

## Four Decades of Scientific Explanation, Cont'd

- Administrative:
  - Comments (scribbled!) returned today ...
  - Please discuss with me next week ...
  - Guest Lecture on the 20th (Strevens)
- “Decade #2” — I–S Explanation
  - I–S as a generalization of D–N
  - Some special problems for I–S
- “Decade #3” — S–R Explanation
  - Salmon, Greeno, *et al* urge probabilistic *relevance*
  - This lead to (early) Statistical Relevance (S–R) Accounts
  - These have special problems of their own ...

## Decade #2 — The Birth of Statistical Explanation

- H & O were aware that their D–N account (as originally stated) left no room for the explanation of either (1) statistical laws, or (2) token events which cannot be derived from any theory (but on which some theory + auxiliaries/initial conditions may *confer a probability*).
- Hempel's first alteration (a very minor one) was to expand the explanation of *laws* (by more general laws) in D–N to the case of *statistical laws*. This led to the *Deductive-Statistical* (D–S) model.
- On the D–S model, a statistical law may be explained by appeal to more general laws (which may be statistical or universal).
- **Example:** we may derive the half-life of uranium-238 from the basic laws of quantum mechanics (together with the height of the potential barrier surrounding the nucleus and the kinetic energies of the alpha particles within the nucleus). D–S as a mere *variant* of D–N.
- Same problems as D–N (*plus* statistical laws & qualitative predicates!)

## Inductive–Statistical Explanation I

- Hempel offered an inductive generalization of his D–N model of explanation. He called this model the Inductive–Statistical (I–S) model.
- Hempel updated his four high-level desiderata accordingly:
  1. An explanation is an argument having correct logical form (either deductive or inductive — Skyrmsian “strength” idea).
  2. The explanans must contain, essentially, at least one general law (either universal or statistical).
  3. The general law must have empirical content.
  4. The statements in the explanans must be true.
- Hempel quickly realized that a fifth adequacy condition must be added, because of the *non-monotonicity* of “inductively strong” arguments:
  5. (RMS) The requirement of *maximal specificity*.

## Inductive–Statistical Explanation II

- The simplest schema for an I–S explanation would be:

$$\Pr(Gx \mid Fx) = r$$

$$\frac{Fb}{Gb} \quad [r]$$

- Here, “ $\Pr(Gx \mid Fx) = r$ ” is a statistical law which says that the relative frequency of  $G$ s among  $F$ s is  $r$ . And, “[ $r$ ]” indicates the “inductive strength” (in a Skyrmsian sense) of the argument.
- **Example:** John Jones ( $b$ ) recovers quickly ( $Gb$ ) from Strep ( $Fb$ ). Most strep infections ( $Fx$ ) clear up quickly ( $Gx$ ) when treated with penicillin ( $Hx$ ). Thus, we have the following I–S explanation of the fact that  $Gb$ :

$$\Pr(Gx \mid Fx \ \& \ Hx) = r \approx 1$$

$$\frac{Fb \ \& \ Gb}{Gb} \quad [r \approx 1]$$

## Inductive–Statistical Explanation III

- But, what if we were to subsequently learn that John Jones was infected with a *penicillin-resistant* strain of Strep ( $Jb$ )? Plausibly, this would lead to a “strong” I–S explanation of  $\sim Gb$ , as follows:

$$\Pr(\sim Gx \mid Fx \ \& \ Hx \ \& \ Jx) = r_1 \approx 1$$

$$\frac{Fb \ \& \ Gb \ \& \ Jb}{\sim Gb} \quad [r_1 \approx 1]$$

- This illustrates the (well-known to you all by now!) *non-monotonicity* of inductive inferences. We now have two strong arguments whose conclusions are logically incompatible! Such a thing is *unheard of* in the realm of deductive inference (and D–N explanation)!
- In inductive *logic*, we usually add the requirement of *total evidence*. That is, we usually add the requirement that *no additional evidence that would change the degree of support is available at the time*. **This will not work here**, since the *explanandum* is *known*! What to do?

## Inductive–Statistical Explanation IV

- Hempel adds the *requirement of maximal specificity* (RMS). Here, we assume that  $\mathbf{P}$  is the conjunction of all of the premises of the I–S explanation, and  $K$  is the available background knowledge.

(RMS) If  $\mathbf{P} \ \& \ K$  implies that  $b$  belongs to a class  $F_1$  and that  $F_1$  is a subclass of  $F$ , then  $\mathbf{P} \ \& \ K$  must also imply a statement specifying the statistical probability of  $G$  in  $F_1$ , say

$$\Pr(G \mid F_1) = r_1.$$

Here,  $r_1 = r$  unless the probability statement in question is simply a theorem of probability theory (proper).

