# Basic Analytical Tools & Framework

- Administrative: new room, new students (cards)?, pictures?, please consult website (or see me) for syllabus, paper topics, grading, etc.
- Claims, propositions, and arguments (Skyrms ch. 1)
- The deductive support relation between claims (Skyrms ch. 1)
- Deductively valid arguments (Skyrms ch. 1)
- The inductive support relation between claims (Skyrms ch. 2)
- Inductively strong arguments (Skyrms ch. 2)
- The Probability Calculus (Skyrms ch. 6)
- Venn diagrams — a unified formal framework (not in readings)
- Time Permitting: Skyrms' Chapter 8 (applying of inductive logic)

## Claims (Propositions) — The Basic Building Blocks

- A claim (or proposition) is that which is expressed by a (sufficiently precise) declarative sentence (*e.g.*, ‘The earth is flat.’).
- Claims are either true or false, but not both.
- We will use lower-case italic letters (possibly with integer subscripts) ‘$p$’, ‘$q$’, ‘$p_1$’, ‘$p_2$’ ... to denote (basic) claims.
- The connectives ‘&’, ‘$\vee$’, ‘$\sim$’ will be used to denote the *truth-functional* (Boolean) relations ‘and’, ‘or’, ‘not’.
- We have the following Boolean truth-conditions for complex claims:
  - $\ulcorner p \,\&\, q \urcorner$ is true iff both $p$ and $q$ are true.
  - $\ulcorner p \vee q \urcorner$ is true iff either $p$ or $q$ (or both) are true.
  - $\ulcorner {\sim} p \urcorner$ is true iff $p$ is false.

## The Deductive Support Relation Between Claims

- The deductive support (*viz.*, entailment) relation is a qualitative (*i.e.*, yes/no – no degrees) relation between claims or propositions.
- $p$ deductively supports $q$ iff
  - every possible world in which $p$ is true is a world in which $q$ is true.
  - there is no possible world in which $p$ is true and $q$ is false.
  - the conjunction $\ulcorner {\sim} q \,\&\, p \urcorner$ is *impossible* (in the strongest sense).
- $\ulcorner p$ deductively supports $q\urcorner$, $\ulcorner p$ entails $q\urcorner$, $\ulcorner p \vDash q \urcorner$ are synonymous.
- The deductive entailment relation $\vDash$ has several key properties:
  - $p \vDash p$ (reflexivity)
  - If $p \vDash q$ and $q \vDash r$, then $p \vDash r$ (transitivity)
  - If $p \vDash q$, then $p \,\&\, r \vDash q$ (monotonicity)

# Deductively Valid Arguments

- An *argument* $\mathcal{A}$ is a set of claims, one of which ($q$) is the conclusion

$$(\mathcal{A}) \qquad \frac{p_1 \mathbin{\&} \cdots \mathbin{\&} p_n}{\therefore q} \qquad \left[ \text{abbreviated as } \frac{\mathbf{P}}{\therefore q} \right]$$

  the rest ($p_1 \ldots p_n$) are premises (conjunction of $p_i$ abbreviated $\mathbf{P}$).

- $\mathcal{A}$ is *deductively valid* iff $\mathbf{P}$ deductively supports $q$ (*i.e.*, iff $\mathbf{P} \vDash q$).

- Deductive validity inherits properties of the $\vDash$ relation:
  - $\dfrac{p}{\therefore p}$ is valid (reflexivity).
  - If $\dfrac{p}{\therefore q}$ is valid and $\dfrac{q}{\therefore r}$ is valid, then $\dfrac{p}{\therefore r}$ is valid (transitivity).
  - If $\dfrac{\mathbf{P}}{\therefore q}$ is valid, then $\dfrac{\mathbf{P} \mathbin{\&} r}{\therefore q}$ is valid (monotonicity).

- Well-known valid forms (*modus ponens*, disjunctive syllogism, *etc.*).

