Reasonable inference. The inference from a sequence of sentences of **L**, σ, to a sentence of **L**, P is *reasonable-in-***L** if and only if for all interpretations and all contexts k such that $A(\sigma, k)$, $S(g(\sigma, k))$ entails $\|P\|_{g(\sigma, k)}$.

Note that there is no language independent concept of reasonable inference analogous to the language independent notion of entailment. The reason is that, while we have in the theory a notion of proposition that can be characterized independently of any language in which propositions are expressed, we have no corresponding non-linguistic concept of statement, or assertion. One could perhaps be defined, but it would not be a simple matter to do so, since the identity conditions for assertion types will be finer than those for propositions. The reason for this is that different sentences may have different appropriateness conditions even when they express the same proposition.

TWO RECENT THEORIES OF CONDITIONALS

In recent years, two new and fundamentally different accounts of conditionals and their logic have been put forth, one based on nearness of possible worlds (Stalnaker, 'A Theory of Conditionals', 1968, this volume, pp. 41–55; Lewis, *Counterfactuals,* 1973) and the other based on subjective conditional probabilities (Adams, *The Logic of Conditionals,* 1975). The two accounts, I shall claim, have almost nothing in common, They do have a common logic within the domain on which they both pronounce, but that, as far as I can discover, is little more than a coincidence. Each of these disparate accounts, though, has an important application to natural language, or so I shall argue. Roughly, Adams' probabilistic account is true of indicative conditionals, and a nearness of possible worlds account is true of subjunctive conditionals. If that is so, the apparent similarity of these two 'if' constructions hides a profound semantical difference.

1. THE TWO ACCOUNTS

I begin with a rough and simplified sketch of the two accounts and relationships between them. First, some terminology: I shall use 'proposition' as a theory-laden word, the theory being a representation of subjective probability, or *credence.* On this representation, we start with a set t of all *epistemically possible worlds* (or *worlds*). Any proposition is identified with a set of worlds, the worlds *in* which the proposition is *true.* Not every subset of t need be a 'proposition'; rather the 'propositions' comprise a fixed set $\mathscr{F}$ of subsets of t. $\mathscr{F}$ is required to be a 'field of sets' whose 'universal set' is t. $\mathscr{F}$ is a *field of sets* iff $\mathscr{F}$ is a set of sets and, where $t = \bigcup \mathscr{F}$, $t \in \mathscr{F}$ and $\mathscr{F}$ is closed under the operations of union and t-complementation. t is called the *universal set* of $\mathscr{F}$. Members of $\mathscr{F}$ are called *propositions* of $\mathscr{F}$, and members of t are called *worlds* of $\mathscr{F}$. A person's *credences,* or degrees of belief, are represented by real numbers from zero to one, and the function ρ which gives them is a probability measure on $\mathscr{F}$: a non-negative real-valued function whose domain is $\mathscr{F}$, such that $\rho(t) = 1$ and where propositions a and b are disjoint, $\rho(a \cup b) = \rho(a) + \rho(b)$. (See Kyburg, 1980, pp. 14–18). When $\rho(a) \neq 0$, $\rho(b/a)$ is defined as the quotient

$\rho(a \cap b)/\rho(a)$; when $\rho(a) = 0$, $\rho(b/a)$ need not be defined for purposes here. Where ρ gives a subject's credences, $\rho(b/a)$ is his conditional credence in b given a; it is the credence he would give b were he to learn a and nothing else. Logical notation will be used: '&' or juxtaposition for intersection, '$\vee$' for union, '$-a$' or '$\bar{a}$' for the t-complement of a, and '$a \supset b$' for $\bar{a} \vee b$. We let f be the empty set, and represent entailment set-theoretically: a *entails* b iff $a \subseteq b$.

On the Adams account (1975), an indicative conditional need not express a proposition in this sense. Indicative conditionals are rather to be understood through their conditions of acceptance or assertability, and where a and b are propositions, one accepts the indicative conditional 'If a, then b' iff one's conditional credence in b given a is sufficiently high. On this basis, a logic for conditionals can be constructed.

Consider first the logic of propositions. For finite sets of propositions, the notions of consistency and consequence can be formulated in terms of probabilities, and the notions so formulated turn out to be equivalent to the notions in their standard formulation. Where $\mathscr{F}$ is a field of sets and $\mathscr{A}$ is a finite set of propositions of $\mathscr{F}$, we can define

> $\mathscr{A}$ is *p-consistent* iff for every $\delta > 0$, there is a probability measure ρ on $\mathscr{F}$ such that for every $A \in \mathscr{A}$, $\rho(A) > 1 - \delta$ (Adams, 1975, p. 51).

Where $\mathscr{A}$ is a finite set of propositions of $\mathscr{F}$ and B is a proposition of $\mathscr{F}$, we can define

> B is a *p-consequence* of $\mathscr{A}$ iff for every $\epsilon > 0$ there is a $\delta > 0$ such that for any probability measure on ρ on $\mathscr{F}$ with $\rho(A) > 1 - \delta$ for each $A \in \mathscr{A}$, we have $\rho(B) > 1 - \epsilon$.

(If there is such a δ, it turns out, then ϵ/n, where n is the number of propositions in $\mathscr{A}$, is such a δ.) It can then be shown that $\mathscr{A}$ is p-consistent iff it is consistent, and B is a p-consequence of $\mathscr{A}$ iff B is a consequence of $\mathscr{A}$. (Adams, 1975, pp. 57–58).

Turn now to conditionals, and consider $\rho(a \to b)$ just to be the conditional probability $\rho(b/a)$. The definitions of 'p-consistent' and 'p-consequence' can then be applied without change to conditionals and sets of conditionals, or to mixed sets of conditionals and propositions. (Indeed in these definitions, we can represent any proposition a by the conditional $t \to a$, since for any probability measure ρ, $\rho(a) = \rho(a/t) = \rho(t \to a)$.)

Note that this account applies only to conditionals constructed from propositions, with $\to$ the main connective. Where a, b, and c are propositions, the account deals with $a \to b$, but not with $(a \to b) \to c$, $a \to (b \to c)$, $a \,\&\, (b \to c)$, $a \vee (b \to c)$, $-(a \to b)$, and the like. The account is not one of conditionals embedded in longer sentences. Formally, we might consider a conditional simply to be an ordered pair of propositions; in any case, for all the Adams account tells us, a conditional is not itself a proposition and cannot be treated as one.[1]

On the Stalnaker nearness account (1968), in contrast, a conditional is a proposition. What proposition it is is determined by a *selection function* (or *s-function*), which we may think of as picking out, for each non-empty proposition a and world w, the world in a (or *a-world*) nearest to w. The conditional 'If a then b' says that the nearest a-world to the actual world is a b-world. In other words, for a given s-function σ, the conditional $a \mathbin{\Box\!\!\to}_\sigma b$ determined by σ is the set $\{w \mid \sigma(a, w) \in b\}$. Formally, σ is an *s-function* for $\mathscr{F}$ iff to every non-empty proposition a and world w, σ assigns a world, and these conditions are satisfied for every world w, proposition $a \neq f$, and proposition b.

(S1) $\sigma(a, w) \in a$.

(S2) If $w \in a$, then $\sigma(a, w) = w$.

(S3) If $a \subseteq b$ and $\sigma(b, w) \in a$, then $\sigma(a, w) = \sigma(b, w)$.

A Stalnaker conditional as I have defined it is a set of worlds but may not be a proposition. That is to say, let $\mathscr{F}$ be a field of sets, a and b be propositions of $\mathscr{F}$ with $a \neq f$, and σ be an s-function for $\mathscr{F}$; then $a \mathbin{\Box\!\!\to}_\sigma b$ is a subset of t but may not be a proposition of $\mathscr{F}$. σ will be called an *internal s-function* for $\mathscr{F}$ if σ is an s-function for $\mathscr{F}$ and for every a, $b \in \mathscr{F}$ with $a \neq f$,

(S4) $a \mathbin{\Box\!\!\to}_\sigma b$ is a proposition.