- The “unless” clause in (RMS) is there to block the trivialization which might otherwise arise from the fact that we know from  $K$  that  $b$  is both  $F$  and  $G$ . But, of course, probability theory (proper) implies that  $\Pr(Gx \mid Fx \ \& \ Gx) = 1$ . But, this should not undermine the explanation.

## Inductive–Statistical Explanation V

- Note, deductive arguments *automatically* satisfy (RMS). *Why?*
- According to Hempel, the *non-monotonicity* of inductive inferences leads, inevitably, to the *epistemic relativity of statistical explanation*:  
“The concept of statistical explanation for particular events is essentially relative to a given knowledge situation as represented by a class *K* of accepted statements.”
- Skyrms makes a similar move. I concede that inductive inferences are *contextual* (or *indexical*), but why does *that* force them to be *epistemically* relative? We don't take this attitude toward special relativity (*versus* Newtonian theory). We would say that things like “simultaneity” are *contextual* (*i.e.*, that they are relative to a *frame of reference*), but we *don't* seem to think they depend on *what we know*. Why the different attitude when it comes to inductive inference (*versus* deductive inference)? This is a *serious* (and *deep*) topic for a paper!

- Alberto Coffa's discussion (as quoted Salmon) is quite illuminating on this issue of "epistemic relativity". He says, about "confirmation", that:

Although the syntactic form of expressions like "hypothesis  $h$  is well-confirmed" may mislead us into believing that confirmation is a property of sentences, closer inspection reveals the fact that it is a relation between sentences and knowledge situations and that the concept of confirmation cannot be properly defined . . . without reference to sentences intended to describe a knowledge situation.

But, interestingly, Coffa sings a different tune about "explanation":

. . . the possibility of a notion of true explanation . . . is not just a desirable but ultimately dispensable feature of a model of explanation: it is the *sine qua non* of its realistic, non-psychologistic inspiration. It is because certain features of the world can be deterministically responsible for others that we can describe a concept of true deductive explanation . . . If there are features of the world which can be non-deterministically responsible for others, then we should be able to define a model of true inductive explanation.

- Why not say the same thing about inductive *arguments/logic*? Isn't the notion of a "good argument" the *sine qua non* of the realistic, non-psychologistic inspiration of *logic* (inductive or otherwise)?



## Inductive–Statistical Explanation VI

- You can already guess what some of the problems with I–S explanation are going to be. As we saw with Skyrms' discussion of “strength”, the most obvious difficulties involve the issue of *relevance*.
- “ $\Pr(X | Y)$  is high” is neither necessary nor sufficient for “ $Y$  is relevant to  $X$ ”. The Fred Fox example, and the paresis/syphilis example (above) show that *relevance* is an important aspect of explanations.
- Both the D–N and the I–S accounts suffer from the relevance problem. The hexed salt example shows that D–N is also vulnerable here.
- While high probability is neither necessary nor sufficient for explanatory power, it may still be plausible that (*other things being equal* — *e.g.*, assuming that there *is* relevance, *etc.*) higher probabilities tend to lead to *better* explanations. Our own Michael Strevens discusses this point nicely in his recent paper “Do Large Probabilities Explain Better?”. He will be guest lecturing on T 11/20.

## Statistical–Relevance Explanation I

- Salmon, Greeno, Jeffrey, and others were among the first to question the high probability requirement of the I–S account.
- Jeffrey argued that stochastic processes generate outcomes with varying probabilities, and that we understand the low probability outcomes as well as we understand the high probability outcomes. [NOTE: It's not clear to me from this argument why we should move from I–S to *relevance*. Is Jeffrey talking about *relevance* here?]
- Salmon and Greeno formulated theories in which the key probabilistic fact is a fact about probabilistic *relevance* — not just the (high, posterior) probability of the explanandum.
- Accounts involving relevance as the key attribute face special problems of their own — problems not faced by the I–S account.
- The most important of these is known as *Simpson's Paradox*. Nancy Cartwright describes a good example illustrating this paradox.

## Statistical–Relevance Explanation II

- In the early 80's there was a positive correlation between being female (F) and being rejected from Berkeley's graduate school (R).
- This (initially) raised some suspicions about the possibility of sexual discrimination in the admissions process for Berkeley's grad school.
- Symbolically,  $\Pr(R | F) > \Pr(R)$ . That is, being female is *statistically relevant* to being rejected from Berkeley grad school. Or is it?
- If we *partition* the applicants according to the department to which they applied:  $\{D_1, \dots, D_n\}$ , then the correlation disappears!
- That is,  $\Pr(R | F \ \& \ D_i) = \Pr(R | D_i)$ , for all  $i$ .
- Should we still be suspicious about sexual discrimination? Or, more relevantly here, should we still think that the gender of the applicant is *explanatorily relevant* to why they got rejected (or accepted)?

