

Philosophy 142: Why S2?

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C. I. Lewis decided that the system **S2**, or N_ρ , was to be regarded as the correct system for strict implication. For us, having just worked through the non-normal interpretations of **S2**, this may seem odd. Why not choose a standard system like K_ρ instead? But recall that in the development of modal logic, axiomatizations came first and the ‘non-standard’ semantics of **S2** was only introduced by Kripke much later. Moreover, Lewis felt that the acceptable version of transitivity for strict implication had the form: $((P \rightarrow Q) \wedge (Q \rightarrow R)) \rightarrow (P \rightarrow R)$. The stronger ‘all arrow’ exportation version $(P \rightarrow Q) \rightarrow ((Q \rightarrow R) \rightarrow (P \rightarrow R))$ was to be avoided. The following exercises now demonstrate why Lewis chose **S2** over **S3** (*i.e.*, $N_{\rho\tau}$).

- 1.** Show $\vdash_{N_{\rho\tau}} (P \rightarrow Q) \rightarrow ((Q \rightarrow R) \rightarrow (P \rightarrow R))$.
- 2.** Show $\not\vdash_{N_\rho} (P \rightarrow Q) \rightarrow ((Q \rightarrow R) \rightarrow (P \rightarrow R))$. Find a finite counter-model.
- 3.** Show $\vdash_{N_\rho} ((P \rightarrow Q) \wedge (Q \rightarrow R)) \rightarrow (P \rightarrow R)$.