Given propositions a and b, different s-functions σ will yield different sets of worlds as the value of $a \mathbin{\Box\!\!\to}_\sigma b$. The choice of an s-function, Stalnaker says, is a pragmatic matter, which is determined by context (1968), pp. 109–111; this volume, pp. 51–52; 1975)

Stalnaker's account allows conditionals to be embedded, so that $(a \to b) \to c$, $a \to (b \to c)$, $a \vee (b \to c)$, and the like are allowed. Let us confine ourselves for the moment, though, to expressions of the kind Adams countenances, and return to treating conditionals as ordered pairs of propositions. That way, we can compare what the Stalnaker theory has to say with what the Adams theory has to say within the more limited domain of the Adams theory. Where $\mathscr{F}$ is a field of sets, then, a *conditional* $a \rightarrow b$ of $\mathscr{F}$ will be an ordered pair $\langle a, b\rangle$ of propositions of $\mathscr{F}$, with $a \neq f$.

Note first that, as with the Adams theory, we may identify a proposition a with a conditional $t \rightarrow a$. For any *s*-function σ, $a = (t \Box\!\!\rightarrow_\sigma b)$; that follows from (S2). Thus instead of talking about logical relations among propositions and conditionals here, we may speak simply of conditionals.

Now using the Stalnaker machinery, we can give new characterizations of consistency and consequence of sets of conditionals. Let $A_1, \ldots, A_n$, C be conditionals of $\mathscr{F}$, and let $\mathscr{A} = \{A_1, \ldots, A_n\}$. Henceforth, where A is a conditional $a \rightarrow b$, write $a \Box\!\!\rightarrow_\sigma b$ as A^σ. $\mathscr{A}$ is *s-consistent* iff for some *s*-function σ for $\mathscr{A}$, the set $\mathscr{A}^\sigma = \{A_1^\sigma, \ldots, A_n^\sigma\}$ is consistent. Here for any given σ, $A_1^\sigma, \ldots, A_n^\sigma$ are sets of epistemically possible worlds, and so to say that $\mathscr{A}^\sigma$ is consistent is just to say that it has a non-empty intersection. C is an *s-consequence* of $\mathscr{A}$ iff for every *s*-function σ for $\mathscr{F}$, C^σ is a consequence of $\mathscr{A}^\sigma$, or in other words, iff for every such σ,

$$A_1^\sigma \;\&\; \ldots \;\&\; A_n^\sigma \subseteq C^\sigma.$$

Modified versions of these definitions restrict consideration to *s*-functions which are internal: $\mathscr{A}$ is *s-consistent in the strong sense* iff for some internal *s*-function σ for $\mathscr{F}$, $\mathscr{A}^\sigma$ is consistent, and C is an *s-consequence of $\mathscr{A}$ in the weak sense* iff for every internal *s*-function σ for $\mathscr{F}$, C^σ is a consequence of $\mathscr{A}^\sigma$. Theorem 2 of the Appendix shows that these definitions are equivalent to the ones they modify.

Now at least for finite sets of conditionals, the relations of *p*-consequence and *s*-consequence coincide, as do the properties of *p*-consistency and *s*-consistency. That I shall prove in Section 3 and the Appendix. Thus even if the two theories are incompatible with each other, if one explains the logic of conditionals on their common domain, the other will appear to do so equally well.

2. NEARNESS AND PROBABILITIES: A PRIMER

Why do the two accounts yield the same logic? It might seem that they do so because both employ the same fundamental idea, that of a minimal change or minimal revision. They do so, though, in different ways. Here are the maxims that guide the two treatments. Adams' maxim: to decide whether to believe a conditional $a \rightarrow b$, hypothetically revise your beliefs in a minimal way so as to believe a, and then see if you believe b. Stalnaker's maxim:

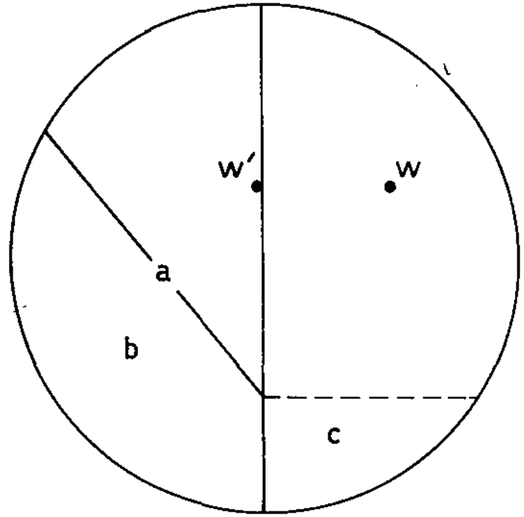

Fig. 1.

to decide whether a conditional $a \rightarrow b$ is true in a world w, change w in a minimal way so that a is true, and then see whether b is true in that changed world. Adams' system, then, involves changes in states of belief, whereas Stalnaker's involves changes in a world – that is, in an epistemically possible world. Now a world, as the term is used here, is not a state of belief. A world is rather a maximally specific way things might be. A state of belief, in contrast, is no more specific than one's beliefs are opinionated; it can be represented by a probability measure. Take an illustrative contrast: any world is either one in which Richard III had the two young princes killed or one in which he did not, but I give some credence to both possibilities. Thus my credence measure – the probability measure that represents my state of belief – assigns positive credence both to the set of worlds in which he had them killed and to the set of worlds in which he did not.[2]

The logic of states of belief can be illustrated by Venn diagrams.[3] In a Venn diagram, worlds are represented by points and propositions by regions, with the area of each region proportional to its probability. The entire region of the diagram is a proposition k of probability one. In Figure 1, the entire circle represents k, the left half of the circle represent a proposition a, and the lower left part of that half-circle represents a proposition b which entails a. A minimal revision of k to accomodate a simply involves erasing the right half of the circle and expanding the left half uniformly. A minimal change in w to make a true is a shift from world w to that a-world w' which, in some sense, is most like w.

Likeness of worlds need not, in a Venn diagram, be represented by geometric nearness. Suppose, however, that σ indeed is an s-function for which $\sigma(a, w)$ is always the a-world geometrically nearest to w in Figure 1. Then c is the set of $\bar{a}$-worlds the nearest a-world to which is a b-world. Thus $b \vee c$ is the Stalnaker conditional $a \Box\!\!\rightarrow_\sigma b$ for this σ. Interesting facts can now be read off the diagram. In Figure 1, the conditional probability $\rho(b/a)$ is approximately $\frac{1}{2}$, since ab is roughly half the area of a. $\rho(b \vee c)$, which is $\rho(a \Box\!\!\rightarrow_\sigma b)$, on the other hand, is considerably less than $\frac{1}{2}$. We thus see that nothing in the Stalnaker logic requires that $\rho(a \Box\!\!\rightarrow_\sigma b) = \rho(b/a)$ when $\rho(a) \neq 0$; the probability of a Stalnaker conditional may be distinct from the corresponding conditional probability.[4]

Moreover, for any conditional proposition $a \Rightarrow b$ to be such that

(1) $\rho(a \Rightarrow b) = \rho(b/a),$

$a \Rightarrow b$ must divide the $\bar{a}$-worlds in exactly the proportion in which b divides the a-worlds. In other words, if in Figure 2, c is the $\bar{a}$-part of $a \Rightarrow b$, we must have

(2) $\rho(c)/\rho(\bar{a}) = \rho(b)/\rho(a).$

That should be obvious from Figure 2. The proof assumed only that $a \Rightarrow b$ is a genuine proposition, and that it is true in every ab-world and false in every $a\bar{b}$-world, so that