## Statistical–Relevance Explanation III

- Simpson’s Paradox forces the statistical relevance theorist to make some special maneuvers. Enter the notion of “homogeneous partition”.
- Salmon’s original S–R account is intended to provide an answer to the question “Why does this (member of the reference class)  $A$  have the attribute  $B$ ?” If the question is not stated this precisely, then Salmon suggests using “pragmatics” to determine the reference class.
- An S–R explanation (of why this  $A$  is a  $B$ ), consists of the prior probability of  $B$  (given  $A$ ), a homogeneous relevant partition  $\{A \& C_i\}$  of  $A$  with respect to  $B$ , the posterior probabilities of  $B$  in each member  $A \& C_i$  of the partition, and a statement of the location of the individual in question in a particular cell  $A \& C_k$  of the partition:
  - $\Pr(B | A) = p$
  - $\Pr(B | A \& C_i) = p_i$
  - $\{A \& C_i\}$  is a homogenous relevant partition of  $A$  with respect to  $B$
  - $b$  is a member of  $A \& C_k$

## Statistical–Relevance Explanation IV

- What is a “homogeneous relevant partition of  $A$  with respect to  $B$ ?”  
Let’s return to the Berkeley graduate school example. Let  $b$  be some applicant ( $A$ ) who was rejected ( $B$ ). And, we want to know why this applicant ( $b$ ) was rejected (*i.e.*, why this  $A$  ( $b$ ) is a  $B$ ).

- In order to give an S–R explanation of  $Bb$  here, we would need to appeal to some partition of  $A$  which is *relevant* with respect to  $B$ .
- We know such a partition  $\{A \& C_i\}$ : the partition in which the  $C_i$  are the *genders* of the applicants. If  $C_1 = \text{Male}$ , and  $C_2 = \text{Female}$ , then:

$$\Pr(B \mid A) = p, \Pr(B \mid A \& C_1) = p_1, \Pr(B \mid A \& C_2) = p_2$$

Here,  $p_2 < p < p_1$ . Therefore, the partition  $\{A \& C_i\}$  is *relevant* to  $B$ .

- But, the partition  $\{A \& C_i\}$  is *not homogeneous* with respect to  $B$ .  
This is because there is a further set of *relevant factors*  $\{D_i\}$  such that:

$$\Pr(B \mid A \& C_1 \& D_i) = \Pr(B \mid A \& C_2 \& D_i), \text{ for all } i$$

## Statistical–Relevance Explanation V

- To summarize, a *partition*  $\{A \& C_i\}$  of a class  $A$  is a collection of mutually exclusive and exhaustive subsets of  $A$ . Each subclass  $A \& C_i$  in the partition  $\{A \& C_i\}$  is called a *cell* of the partition.
- A partition  $\{A \& C_i\}$  of  $A$  is *relevant* with respect to  $B$  if the probability of  $B$  in each cell  $A \& C_i$  of the partition is different from each other cell. That is, if  $\Pr(B \mid A \& C_i) \neq \Pr(B \mid A \& C_j)$ , for all  $i \neq j$ .
- Salmon:  $F$  is *homogeneous* with respect to  $B$  if *no relevant partition (wrt  $B$ ) can be made within  $F$* . *Objectively* homogeneous = no relevant partition can be made *in principle*; *epistemically* homogeneous = no relevant partition is *known* (presumably, by the explainer).
- The partition of  $A$  according to gender ( $C_i$ ) is *relevant* to acceptance ( $B$ ). But,  $\Pr(B \mid A \& C_1 \& D_i) = \Pr(B \mid A \& C_2 \& D_i)$ . So, the further partition into  $D_i$  is *not* relevant, according to Salmon. But, this means Salmon has no way to *block* the explanatoriness of gender! Problem.

## Statistical–Relevance Explanation VI

- Intuitively, “homogeneity” is *intended* to block the explanatoriness of gender for acceptance in the Berkeley grad school example. But, Salmon’s definition of “homogeneity” does not do the trick.
- Another way to see the problem is to ask yourself *why* we think the further partitioning on the  $D_i$  *undermines* the original judgment that gender *is* explanatorily relevant to acceptance. It is *not* because the probabilities of  $B$  are *different* in each cell of the new partition. On the contrary, it is because the probabilities of  $B$  are *the same* in the male *vs* female cells (within each department) of the new partition.
- Intention?: “all (*causally*) relevant factors” must be included in a partition *before* a final judgment of relevance can be made.
- Really, “homogeneous” means “including all relevant factors”. Here, I think we need to *presuppose* a “causally relevant factor” concept.