Four Decades of Scientific Explanation, Cont'd

- Administrative:
  - Let's talk about papers this week (today?) ...
  - I will be away next week (but on email always) ...
  - Guest Lecture on the 20th (Strevens)
- Determinism, Indeterminism, and Explanation
- Interpretations of Probability
  1. Subjective Interpretations
     (a) *Justified* / *rational* degrees of belief
     (b) (Logical probability is often put here ...)
  2. Objective Interpretations
     (a) Classical accounts (could also go in 1(a), above)
     (b) Frequency accounts
     (c) Propensity accounts
     (d) (Logical probability is sometimes put here ...)

Determinism, Indeterminism, and Explanation

- According to Salmon, Coffa, and many others, if the universe were deterministic, then all explanations would be deductive. That is, there would be no *irreducibly* probabilistic explanations in a deterministic world. Any probabilities that remained would be "purely epistemic". Such explanations would merely be *incomplete*.
- In Salmon's S–R theory, this means that in deterministic worlds, further partitions will always be possible, until a partition which *determines* the outcome is finally reached.
- In Coffa's theory, all "propensities" would be *perfect* in deterministic worlds, and so we would always have $\mathcal{P}(Ga \mid Fa) = 1$.
- In Hempel's theory, an omniscient modeler would always end-up with extreme epistemic probabilities in their I–S explanations.
- Are all incomplete explanations bad explanations? Isn't explanation also *pragmatic* in nature (meeting a *demand* for explanation)?

Interpretations of Probability I – Overview

- There are many different interpretations of probability.
  1. Subjective Interpretations
     (a) Probabilities as *justified* degrees of
     (b) Probabilities as *rational* degrees of belief
     (c) (Logical probability is often put here ...)
  2. Objective Interpretations
     (a) Classical accounts (could also go in 1(a), above)
     (b) Actual frequency accounts
     (c) Hypothetical frequency accounts
     (d) Propensity accounts
     (e) (Logical probability is sometimes put here ...)
- I would say that there are in fact many different *theories* of probability, each of which is more or less appropriate for certain applications. See Gillies' good recent book *Philosophical Theories of Probability*.

Interpretations of Probability II – Justified *vs* Rational I

- *Justified* belief and *Rational* belief are not (necessarily) the same thing. In fact, they are distinct. Consider the following counterexamples.
- Justified $\not\Rightarrow$ Rational: I may have overwhelming evidence that my wife has big feet. But, I may also know that I'm the kind of person who can't keep his mouth shut! So, it may *not* be rational for me to believe this — even though it would be *justified*, given my evidence.
- Rational $\not\Rightarrow$ Justified: Someone has offered me 1 million dollars to believe that there are 52,678 marbles in a jar in my attic (and I *love* money!). My evidence suggests there are 52,679 marbles in the jar (I counted them several times, and had others count them, etc.). So?
- The same point holds for degrees of belief. Rational degrees of belief are those which would be most well advised for me to have, all things considered in my complete life, and justified degrees of belief are those that are the most accurate, etc., given my total evidence.

Interpretations of Probability II – Justified *vs* Rational II

- Arguments are made which aim to show that rational degrees of belief and justified degrees of belief should satisfy the probability axioms.
- The "rationality" approaches usually involve either (1) "Dutch Books" which are supposed to show that if you're degrees of belief are not probabilistic, then you are susceptible to a "sure loss" (and, hence, that you are irrational), or (2) "representation theorems" which aim to show that if you're preferences satisfy certain "rationality constraints" (*e.g.*, intransitivity, etc.), then your degrees of belief will be probabilistic.
- The "justification" approaches usually involve some measure of "accuracy" of degrees of belief, and that the "most accurate" degrees of belief will (in general) be those which satisfy the probability axioms.
- All of these arguments have interesting variations and potential problems. See Rosenkrantz's book *Foundations and Applications of Inductive Probability* for a nice discussion of both types of accounts.

Interpretations of Probability II – Justified *vs* Rational III

- Most subjectivists do not require anything more of an agent's degrees of belief than mere satisfaction of the probability axioms. So, for them, the distinction between rational and justified is of no consequence.
- Most people agree that it is the *justified* degrees of belief that are relevant to *epistemic* applications like confirmation are explanation (if you think those things are purely subjective and epistemic).
- What are *rational* degrees of belief good for? It doesn't seem that rational degrees of belief are relevant to decision making either (shouldn't you act on the basis of your *most accurate* assessment of probability — or on your "happiest" assessment of probability, etc.?).
- I do *not* think that *logical* interpretations of probability belong here. Logical probabilities are quantitative generalizations of notions of deductive logical consequence. See Roeper and Leblanc's book *Probability Theory and Probability Logic* for how this can be done.