(3) $a \,\&\, (a \Rightarrow b) = ab.$

The question is then how $a \Rightarrow b$ should divide the $\bar{a}$-worlds. For to treat a conditional $a \rightarrow b$ as a proposition $a \Rightarrow b$ is to suppose that in every possible world in which $\bar{a}$ holds, there is a fact of the matter whether $a \Rightarrow b$ holds in that world: the conditional is either true or false in that world. Now we have $(a \Rightarrow b) \doteq (b \vee c)$; thus since b and c are disjoint, $\rho(a \Rightarrow b) = \rho(b) + \rho(c)$. Since $b \subseteq a$, we have $\rho(b/a) = \rho(b)/\rho(a)$. Thus since $\rho(a) \neq 0$, (1) becomes

$$\rho(b) + \rho(c) = \rho(b)/\rho(a).$$

Given that $1 - \rho(a) \neq 0$, this is algebraically equivalent to

$$\frac{\rho(c)}{1 - \rho(a)} = \frac{\rho(b)}{\rho(a)},$$

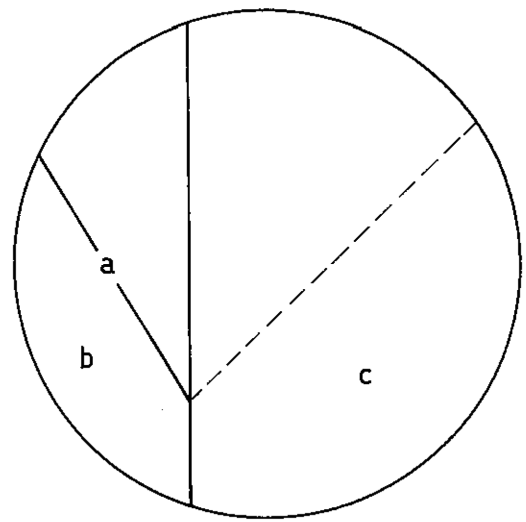

Fig. 2.

and with $\rho(\bar{a})$ substituted for $1 - \rho(a)$, this is (2), which was to be proved. c, then, is carved out of $\bar{a}$ in such a way as to make th ratio $\rho(c)/\rho(\bar{a})$ equal to $\rho(b)/\rho(a)$. I shall call this the *Fundamental Consequence* of requirements (1) and (3). A like argument shows that (2) and (3) entail (1); thus given (3), we have that (2) and (1) are equivalent.

It is hard to imagine a natural way of choosing c that would satisfy the Fundamental Consequence. We have already seen that not all Stalnaker conditionals do: it is not the case that for every ρ, a, b, and σ,

(4) $\rho(a \Box\!\!\rightarrow_\sigma b) = \rho(b/a)$ if $\rho(a) \neq 0.$

David Lewis (1976, pp. 300–303; this volume, pp. 131–134) has proved something stronger: that except in utterly trivial cases, there is no σ such that for every ρ, a, and b, (4) holds. Lewis's result, indeed, is even stronger than this. Let $\Rightarrow$ be any two-place propositional function: function which, to any two propositions $a \neq f$ and b assigns a proposition $a \Rightarrow b$. We do not require that $\Rightarrow$ be a Stalnaker conditional. We do suppose, for some fixed ρ, that $\rho(a \Rightarrow b) = \rho(b/a)$ whenever $\rho(a) \neq 0$, and that the same holds for the probability measure obtained from ρ by conditionalizing on any proposition c of non-zero probability. In other words, we suppose the following.

(PC) For any $a, b, c \in \mathscr{F}$, if $\rho(ac) \neq 0$, then $\rho(a \Rightarrow b/c) = \rho(b/ac).$

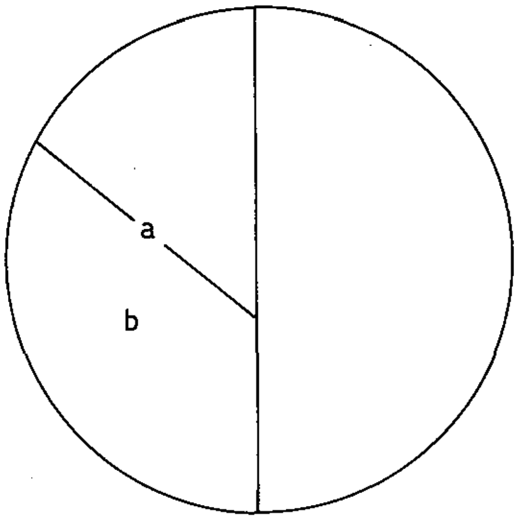

Fig. 3.

A probability measure will be called *non-trivial* iff there are at least three mutually disjoint propositions to which it assigns non-zero probability. Lewis showed[5] that (PC) holds for no non-trivial probability measure ρ.

The proof is best seen by means of Figure 3. There $\bar{a}$ and b are two of the three mutually disjoint propositions whose existence is assured by the non-triviality of ρ; thus $a\bar{b}$, b, and $\bar{a}$ partition the space, all have non-zero probability, and $ab = b$. Now from (PC),

$$\rho(a \Rightarrow b/\bar{b}) = \rho(b/a\bar{b}) = 0.$$

Thus $a \Rightarrow b$ and $\bar{b}$ intersect at most in a set of measure zero, so that $\rho(a \Rightarrow b) \leqslant \rho(b)$. But this is absured, since from (PC) and $ab = b$,

$$\rho(a \Rightarrow b) = \rho(b/a) = \rho(b)/\rho(a),$$

and since $\rho(a) < 1$, it follows that $\rho(a \Rightarrow b) > \rho(b)$.

We began with the question of why two fundamentally different theories of conditionals, Adams' and Stalnaker's, yield the same logic on their common domains. From Lewis' proof, we know that the answer cannot be this: that for some *s*-function σ, we have $\rho(a \mathrel{\Box\!\!\rightarrow} \sigma\, b) = \rho(b/a)$ for every probability measure ρ and pair of propositions a and b with $\rho(a) \neq 0$. The answer cannot even be this: that for some *s*-function σ and probability measure ρ,

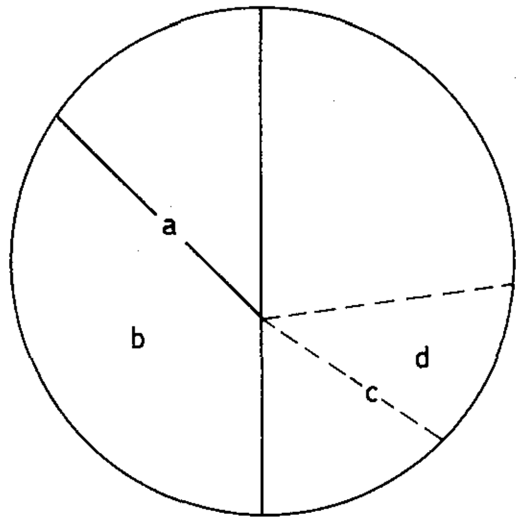

Fig. 4.

we have $\rho_c(a \mathrel{\Box\!\!\rightarrow}_\sigma b) = \rho_c(b/a)$ for all propositions a, b, c with $\rho(ac) \neq 0$, where ρ_c is the probability measure obtained from ρ by conditionalizing on c. We might, however, look for a weaker ground for the sameness of the two logics. Might it be the case, for at least some internal *s*-function σ, that there is a fixed probability measure ρ such that, for any two propositions a and b with $\rho(a) \neq 0$, we have $\rho(a \mathrel{\Box\!\!\rightarrow}_\sigma b) = \rho(b/a)$?

Stalnaker (1976, pp. 303–304) has shown that even to this question the answer is no.[6] For no non-trivial probability measure ρ on field of sets $\mathscr{F}$ is there an internal *s*-function σ such that for every conditional $a \to b$ of $\mathscr{F}$, the following hold.

(i) $a \,\&\, (a \mathrel{\Box\!\!\rightarrow}_\sigma b) = ab$

(ii) $\rho(a \mathrel{\Box\!\!\rightarrow}_\sigma b) = \rho(b/a)$ if $\rho(a) \neq 0$.

For suppose, as in Figure 4, there are three mutually disjoint propositions of non-zero probability, two of which are $\bar{a}$ and b. Again let c be $\bar{a} \,\&\, (a \mathrel{\Box\!\!\rightarrow}_\sigma b)$. From (2), the Fundamental Consequence of (i) and (ii), we know that c must have non-zero probability. Now consider the conditional $\bar{c} \mathrel{\Box\!\!\rightarrow}_\sigma a\bar{b}$. For this, the Fundamental Consequence is that where $d = c \,\&\, (\bar{c} \mathrel{\Box\!\!\rightarrow}_\sigma a\bar{b})$, we must have $\rho(d)/\rho(c) = \rho(a\bar{b})/\rho(\bar{c})$. Since $\rho(a\bar{b}) \neq 0$ and $\rho(c) \neq 0$, we have $\rho(d) \neq 0$, and there is a d-world – that is, a world

w in which both c and $\bar{c} \Box\!\!\rightarrow_\sigma a\bar{b}$ obtain. c, though, is precisely the set of $\bar{a}$-worlds whose nearest a-world is a b-world, and since $a \subseteq \bar{c}$, c cannot contain any worlds whose nearest $\bar{c}$-world is an $a\bar{b}$-world. For if the nearest $\bar{c}$-world to a c-world is an a-world, it is the nearest a-world, and hence not a b-world. That completes Stalnaker's proof.