## The Relation of Inductive Support (informal)

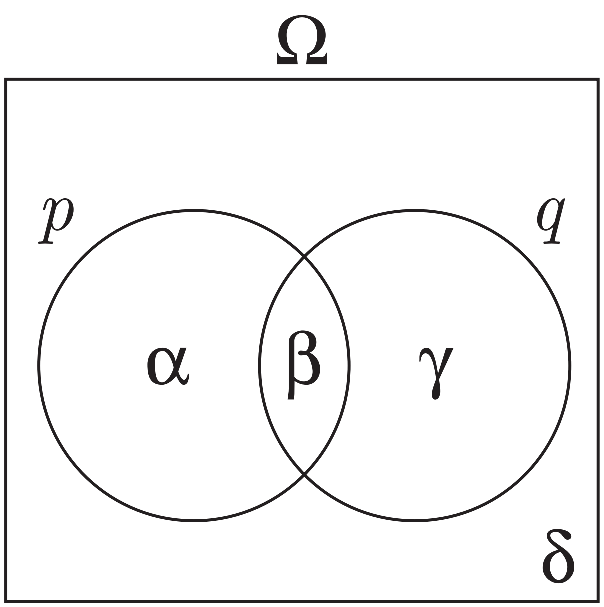
- The inductive support relation is a *quantitative* (*i.e.*, a relation which admits of *degrees*) between claims or propositions.
- $p$ inductively supports $q$ (on Skyrms' definition) iff
  - 'Most' possible worlds in which $p$ is true are worlds in which $q$ is true.
  - It is improbable that $q$ is false, *given* that $p$ is true.
  - ***NOT*** the same as: the conjunction $\ulcorner {\sim} q \,\&\, p \urcorner$ is *improbable.*
  - $\therefore$ ***NOT*** (strictly) analogous to definition of deductive support.[a]
- Skyrms presents two examples to illustrate the differences between:
  - $\ulcorner {\sim} q \,\&\, p \urcorner$ is improbable, and
  - "${\sim} q$ *given* $p$ is improbable"

  [This is the subject of two paper topics — we shall return to this!]

[a]But, deductive entailment *is* a "limiting case" of inductive strength. *Why?*

## Digression: The Probability Calculus I

- We can't really be precise about the relation of inductive support without using (formal) *probability calculus* (*i.e.*, Ch. 6 of Skyrms).
- Probabilities (inductive ones) are functions from claims onto the real unit interval. These functions can be thought of (roughly!) as measures of *proportions of possible worlds in which claims are true.*
- It helps to use *Venn diagrams* to picture probabilities:




- The circles represent the sets of possible worlds in which the claims *p* and *q* are true.
- The *reference class* Ω is the set of possible worlds with respect to which Pr is defined.
- $\Pr(\Omega) = 1$ (*total area* of the r.c. 'box' is 1)
- $\Pr(p \& {\sim}q) = \alpha$, $\Pr(p \& q) = \beta$, $\Pr({\sim}p \& q) = \gamma$
- $\Pr({\sim}p \& {\sim}q) = \delta = 1 - (\alpha + \beta + \gamma)$

# The Probability Calculus II

- Thinking of probabilities as (normalized) *areas* in Venn diagrams of this kind *automatically* gives us Skyrms' first 6 rules for $\Pr(\cdot)$:
  - If $p$ is a logical truth, then $\Pr(p) = 1$.
  - If $p$ is a logical falsehood, then $\Pr(p) = 0$.
  - If $p$ and $q$ are mutually exclusive (*i.e.*, if there are no possible worlds in which $\ulcorner p \,\&\, q \urcorner$ is true), then $\Pr(p \vee q) = \Pr(p) + \Pr(q)$.
  - If $p$ and $q$ are logically equivalent, then $\Pr(p) = \Pr(q)$.
  - $\Pr({\sim}p) = 1 - \Pr(p)$
  - $\Pr(p \vee q) = \Pr(p) + \Pr(q) - \Pr(p \,\&\, q)$
- Can you see (using Venn diagrams) why the following is true?
  - If $p \vDash q$, then $\Pr(p) \leq \Pr(q)$.
- Venn d's can model *both* deductive *and* inductive relationships.