Interpretations of Probability IV – Dutch Books

- For each event E in a collection of events $E_1, \ldots, E_n$, Mr. B has to announce a number $q(E)$ (called his *betting quotient* on E), and then Ms. A will choose the *stake* S. Mr. B must pay Ms. A \$ $[q(E) \times S]$ in exchange for S if E occurs. S can be positive or negative, but $|S|$ must be small in relation to Mr. B's wealth. Under these circumstances, $q(\cdot)$ is taken to be a measure of Mr. B's degree of belief in E.
- We say that $q(\cdot)$ is *coherent* iff it is impossible for Ms. A to choose stakes S such that she wins *no matter what happens*. That is, $q(\cdot)$ is *coherent* iff Ms. A cannot construct a "Dutch Book" against Mr. B.
- **Theorem**. $q(\cdot)$ is *coherent* iff $q(\cdot)$ is a *probability function*.
- **Claim** (*not* a theorem!). $q(\cdot)$ is *rational* iff $q(\cdot)$ is a *probability function*.
- **Theorem** $\overset{?}{\Longrightarrow}$ **Claim**? *Betting* behavior $\overset{?}{\approx}$ *rational* behavior?
- **Theorem***. Mr. B is open to a *sure win* iff $q(\cdot)$ is *non*-probabilistic!

Interpretations of Probability V – Representation Theorems I

- Let f, g, h be acts which Mr. B could choose to perform. And, let $f(x)$ be the consequence (for Mr. B) which obtains if he chooses to perform act f, and the resulting state of the world is x. The *expected utility* of choosing to perform act f is defined as: $EU(f) = \sum_x q(x) \cdot u(f(x))$, where $q(\cdot)$ are Mr. B's degrees of belief, and $u(\cdot)$ are Mr. B's utilities.
- $f \precsim g$ iff Mr. B (weakly) *prefers* g to f. $f \precsim g$ iff either Mr. B *strictly* prefers g to f ($f \prec g$), or is *indifferent between* g and f ($f \sim g$). $f \precsim_A g$ iff Mr. B would prefer g to f, were he to learn that A is true.
  1. **Normality**. Either $f \precsim g$ or $g \precsim f$ (or both).
  2. **Transitivity**. If $f \precsim g$ and $g \precsim h$, then $f \precsim h$.
  3. **Sure-Thing Principle**. If $f \precsim_A g$ and $f \precsim_{\sim A} g$, then $f \precsim g$. And, if Mr. B thinks A is possible, then if $f \prec_A g$ and $f \precsim_{\sim A} g$, then $f \prec g$. [What you choose at a node in a tree does not depend on what would have happened if you had not reached that node.]

Interpretations of Probability VI – Representation Theorems II

- **Theorem**. If Mr. B's preferences satisfy (1)–(3), then there is a *unique* representation of Mr. B's preferences in terms of his degrees of belief $q(\cdot)$ and his utilities $u(\cdot)$, such that $f \precsim g$ iff $EU(f) \leq EU(g)$. Moreover, the degrees of belief $q(\cdot)$ in this representation will obey the probability axioms. So, under these assumptions, Mr. B will act (if he acts rationally!) *as if* his degrees of belief are probabilistic.
- **Theorem***. If Mr. B's preferences satisfy (1)–(3), then there is a *unique* representation of Mr. B's preferences in terms of his degrees of belief $q(\cdot)$ and his utilities $u(\cdot)$, such that $f \precsim g$ iff $EU^*(f) \leq EU^*(g)$. Moreover, the degrees of belief $q(\cdot)$ in *this* representation will *not* obey the probability axioms (but, $EU^*(f)$ is defined *non*-standardly).
- Rational agents *can be represented as having* probabilistic degrees of belief $\overset{?}{\Longrightarrow}$ rational agents *do* have probabilistic degrees of belief?
- See Maher's *Betting On Theories* for discussion of DB's and RT's.

Interpretations of Probability VII – Actual Frequency Accounts

- Say we are interested in determining probabilities involving certain factors X, Y, Z, etc. in a (finite) population P of size N.
- One suggestion is to define $\Pr(X)$ as the *actual frequency* of X's in P. Let $\#(\alpha)$ be the size of α. Then, the actual frequency account suggests:

$$\Pr(X) = \frac{\#(X)}{N}$$

- On this account, *all probabilities must be rational numbers*. What if the bias of a coin is $\frac{1}{\sqrt{2}}$ or $\frac{1}{\pi}$? Moreover, our physical theories (*e.g.*, QM) imply *irrational* probabilities for many events (*e.g.*, the probability of finding en electron in a certain region around a hydrogen nucleus).
- Actual frequencies can often be *misleading* about the underlying *causal* (or just *probabilistic*!) facts. Small samples may be *unrepresentative* of underlying structure. We think that a streak of 1000 heads in a row (with a fair coin) will eventually get "washed out" in the long run.