Note that the proof involves treating conditionals, and truth-functions involving conditionals, as themselves components of conditionals to which the requirement that the probability of a conditional equal the corresponding conditional probability applies. Van Fraassen succeeds in giving the requirement a more narrow scope. Start off with a finite field $\mathscr{F}$ of sets and a fixed probability measure ρ over $\mathscr{F}$. The resulting probability space, Van Fraassen shows, can be embedded in a larger probability space with an internal s-function σ such that the probability of any conditional $a \Box\!\!\rightarrow_\sigma b$ formed from propositions in the original set equals the corresponding conditional probability.[7] This result provides the key to answering the question of why the Adams and Stalnaker logics are the same in their common domain.

The rough idea of the Van Fraassen construction is this: Let there be many possible worlds, and apart from the requirement that each world be nearest to itself, decide nearness by chance. That is to say, where w is a world, whenever $w \notin a$ and $b \subseteq a$, let the chance that the nearest a-world to w is a b-world simply be $\rho(b/a)$. I shall call such an s-function a *ρ-random* s-function. Then indeed where σ is such an s-function and $c = \bar{a} \;\&\; (a \Box\!\!\rightarrow_\sigma b)$, we have $\rho(c)/\rho(\bar{a}) = \rho(b)/\rho(a)$: the Fundamental Consequence is satisfied. For $\rho(c)/\rho(\bar{a})$ is just the proportion of $\bar{a}$ worlds whose nearest a-world is a b-world, and by selecting the nearest a-world to any $\bar{a}$-world randomly, we guarantee that this proportion is just $\rho(b)/\rho(a)$.

3. THE EQUIVALENCE OF THE TWO LOGICS

The possibility of a ρ-random s-function helps to explain why the logic of p-consequence and s-consequence are the same: using such s-functions, we can show that any s-consequence is a p-consequence. Let $A_1, \ldots, A_n$, C be conditionals, let $\mathscr{A} = \{A_1, \ldots, A_n\}$, and let C be an s-consequence of $\mathscr{A}$. We are to prove that C is a p-consequence of $\mathscr{A}$. First we need to prove that embedding a field of sets in a larger one does not affect the relation of s-consequence; in the Appendix, that is Theorem 1 and the main result is Theorem 3. Now given $\epsilon > 0$, let $\delta = \epsilon/n$, and suppose $\rho(A) > 1 - \delta$ for every $A \in \mathscr{A}$. What we are to show is that $\rho(C) > 1 - \epsilon$.

Here is a sketch of the proof, with questions of embedding ignored. Let σ be a ρ-random s-function. Since C is an s-consequence of $\mathscr{A}$, C^σ is a consequence of $\{A_1^\sigma, \ldots, A_n^\sigma\}$, and so since $\rho(A^\sigma) > 1 - \delta$ for each $A \in \mathscr{A}$, we have $\rho(C^\sigma) > 1 - \epsilon$, and hence $\rho(C) > 1 - \epsilon$. Supposing that C is an s-consequence of $\mathscr{A}$, we have shown that C is a p-consequence of $\mathscr{A}$.

One way to prove the converse is this. Adams (1975, pp. 60–61) gives a set of inference rules that are complete for p-consequence: any p-consequence is derivable by those rules. Inspection of those rules reveals that each is sound for s-consequence: any conclusion derived by those rules is an s-consequence of its premises. Therefore any p-consequence is an s-consequence.[8]

I now sketch an independent proof that any p-consequence is an s-consequence. In the first place, for any finite set $\mathscr{A}$ of conditionals and conditional $c \rightarrow d$, $c \rightarrow d$ is a p-consequence of $\mathscr{A}$ iff $\mathscr{A} \cup \{c \rightarrow \bar{d}\}$ is not p-consistent, and $c \rightarrow d$ is an s-consequence of $\mathscr{A}$ iff $\mathscr{A} \cup \{c \rightarrow \bar{d}\}$ is not s-consistent. The first is noted by Adams, and both are straightforward consequences of the definitions. Therefore to show that any p-consequence is an s-consequence, it will suffice to show that any s-consistent set of conditionals is p-consistent. For suppose we have shown this, and suppose $c \rightarrow d$ is a p-consequence of $\mathscr{A}$. Then $\mathscr{A} \cup \{c \rightarrow \bar{d}\}$ is not p-consistent. Hence by supposition, $\mathscr{A} \cup \{c \rightarrow \bar{d}\}$ is not s-consistent, and so $c \rightarrow d$ is an s-consequence of $\mathscr{A}$. Hence to complete the proof that p- and s-consequence coincide, we need only prove that any s-consistent set of conditionals is p-consistent, which is Theorem 5 in the Appendix.

The idea of its proof is to show how, given a $\delta > 0$ and an s-function σ such that the set $\mathscr{A}^\sigma$ is consistent, to construct a probability measure ρ such that all the antecedants have non-zero probability and all the conditional probabilities $\rho(b_i/a_i)$ are greater than $1 - \delta$. Here is a sketch of the procedure. Since $\mathscr{A}^\sigma$ is consistent, we can let w^* be a world in its intersection; thus all of $a_1 \Box\!\!\rightarrow_\sigma b_1, \ldots, a_n \Box\!\!\rightarrow_\sigma b_n$ hold at w^*. We now order the antecedants $a_1, \ldots, a_n$ by their distance from w^*, and consider the sequence $w_1, \ldots, w_m$ of worlds nearest to w^* in the sequence of antecedants so ordered. We can let our probability measure ρ give non-zero probability only to worlds $w_1, \ldots, w_m$ in such a way that the probability of each world in the sequence dwarfs the combined probability of the remaining worlds in the sequence. That is, for each w_i, $\rho(w_i) \geqslant (1 - \delta)\rho(w_i \vee \ldots \vee w_m)$. Then for each a_k and b_k, we have $\rho(b_k/a_k) > 1 - \delta$. For suppose w_j is the nearest a_k-world to w^*. Then a_k holds at w_j, and holds at no w_i that is nearer than w_j to w^*. Moreover, since $a_k \Box\!\!\rightarrow_\sigma b_k$ holds at w^*, b_k holds at w_j. Thus the

a_k-worlds consist of some subset of $\{w_j, \ldots, w_m\}$, and the a_kb_k-worlds consist of at least w_j. Hence since $\rho(a_kb_k) \geqslant (1-\delta)\rho(w_j \vee \ldots \vee w_m) \geqslant (1-\delta)\rho(a_k)$, we have $\rho(a_kb_k)/\rho(a_k) \geqslant 1-\delta$, or in other words, $\rho(b_k/a_k) \geqslant 1-\delta$. The construction has succeeded, and the Theorem is proved.

We have shown that any *s*-consequence is a *p*-consequence and conversely: the Adams and Stalnaker logics are equivalent on their common domain. A striking aspect of both halves of the equivalence proof is that they draw on no important similarity of the pictures that motivate the two logics. Rather, each proof is based on a trick. The trick behind the proof that any *s*-consequence is a *p*-consequence is to find, for any probability measure ρ, a Van Fraassen *s*-function: a function σ which has the formal properties of a Stalnaker selection function, but is so far from reflecting any intuitive idea of 'nearness' of possible worlds that, for proposition a and world w in which a does not hold, it selects an a-world as formally 'nearest' to w at random. The trick behind the proof that any *p*-consequence is an *s*-consequence is to find, for any *s*-function σ, a probability measure ρ which corresponds in the needed way, but whose correspondance to σ is contrived and unnatural. We took an arbitrary world w^* in the intersection of a set of Stalnaker conditionals, ordered the antecedants by distance from w^*, and concentrated all the probability in the antecedant worlds nearest to w^*, in decreasing orders of magnitude as the antecedants grew more distant from w^*. The proofs in both directions, in short, match *s*-functions and probability measures in a contrived way.

