (what *kind*)? Could "*given*" be a *truth-functional* connective?
- Do our definitions of conditional probability and independence make sense? Consider the  $\Pr(q) = 0$  case. What should  $\Pr(q \text{ given } q)$  be here? Should  $q$  be *independent of itself* here?

### Skyrms' Chapter 8: Applying Inductive Logic

- In chapter 8, Skyrms starts talking about applications of inductive logic to philosophy of science (basically, to "confirmation").
- How does Skyrms suggest we should capture Popper's relation of "corroboration" — using inductive probability?
- How does Skyrms unpack the comparative relation: " $p$  is better evidence for  $q$  than  $r$  is for  $s$ " in chapter 8?
- Are these concepts (*i.e.*, "corroborative evidence" and "better evidence") already implicit in his definition of inductive strength?
- If not, might this be a *weakness* of his account of inductive strength? Can we give problematic *examples* here?
- Can you think of alternative ways to define inductive strength that might overcome these weaknesses (*i.e.*, that might capture all of these notions under the single umbrella of "inductive strength")?