Interpretations of Probability VIII – Hypothetical Frequency I

- Think of the actual population P as the result of one *experiment*, an experiment that could (theoretically) be *repeated*. The experiment is:
  - We endow a set of N individuals with a certain distribution of initial conditions. These initial conditions are factors that are causally relevant to the way individuals in P are, with respect to having or lacking X, Y, Z, etc. (excluding the properties themselves).
  - A distribution of initial conditions gives rise to an *experimental set-up*, which gives the individuals in the population "propensities" of various strengths to have or lack the factors X, Y, Z, etc..
  - The result of an experiment is the resulting conditional and unconditional *frequencies*, involving the factors X, Y, Z, etc..
  - A repetitions of an experiment is a running of the experiment with the same initial conditions but a possibly different population of individuals (*i.e.*, this could be a *hypothetical* population).

Interpretations of Probability IX – Hypothetical Frequency II

- Let us assume that the experiment is repeated on populations of the same size N as the actual population P. Specifically, let P_0 (= P), P_1, P_2, … be the infinite sequence of hypothetical populations that would result from conducting this experiment infinitely may times.
- And, let Fr_0, Fr_1, Fr_2, … be functions giving the values of frequencies involving the factors X, Y, Z, etc. in the populations P_0, P_1, P_2 … respectively. Then, according to this sketch of a hypothetical frequency (plus propensity?) account, we have:

$$\Pr(X) = \lim_{n \to \infty} \frac{\sum_{i=0}^{n} Fr_i(X)}{n}$$

- Q: Why consider the different populations P_i one by one and then average their Fr_i's rather than combining the P's into one overall population and then looking for limits of frequencies in the combined population? A: we need to have an *ordering* over the P_i …

Interpretations of Probability X – Hypothetical Frequency III

- The limit (if it exists — see below!) depends on the *order* in which the (partial sums of the) frequencies are taken. So, we need to have an ordering of the P_i's, and the "temporal" ordering (of the hypothetical populations!) seems most natural (there would be no principled way of ordering the frequencies in the "combined" infinite set of populations).
- Also, the *size* of a population might *itself* be a relevant factor. So, we want to first identify the Fr_i — all defined on populations *of size* N — and then "average" them, rather than combining into an *infinite* population, and then looking for limiting frequencies within it.
- This approach avoids the two problems of the actual frequency account. On this account, probabilities may take on any real number on $[0, 1]$. And, intuitively, we think contingencies of the actual population will "wash out" as we go to the infinite (hypothetical) limit — especially, since we're assuming the initial conditions are *causally complete* with respect to the factors in which we are interested.

Interpretations of Probability XI – Hypothetical Frequency IV

- We can see now why we need to specify what *kind* of population P is. Presumably, we are interested not just about the probabilities of X, Y, etc. in the actual population P. We want to know what the probabilities are in populations of a certain *kind*, which P exemplifies.
- Different kinds of populations would, presumably, have different sets of initial-condition-features, and so would (in an indeterministic world!) determine different probabilities for the properties in question. This can be generalized to cases in which the populations are of different sizes.
- Problems: Is there such a thing as *THE* sequence of (hypothetical) repetitions P_i that would have obtained, if we had repeated the experiment over and over again *ad infinitum*? This matters because even a different order of the same P_i's would undermine the limit definition. There are uncountably many sequences (just re-orderings of the P_i's) — and *any* limit value can be generated in infinitely many ways ... See Eells' *Probabilistic Causality* chapter 2 for much more ...

Interpretations of Probability XII – Propensity I

- Propensity interpretations are similar to hypothetical frequency interpretations, in that they assume there is an experimental set-up which fixes (presumably, *via* fixing relevant causal factors) the limiting-frequencies that will be observed in repetitions of experiments.
- The difference is that propensity accounts do not define probability as limiting relative frequency. Rather, they take the probability to be the propensity or the disposition itself, and the frequencies are just emergent properties of the underlying structure in the set-up.
- Problems: Are propensities probabilities at all? Presumably, the experimental set-up S confers probabilities on factors, but is the converse also true? In probability theory, there is no asymmetry. If $\Pr(X \mid S)$ is well-defined in a probability space, then so is $\Pr(S \mid X)$. But this seems nonsensical on the propensity account (propensities have a directionality imposed by causal structure). How dow we learn about propensities? Is our only access through *frequencies*?