Here, then, is our situation. We saw earlier that the superficial connection between the Stalnaker and Adams accounts – that both depend in some way on a 'nearest' revision – masked a fundamental disparateness of the accounts. Still, the two accounts yield the same logic in the domain on which they both pronounce. That suggests that in some deep way the two accounts indeed are connected. The proofs I have given, though, display the sameness of logics as resting not on a deep connection between the two accounts, but on contrivances. That does not preclude there being a deep connection which some other proof might display, but at this point we have no reason to regard the sameness of logics as anything but coincidence.

4. GRAMMATICALLY INDICATIVE AND SUBJUNCTIVE CONDITIONALS

I turn now to natural language. Semantically, I shall argue, conditionals are of two major kinds, which are alike only superficially. The Adams account applies to conditionals of one kind, which I shall call 'epistemic conditionals'; the Stalnaker account applied to conditionals of the other kind, which I shall call 'nearness conditionals'. To this semantical distinction there roughly corresponds a syntactical distinction. For the most part, what I shall call 'grammatically indicative' conditionals are epistemic conditionals and what I shall call 'grammatically subjunctive' conditionals are nearness conditionals. The grammatical distinction has little to do with the indicative and subjunctive moods, and I use the terms I do only for want of better.

As grammatical paradigms, take the pair:

(5) If Oswald hadn't shot Kennedy, no one would have shot Oswald.

(6) If Oswald didn't shoot Kennedy, no one shot Oswald.

(5) I shall treat as a paradigm of a grammatically subjunctive conditional; (6), as a paradigm of a grammatically indicative conditional. There are two grammatical differences between them, one in the antecedent and one in the consequent. In the antecedent, (5) uses the past perfect 'hadn't shot' where (6) uses the simple past 'didn't shoot'. In the consequent, (5) uses 'would have shot' where (6) uses 'shot'. Consider each of these in turn.

Although grammatically, only the antecedent of (6) is in the simple past, semantically both antecedents concern the simple past. The antecedent of (5) is thus grammatically prior to its time of reference. That can be seen most clearly from a variant of (5) with the same meaning and general grammatical form,

(7) If Oswald hadn't shot Kennedy when he did, no one would have shot Oswald.

The situation posited in the antecedent of (7) is one in which Oswald didn't shoot Kennedy at the time when in actual fact he did. It is not one in which Oswald hadn't shot Kennedy at the time when in fact he did; that could only mean that he hadn't already shot Kennedy at the time when in fact he shot Kennedy, which is true – or at least would be true in fact, Oswald shot Kennedy only once. The antecedent of (7), though, clearly posits a contrary to fact situation in which Oswald didn't shoot Kennedy at the very time when in fact he shot Kennedy. I shall call this use of a grammatical tense prior to the one ordinarily appropriate for the time of the antecedent an *antecedent tense shift*.

The second feature by which (5) differs from (6) is in its use of 'would' in the consequent. It will turn out that grammatically, 'would' acts as the past tense of 'will', and so the feature of (5) to note is that there is a form of 'will' in the consequent.

I shall call a conditional with antecedent tense shift and a model auxilliary such as 'will' in the consequent *grammatically subjunctive*, and other conditionals *grammatically indicative*. The terms are unfortunate in a way, since the antecedent of a grammatically subjunctive conditional, we shall see, need not be in the subjunctive mood. I use these terms because I can find no simple, familiar terms to mark the systematic distinction I want to make which are not at least equally misleading.

Here are general directions for constructing conditionals with the features I have noted. Conditionals which are constructed in this way and which, as a result, exhibit antecedent tense shift, form, I shall claim, a significant grammatical class. Begin with a stem conditional, which, because its verbs lack tense and person, is not itself a piece of English.

(8) *If he be upset, she comfort him.*

Optionally, either stem may be transformed into a perfect, progressive, or perfect progressive.

(9) *If he have been upset, she have comforted him.*

(10) *If he be upset, she be comforting him.*

(11) *If he have been upset, she comfort him.*

A model auxilliary[9] such as 'will' is now applied to the verb stem of the consequent, so that (8)–(11) become

(12) *If he be upset, she will comfort him.*

(13) *If he have been upset, she will have comforted him.*

(14) *If he be upset, she will be comforting him.*

(15) *If he have been upset, she will comfort him.*

Finally, either present tense, with appropriate person, is applied to both clauses, or past tense is applied to both clauses. (12)–(15) now become

(16) If he is upset, she will comfort him.

(17*) If he was upset, she would comfort him.*

(18) If he has been upset, she will have comforted him.

(19) If he had been upset, she would have comforted him.

(20) If he is upset, she will be comforting him.

(21*) If he was upset, she would be comforting him.*

(22) If he has been upset, she will comfort him.

(23) If he had been upset, she would comfort him.

(17*) and (21*) are not securely part of standard English; their antecedent need to be in the subjunctive mood.

(17) If he were upset, she would comfort him.

(21) If he were upset, she would be comforting him.

Present antecedents in (16), (18), (20), and (22), on the other hand, seem quaint or worse in the subjunctive mood; see (12)–(15). The subjunctive mood is now vestigial in English, and applies to antecedents of what I am calling 'grammatically subjunctive conditionals' only in the past tense.

The instructions I have given allow thirty-two grammatically subjunctive 'will' conditionals to be constructed from the stem conditional (8). Optional features are past or present tense, perfect antecedent, progressive antecedent, perfect consequent, and progressive consequent. Perhaps not all thirty-two are easily interpretable, but I think that for each, some imaginable context can be found in which it can be read with antecedent tense shift.

The rules for the formation of indicative conditionals are much more flexible: a modal auxiliary in the consequent is optional, and past and present tenses can apply separately to antecedent and consequent. The following, for instance, are allowable.

(24) If he was upset, she is comforting him.

(25) If he is upset, she had been yelling at him.

(26) If he was upset, she will learn about it.

These rules allow the construction both of sentence like (24)–(26) which cannot be constructed with the rules for subjunctive conditionals, and of sentences which can. In the case of sentences like (22), which can be constructed by either set of rules, the sentence is syntactically ambiguous: it is read as a grammatically subjunctive conditional if it is read as having antecedent tense shift, and otherwise it is read as a grammatically indicative conditional.

I propose that what I have been calling 'grammatically subjunctive conditionals' form a significant grammatical class. Grammatically subjunctive conditionals are those conditionals with the following two features.

(i) A modal auxilliary is the stem verb of the consequent, its tense agreeing with that of the antecedent.

(ii) Feature (i) induces an antecedent tense shift: the time to which the antecedent is understood as referring is after the time to which if would refer if uttered as an independent sentence . .

Indicative conditionals lack feature (ii) and may lack feature (i). Only grammatically subjunctive conditionals have antecedents in the subjunctive mood, but many have antecedents in the indicative mood. The next question is whether this grammatical distinction has any semantical consequences apart from the antecedent tense shift.

5. THE SEMANTICAL DISTINCTION

Sly Pete and Mr. Thomas Stone are playing poker aboard a Mississippi River boat. Both Pete and Stone are good poker players, and Pete, in addition, is unscrupulous. Stone has bet up to the limit for the hand, and it is now up to Pete to call or fold. Zack has seen Stone's hand, which is quite good, and signalled its contents to Pete. (Call this moment t_0). Stone, suspecting something, demands that the room be cleared. Five minutes later, Zack is standing by the bar, confident that the hand has been played out but ignorant of its outcome. (Call this moment t_1). He now entertains these two conditionals.

(27) If Pete called, he won.

(28) If Pete had called, he would have won.

At t_1, Zack accepts (27), because he knows that Pete is a crafty gambler who knew Stone's hand; thus Zack knows that Pete would not have called unless he had a winning hand. (28), on the other hand, Zack regards as probably false. For he knows that Stone's hand was quite good, and therefore regards it as unlikely that Pete had a winning hand. Thus he regards it as unlikely that if Pete had called, he would have won.

(27) is grammatically indicative, whereas (28) is grammatically subjunctive. To this grammatical difference, we have seen, there corresponds a semantical difference. What is it? Zack knows enough that were he to learn that Pete had called and to learn nothing else, he would come to believe that Pete had won. In other words, because Zack believes that Pete would not have called unless he had a winning hand, Zack's conditional credence ρ (Pete won / Pete called) is close to one. (27), then, seems to fit Adams' theory: Zack's acceptance of (27) depends on the corresponding conditional probability's being high, the probability in question being Zack's credence.