## The Probability Calculus III

- To calculate $\Pr(p \textit{ given } q)$, we treat $q$ *as if* it were the "new" reference class. That is, we "conditionalize" the function $\Pr(\cdot)$ on $q$.
- That is, to calculate $\Pr(p \textit{ given } q)$, we ask ourselves the following question: "What is the proportion of $q$-worlds that are $p$-worlds?"
- Looking at our Venn diagram, we can see that the proportion of $q$-worlds that are $p$-worlds is given (intuitively) by:

$$\frac{\text{`area' of } p \mathbin{\&} q\text{-worlds}}{\text{`area' of } q\text{-worlds}} = \frac{\beta}{\beta + \gamma}$$

- This leads to our definition of $\Pr(p \textit{ given } q)$ (Skyrms' Def. 12):

$$\Pr(p \textit{ given } q) =_{df} \frac{\Pr(p \mathbin{\&} q)}{\Pr(q)}$$

- NOTE: on this def., $\Pr(p \textit{ given } q)$ is *undefined* if $\Pr(q) = 0$.

# The Probability Calculus IV

- Probabilistic (*a.k.a.*, stochastic) independence is a relation between claims or propositions. We abbreviate this relation using the symbol $\perp$. The relation $p \perp q$ is defined (by Skyrms) as follows:
  - $p \perp q$ if $\Pr(p \textit{ given } q) = \Pr(p)$.[a]
- With Skyrms' caveat (p. 121, see footnote), this is equivalent to:
  - $p \perp q$ if $\Pr(p \,\&\, q) = \Pr(p) \cdot \Pr(q)$.
- The intuition behind this definition is (roughly) that *conditionalizing on q has no effect on the probability of p.*
- In this sense, if $p \perp q$, then $q$ is *irrelevant* to $p$ (and *vice versa*, because $\perp$ is a *symmetric* relation! Can you prove this?).
- The $\perp$ relation captures a kind of (ir)relevance, which is *crucial* for our discussions of induction, confirmation, and explanation.

[a] What if $\Pr(q) = 0$? Skyrms, page 121, says $p \perp q$ in this case! See paper topics.

# What *is* (Inductive) Probability? I

- Skyrms (pp. 26–28) seems skeptical about the prospects for an objective account of inductive probability and inductive logic.
- He laments that "There are no universally accepted rules for constructing inductively strong arguments; no general agreement on a way of measuring the inductive strength of arguments; no precise, uncontroversial definition of inductive probability."
- Naively, we might try thinking of inductive probability as a quantitative generalization (or measure) of deductive (logical) necessity (or modality). But, this leads to the following problem(s):
- Can we discover (*a priori*?) what the "*logical probabilities*" are? If $\Omega$ is the set of logical truths, then it is not clear what the values of $\Pr(\cdot)$ should be (except for the logical truths and logical falsehoods, the probabilities of which are 'given' by pure deductive intuition).

## What *is* (Inductive) Probability? II

- We do seem to have pretty strong (*a priori*?) intuitions about what kinds of propositions are logically *impossible* (or *necessary*).
- But, when we move to *quantitative* judgments of "logical *probability*," our intuitions seem to be much more shaky.
- There are further subtleties. Claims that are *impossible* are impossible *given any other claim(s)*. That is: if $p$ is impossible, then $p$ is impossible *given* $q$ — for *any* $q$. Not so for *improbability*!
- For, no matter low $\Pr(p \text{ given } \Omega)$ is, $\Pr(p \text{ given } \Omega \,\&\, q)$ can be arbitrarily high, for appropriate choice of $q$ (*e.g.*, $q = p$).
- That is, judgments about (im)probabilities will depend very sensitively on what we take to be part of the "background" (or the "reference class"). (Im)rpobability seems indexical or contextual in a way that (im)possibility is not. This makes things more difficult.