F. P. Ramsey wrote in a footnote (1931, p. 247), "If two people are arguing 'If p will q?' and both are in doubt as to p, they are adding p hypothetically to their stock of knowledge and arguing on that basis about q . . . . We can say they are fixing their degrees of belief in q given p." This test – to see whether you accept q if p, add p hypothetically to your knowledge and note whether you now accept q – will be called the *Ramsey test.*[10] A conditional to which the Ramsey test applies will be called an *epistemic conditional.* A past grammatically indicative conditional like (27), it appears, is an epistemic conditional: it is accepted by anyone whose corresponding conditional credence is sufficiently high.

Although (27) has a clear acceptance condition, it does not have clear truth conditions. Suppose that in fact, as Zack suspects, Pete did not call, because he know he held a losing hand. It is not clear what then has to be true for (27) to be true.

(28), on the other hand, Zack regards as unlikely, because he thinks it unlikely that Pete had a winning hand. It seems, then, that Zack regards (28) as a proposition which is true or false according as Pete has a winning or a losing hand. Clearly his acceptance of (28) does not go by the Ramsey test, since (28) passes the test but Zack does not accept it. Whereas, then, (27) is an epistemic conditional which Zack does not in any obvious way treat as a proposition, the grammatically subjunctive conditional (28) is not an epistemic conditional, but is treated as epistemically equivalent to the proposition that Pete had a winning hand.

In many respects, at least, (28) fits Stalnaker's theory. (28) is treated as a proposition, whose truth or falsity depends on qualities of its subject matter, the game and its players, rather than the state of mind of the person who entertains it. Whether (28) is true depends, it seems reasonable to claim, on whether Pete wins in a world in which Pete calls, and which of all such worlds is nearest to the actual world by the following criteria: it is exactly like the actual world until it is time for Pete to call or fold; then it is like the actual world apart from whatever it is that constitutes Pete's decision to call or to fold, and from then on it develops in accordance with natural laws.[11] I do not mean to commit myself here to the details of Stalnaker's theory, but to note that the picture that guides Stalnaker applies without undue strain to past grammatically subjunctive conditionals like (28). I shall call any conditional to which such a picture applies a *nearness conditional.*

In the past tense, it seems from the example, a grammatically indicative conditional is an epistemic conditional and a grammatically subjunctive conditional is a nearness conditional. What of the future tense? For it, there is no obvious grammatical contrast to make; the salient relevant future conditional is

(29) If Pete calls, he'll win.

This I have classified as grammatically subjunctive; semantically is it an epistemic conditional or a nearness conditional? Indeed can that distinction be made for the future at all?

Consider (29) as uttered by Zack at t_0, when Pete is about to fold or call. If (27) is an epistemic conditional, the Zack accepts it at t_0. For at t_0, he knows everything relevant that he knows at t_1: that Pete is a skilled player who knows Stone's hand, and thus would not call unless he had a winning hand. Thus Zack's conditional credence at t_0, ρ_0 (Pete will win / Pete will call), is close to one, and if he treats (29) as an epistemic conditinal, he accepts (29). If, on the other hand, (29) is a nearness conditional, which is true if an only if Pete has a winning hand, then Zack at t_0 regards (29) as unlikely. For he regards it as highly likely that Stone has the better hand.

We can, then, distinguish a reading of a future tense conditional as an epistemic conditional from a reading of it as a nearness conditional. To see whether you read (29) as an epistemic conditional or as a nearness conditional, put yourself in Zack's epistemic situations at t_0 and see whether you accept (29). If you do, you read it as an epistemic conditional; if you regard (29) as unlikely, then you read it as a nearness conditional.

My informal polls on whether Zack accepts (29) have been inconclusive, but most people I have asked think he does. Thus (29) seems to be read as an epistemic conditional, and thus semantically like the future of an indicative rather than a subjunctive conditional.

If Pete were to call, he would win.

If Pete called, he would win.

are generally treated as nearness conditionals: they are regarded as unlikely, given the information available to Zack at t_0, and as true if and only if Pete has a winning hand. A reading as an epistemic conditional is perhaps most securely elicited by the sentence

If Pete's going to call, he'll win.

Grammatically indicative conditionals seem in general to be epistemic conditionals; these I shall call simply *indicative conditionals.* Grammatically subjunctive conditionals with 'would' are, I have argued, nearness conditionals, and it is these that I shall call *subjunctive conditionals.* Conditionals with antecedent tense shift and 'will' in the consequent I shall leave aside.

6. THOUGHT AND COMMUNICATION WITHOUT CONDITIONAL PROPOSITIONS

Nearness conditionals are propositions, whereas nothing so far in our account of epistemic conditionals requires that they be propositions. We have given acceptance conditions for them but not truth-conditions, and propositions need truth-conditions. Moreover, it looks difficult to interpret epistemic conditionals as propositions, for the most obvious approaches to doing so were ruled out by the results in Section 2. An epistemic conditional $a \to b$ is accepted by a person if and only if, where ρ is his credence measure, we have $\rho(b/a) \approx 1$. If we do suppose that $a \to b$ is a proposition, this amounts to the condition

$$\rho(a \to b) \approx 1 \quad \text{iff} \quad \rho(b/a) \approx 1.$$

If we suppose further that that is because always $\rho(a \to b) = \rho(b/a)$, then the results in Section 2 raise obstacles.

None of this, however, need be disturbing. What a theory of indicative conditionals should do is to explain their role in thought and communication, and that task in no way demands that indicative conditionals be construed as propositions. To see this, let me propose a line of explanation that does without indicative conditional propositions.

Take first thought, and consider belief in propositions. One can correctly be said to 'believe' or 'accept' a proposition, on this line of explanation, iff one's subjective probability for that proposition, or *credence* in it, is as close to one as matters for the purposes at hand.[12] For most purposes, for instance, I can be said to accept that my house will be standing in its usual place when I go home, but for purposes of explaining why I never allow my fire insurance to lapse, even for short periods, I cannot be said fully to accept that my house will not burn down before I go home, and hence cannot be said to accept that my house will be standing in its usual place when I go home.

Acceptance of an indicative conditional can be explained along the same lines, without invoking indicative conditional propositions. One accepts an indicative conditional iff one's corresponding conditional credence is as close to one as matters for the purposes at hand. (Such a high conditional credence I shall call a *conditional belief*, and I shall call sufficiently high conditional credence in b given *a belief in b given a*.

Take next communication, and consider first the communication of a proposition. In the standard, felicitous circumstances of communication, I accept a proposition and express it in a sentence, and my audience, hearing the sentence, comes to accept the proposition. That happens because I exploit certain conventions to get the audience to accept that I have the belief that I do. In felicitous cases, the audience trusts my sincerity and command of language, and for that reason it accepts, on account of my having uttered a sentence S in the circumstances, that I believe a certain proposition a. If the audience accepts a on my authority, it is because the audience supposes that I would not believe a unless I had adequate grounds for a, and takes my having adequate grounds for a as evidence sufficient to warrant accepting a.[14]

Now any such account of communication[15] – take it in your favorite version – will extend naturally to communication of conditional belief. In felicitous cases, I utter an indicative conditional, and thereby insure that the audience comes to accept that I have a certain conditional belief, belief in b given a. The audience does so because it trusts my sincerity and command of language. The audience then infers from my believing b given a that I have some good grounds for so believing, and takes that as a reason for itself believing b given a. Thus is my conditional belief communicated to them.

Conditional propositions, then, need play no role in an account of indicative conditionals. Conditional beliefs – states of high conditional credence – are just as much states of mind as are unconditional beliefs. There is no reason in what has been said here to suppose that a conditional belief constitutes an unconditional belief in a conditional proposition. We have, moreover, no reason to suppose that conditional beliefs must be communicated by means of conditional propositions: the devices which allow the communication of unconditional beliefs, we have seen, could just as well allow the communication of conditional beliefs. Conditional propositions, it seems, are superfluous in the communication of conditional beliefs.

7. PROPOSITIONAL THEORIES WITH CONDITIONAL NON-CONTRADICTION

Suppose we nevertheless do want to treat indicative conditionals as propositions. One way to do so is to adopt the theory that indicative conditionals are truth-functional; I shall discuss that theory in the next section. Other theories that treat conditionals as propositions – Stalnaker's, Lewis's, and Van Fraassen's (1976, p. 276, display line (18)) – share a law of *Conditional Non-contradiction*: that $a \rightarrow \bar{b}$ is inconsistent with $a \rightarrow b$. Now any theory with Conditional Non-contradiction confronts an anomaly which is illustrated by this version of the Sly Pete story.

Sly Pete and Mr. Stone are playing poker on a Mississippi riverboat. It is now up to Pete to call or fold. My henchman Zack sees Stone's hand, which is quite good, and signals its content to Pete. My henchman Jack sees both hands, and sees that Pete's hand is rather low, so that Stone's is the winning hand. At this point, the room is cleared. A few minutes later, Zack slips me a note which says "If Pete called, he won," and Jack slips me a note which says "If Pete called, he lost." I know that these notes both come from my trusted henchmen, but do not know which of them sent which note. I conclude that Pete folded.

If both these utterances express propositions, then I think we can see that both express true propositions. In the first place, both are assertable, given what their respective utterers know. Zack knows that Pete knew Stone's hand. He can thus appropriately assert "If Pete called, he won." Jack knows that Pete held the losing hand, and thus can appropriately assert "If Pete called, he lost." From this, we can see that neither is asserting anything false. For one sincerely asserts something false only when one is mistaken about something germane. In this case, neither Zack nor Jack has any relevant false beliefs. The relevant facts are these: (a) Pete had the losing hand, (b) he knew Stone's hand as well as his own, (c) he was disposed to fold on knowing that he had the losing hand, and (d) he folded. Zack knows (b) and (c), and he suspects (a) and therefore (d). Jack knows (a) and (c), and knowing Pete as he does, may well suspect (b) and therefore (d). Neither has any relevant false beliefs, and indeed both may well suspect the whole relevant truth. Neither, then, could sincerely be asserting anything false. Each is sincere, and so each, if he is asserting a proposition at all, is asserting a true proposition.

It follows that

(27) If Pete called, he won

as uttered by Zack is consistent with

(30) If Pete called, he didn't win

as uttered by Jack. For clearly as uttered by Jack,

(31) If Pete called, he lost

entails (30) in any context, so that if (31) as uttered by Jack is true, then (30) is. Then since both (27) as uttered by Zack and (30) as uttered by Jack are true, they are consistent. The only apparent way to reconcile this with Conditional Non-contradiction is to suppose that the sentence "If Pete called, he won" as uttered by Zack expresses a different proposition from the one the same sentence would express if it were uttered by Jack.

That fits Stalnaker's contention that the selection function is pragmatically determined (1968, pp. 109–111), so that different contexts of utterence invoke different *s*-functions. If the context in which Zack passes his note invokes an *s*-function σ and the context in which Jack passes his note invoke a different *s*-function τ, then (27) and (30) may express Stalnaker conditionals of the form $a \Box\!\!\rightarrow_{\sigma} b$ and $a \Box\!\!\rightarrow_{\tau} \bar{b}$, so that even though Conditional Non-contradiction holds for any fixed *s*-function, (27) as uttered by Zack does not contradict (30) as uttered by Jack. That is to say, even though $a \Box\!\!\rightarrow_{\sigma} b$ contradicts $a \Box\!\!\rightarrow_{\sigma} \bar{b}$ and $a \Box\!\!\rightarrow_{\tau} b$ contradicts $a \Box\!\!\rightarrow_{\tau} \bar{b}$, (27) as uttered by Zack expresses $a \Box\!\!\rightarrow_{\sigma} b$ and (30) as uttered by Jack expresses $a \Box\!\!\rightarrow_{\tau} \bar{b}$, and these do not contradict each other.

The difference in contexts here, though, has a strange feature. Ordinarily when context resolves a pragmatic ambiguity, the features of the context that resolve it are common knowledge between speaker and audience. If the chairman of a meeting announces "Everyone has voted 'yes' on that motion", what the audience knows about the context allows it to judge the scope of 'everyone'. In Example 2, in contrast, whatever contextual differences between the utterances there may be, they are unknown to the audience. I, the audience, know exactly the same thing about the two contexts: that the sentence is the content of a note handed me by one of my henchmen. Whatever differences in the context make them invoke different *s*-functions is completely hidden from me, the intended audience.

That seems strange, for suppose it is so. I trust my henchmen and they are not contradicting each other. I presumably believe Zack's message "If Pete called, he won," and that constitutes believing a proposition *c*. Had Jack instead of Zack slipped me the message, I would have believed that message, but that would constitute believing a different proposition c'. Yet since I don't know which of them slipped me the message, there is no difference in my intrinsic mental state in the two cases: whether my mental state constitutes believing *c* or c' depends not on what that state is intrinsically like, but on who slipped me the note.

Perhaps, though, this feature of the context theory is not as bizarre as I have made it out, for perhaps sentences with indexicals have the very feature I have depicted. Suppose Zack or Jack, I know not which, slips me a note which says

I have swiped Mr. Stone's gold watch chain.

Perhaps this expresses the proposition

(32) Zack has swiped Mr. Stone's gold watch chain

if the note came from Zack and

(33) Jack has swiped Mr. Stone's gold watch chain

if the note came from Jack. I believe the message, but that constitutes believing neither (32) nor (33), but that the writer of the note swiped Stone's gold watch chain.

What is left of the contextual theory is that indicative conditionals act in an indexical-like way, where the workings of the indexical elements can depend on things the audience does not know. Now since the assertability of an indicative conditional depends on the utterer's credences, it seems reasonable to suppose that its propositional content too depends on the utterer's credences. Indeed we are driven to accepting this dependence if we want conditionals to be propositions to which the Ramsey test applies.[16] For suppose that where ρ is the utterer's credence measure, the indicative conditional $a \rightarrow b$ is assertable iff $\rho(b/a) \approx 1$ and it has a propositional content *c* such that $a \rightarrow b$ is assertable iff $\rho(c) \approx 1$. A variant of the Lewis proof will show that there is no ρ-independent propositional function $\rightarrow$ that satisfies these conditions.

(i) ab entails $a \rightarrow b$

(ii) $a \rightarrow b$ is inconsistent with $a \rightarrow \bar{b}$

(iii) For all ρ such that $\rho(a) \neq 0$, $\rho(a \rightarrow b) \approx 1$ iff $\rho(b/a) \approx 1$.

For let $\rho(b/a) \approx 1$ and $\rho(a) \approx 0$. Then by (iii), $\rho(a \rightarrow b) \approx 1$, and hence by (ii), $\rho(a \rightarrow \bar{b}) \approx 0$. Obtain ρ' by conditionalizing on $\overline{ab}$; then $\rho'(\bar{b}/a) = 1$, but since $\rho(\overline{ab}) \approx 1$, $\rho'(a \rightarrow \bar{b})$ remains close to zero.

That leaves only one way for indicative conditionals to be treated as propositions which satisfy Conditional Non-contradiction. Let the propositional content of an indicative conditional depend on the utterer's epistemic state, and do so in such a way that the proposition is accepted by the utterer if and only if the utterer's corresponding conditional credence is sufficiently high. We might, in other words, have a three-place function $\rightarrow$ which yields a proposition $a \rightarrow_\rho b$ as a function of propositions a and b and probability measure ρ, with $\rho(a \rightarrow_\rho b) \approx 1$ when and only when $\rho(b/a) \approx 1$. We can interpret the indicative conditional as a propositional function satisfying Conditional Non-contradiction only at the cost of such radical dependence of the utterer's epistemic state.

8. EMBEDDING AND THE TRUTH-FUNCTIONAL THEORY

Why might we even want indicative conditionals to be propositions? Lewis (1976, p. 305) offers a reason: we then have an account of embedded indicative conditionals – of sentences of such forms as

$$a \rightarrow (b \rightarrow c),\ (a \rightarrow b) \rightarrow c,\ a \mathbin{\&} (b \rightarrow c),\ a \vee (b \rightarrow c), \text{ and } -(a \rightarrow b).$$

Now only a truth-functional theory will account for such embeddings straightforwardly. Here and from now on, let $\rightarrow$ be the indicative conditional connective. It seems[17] to be a logical truth that $a \rightarrow (b \rightarrow c)$ is equivalent to $ab \rightarrow c$. For instance

If they were outside, then if it rained they got wet

seems to be equivalent to

If they were outside and it rained, then they got wet.