## Back to Skyrms on Inductive Strength

- With Pr($p$ *given* $q$) and $p \perp q$ under our belts, we can now return (intelligently) to Skyrms' discussion of inductive strength.
- Now, we can state Skyrms' definition more precisely:
  - An argument $\dfrac{\mathbf{P}}{\therefore q}$ is inductively strong if Pr(~$q$ *given* $\mathbf{P}$) is low.
- It should be clear why this is *not* equivalent to "Pr(~$q$ & $\mathbf{P}$) is low".
- Can you give a *formal reconstruction* the examples Skyrms uses (pages 19–20) to illustrate the difference between these accounts?
- Such a reconstruction is *crucial* for tackling the first paper topics.
- Hint: Skyrms gives an example in which ~$q$ & $\mathbf{P}$ is improbable *merely because* $\mathbf{P}$ is improbable. He claims that $\mathbf{P}$ need not be 'evidentially relevant' in such cases. Thus, he argues, the argument from $\mathbf{P}$ to $q$ need not be strong. Does $\mathbf{P} \perp q$ hold in his example?

# ‘Relevance’ in the *Deductive* Support Relation

- Skyrms’ complaint about the “$\sim q \,\&\, \mathbf{P}$ is improbable” account of inductive strength is (roughly) that $\sim q \,\&\, \mathbf{P}$ can be improbable *even if (intuitively)* $\mathbf{P}$ *has “nothing to do with”* $q$.
- Put another way, Skyrms’ complaint seems to be that $\sim q \,\&\, \mathbf{P}$ can be improbable *merely because* $\mathbf{P}$ (or $q$) *by itself is improbable* — regardless of the *relationship* (or lack thereof) between $\mathbf{P}$ and $q$.
- Some philosophers (but not Skyrms!) have had similar complaints about the “$\sim q \,\&\, \mathbf{P}$ is impossible” account of *deductive* support.
- Such philosophers point out the (intuitive) “irrelevance” of the premises and conclusions in the following *valid* argument forms:

$$\frac{p \,\&\, \sim p}{\therefore\, q} \qquad \frac{p}{\therefore\, q \vee \sim q}$$

- Why not move to something like “$\sim q$ *given* $\mathbf{P}$ is impossible”?

## Various Reflections, Problems, and Issues

- Using our Venn diagram technique to prove *Bayes' Theorem*:

$$\begin{aligned}\Pr(p \textit{ given } q) &= \frac{\beta}{\beta+\gamma} \\ &= \frac{\frac{\beta}{\alpha+\beta}\cdot(\alpha+\beta)}{\beta+\gamma} \\ &= \frac{\Pr(q \textit{ given } p)\cdot\Pr(p)}{\Pr(q)}\end{aligned}$$

- What is "$p$ *given* $q$"? Is it a claim? If so, can " *given* " be thought of as a sentential connective? If so, is it a conditional of some sort (what *kind*)? Could " *given* " be a *truth-functional* connective?

- Do our definitions of conditional probability and independence make sense? Consider the $\Pr(q) = 0$ case. What should $\Pr(q \textit{ given } q)$ be here? Should $q$ be *independent of itself* here?

# Skyrms' Chapter 8: Applying Inductive Logic

- In chapter 8, Skyrms starts talking about applications of inductive logic to philosophy of science (basically, to "confirmation").
- How does Skyrms suggest we should capture Popper's relation of "corroboration" — using inductive probability?
- How does Skyrms unpack the comparative relation: "$p$ is better evidence for $q$ than $r$ is for $s$" in chapter 8?
- Are these concepts (*i.e.*, "corroborative evidence" and "better evidence") already implicit in his definition of inductive strength?
- If not, might this be a *weakness* of his account of inductive strength? Can we give problematic *examples* here?
- Can you think of alternative ways to define inductive strength that might overcome these weaknesses (*i.e.*, that might capture all of these notions under the single umbrella of "inductive strength")?