From this equivalence and a few quite weak additional assumptions, though, it follows that $\rightarrow$, if it is a propositional function at all, is the truth-functional connective $\supset$. The additional assumptions are just that any conditional $a \rightarrow b$ is false in all $a\bar{b}$ worlds, and that if a entails b then $a \rightarrow b = t$. Our assumptions, then, are these: for all non-empty propositions a, b, and c,

(i) $[a \rightarrow (b \rightarrow c)] = (ab \rightarrow c)$

(ii) $(a \rightarrow b) \subseteq (a \supset b)$

(iii) If $a \subseteq b$, then $(a \rightarrow b) = t$.

Given (ii), it remains to be shown that $(a \supset b) \subseteq (a \rightarrow b)$. Consider the sentence

(34) $(a \supset b) \rightarrow (a \rightarrow b)$.

By (i), this is $[(a \supset b) \mathbin{\&} a] \rightarrow b$, or equivalently, $ab \rightarrow b$. By (iii), this is t. By (ii), (34) entails

$$(a \supset b) \supset (a \rightarrow b),$$

and to say that t entails a truth-functional conditional is to say that its antecedent entails its consequent. That completes the proof.

If, then, we want the indicative conditional to be a propositional function, and to account in that way for our readings of embedded indicative conditionals, then the function must be $\supset$. That a non-propositional theory of indicative conditionals fails to account for some embeddings, though, may be a strength. Many embeddings of indicative conditionals, after all, seem not to make sense. Suppose I tell you, of a conference you don't know much about,

(35) If Kripke was there if Strawson was, then Anscomb was there.

Do you know what you have been told? On the truth functional theory, you have been told that either Strawson was there and Kripke wasn't, or Anscomb was there. That seems surprising, though perhaps that is because some feature of conversational implicatures keeps the iterated conditional from being assertable.

Some iterated conditionals of the same form do seem to be assertable, but the way they are read is at odds with the truth-functional theory – at odds with it in a way for which conversational implicatures cannot account. Take, for instance, the conditional $(d \rightarrow b) \rightarrow f$,

(36) If the cup broke if dropped, then it was fragile.

That seems assertable by someone who knows that the cup was being held at a moderate height over a carpeted floor, even if he gives rather low credence to the cup's being dropped or to its being fragile. Suppose, though, that in fact, things are as he thinks likely: the cup was not dropped and was not fragile. Then interpreted truth-functionally, (36) is false. For $(d \supset b) \supset f$ is equivalent to $d\bar{b} \vee f$, and neither disjunct is true. The speaker, indeed, gives low credence to both disjuncts, and hence gives low credence to the disjunction. (36), then, is assertable, even though on a truth-functional interpretation it gets low credence. Conversational implicatures will not save the truth-functional theory from this anomaly. Conversational implicatures, after all, explain only why a sentence believed true may not be appropriately assertable, whereas this is a case of a sentence which is appropriately assertable, but according to the truth-functional theory, false.[18]

The advantage of the truth-functional theory over Adams' theory, according to Lewis, is that it accounts for embedded conditionals. We have seen that it does so incorrectly, and so the alleged advantage turns out to yield a reason for rejecting the truth-functional theory of indicative conditionals.

9. AN ASSESSMENT

If the truth-functional theory will not handle embedding, will a propositional theory with Conditional Non-contradiction? We learned in Section 7 that at best, such a theory will do the job only if the propositional function represented by $\rightarrow$ depends on the utterer's epistemic state. We have now learned that even that kind of dependence will not suffice, for as we saw at the beginning of the last section, if in a fixed epistemic context, $\rightarrow$ represents a fixed propositional function, then to account for some of the behaviour of the indicative conditional, we must conclude that $\rightarrow$ is truth-functional. Since we know that the indicative conditional is not truth-functional, we can eliminate that possibility.

One other possibility remains: that $\rightarrow$ always represents a propositional function, but that what that function is depends not only on the utterer's epistemic state, but on the place of the connective in the sentence. In $a \rightarrow (b \rightarrow c)$, for instance, we might suppose that the two different arrows represent two different propositional functions. Nothing we have seen rules that out.

The pursuit of such a theory, though, has now lost its advantage. A theory of indicative conditionals as propositions was supposed to give, at no extra cost, a general theory of sentences with indicative conditional components: simply add the theory of conditionals to our extant theory of the ways truth-conditions of sentences depend on the truth-conditions of their components. The alternative was to develop a new theory to account for each way indicative conditionals might be embedded in longer sentences, and that seemed costly. Now it turns out that for each way indicative conditonals might be embedded in longer sentences, a propositional theory will have to account for their propositional content, and do so in a way that is sensitive

to the place of each indicative conditional in its sentence. In $a \rightarrow (b \rightarrow c)$, the right and left arrows must be treated separately. What must be done with the left and right arrow in $(a \rightarrow b) \rightarrow c$ or with the arrows in $a \,\&\, (b \rightarrow c)$ and $a \vee (b \rightarrow c)$ we do not yet know. Thus, for instance, no account of sentences of the form $(a \rightarrow b) \rightarrow c$ will fall out of a simple general account of indicative conditionals as propositions; rather the account of indicative conditionals itself will have to confront separately the way left-embedded arrows work. A propositional theory would not save labor; instead it would demand all the labor that would have to be done without it.

What, then, of the alternative: to deal *ad hoc* with each kind of embedding without treating indicative conditionals as propositions? Here I think the prospects are not so bleak as might be supposed. In the first place, some sentences with indicative conditional components, such as (35), make no sense. An *ad hoc* treatment is more likely to account for this fact than is a theory which systematically assigns truth-conditions to every sentence. In the second place, various *ad hoc* accounts do turn out to work. Sentences with the apparent form $a \rightarrow (b \rightarrow c)$ can be read as really having the form $ab \rightarrow c$. A sentence with the apparent form $(a \rightarrow b) \,\&\, (c \rightarrow d)$ can be read as expressing a combination of two conditional beliefs, belief in b given a and belief in d given c.

More difficult is the conditional $(d \rightarrow b) \rightarrow f$,

(36) If the cup broke if dropped, then it was fragile.

I think, though, that an account can be given that explains why this sentence seems to make sense and others of the same form do not. Consider first the antecedent $d \rightarrow b$, "If the cup was dropped, it broke." Such an indicative conditional may have an *obvious basis*: a proposition c such that it is presupposed, for both utterer and audience, that he will believe the consequent given the antecedent iff he believes c. The obvious basis for the conditional $d \rightarrow b$ is c,

(37) The cup was disposed to break on being dropped.

c might hold either because the cup was fragile, or because it was being held over an especially hard floor or at an especially great height. Now when a conditional of the form $(d \rightarrow b) \rightarrow f$ is understandable, I propose, it is because the antecedent $d \rightarrow b$ has an obvious basis c and its obvious basis is understood in its place. The compound conditional $(d \rightarrow b) \rightarrow f$, then, is ordinarily understood as $c \rightarrow f$; (36) is read as

(38) If the cup was disposed to break on being dropped, then it was fragile.

This is indeed assertable by someone who knows that the cup was being held at a moderate height above a carpeted floor and does not know whether it was fragile. For were he to learn that it was disposed to break on being dropped, be would come to believe it fragile.

The obvious basis for an indicative conditional is not synonymous with the conditional itself. 'The cup broke if dropped' is assertable by me if I believe this:

(i) The cup was being held by someone who would not have dropped it unless it was highly fragile, but who might well have dropped it if it was highly fragile.

It is also assertable by me if I believe this:

(ii) A trusted assistant was under order to inform me if the cup dropped without breaking, and not to bother me otherwise. The cup may have been dropped, for all I know, but I have not been informed that it dropped without breaking.

Neither of these entails that the cup was disposed to break on dropping but belief in either would give me grounds for asserting that the cup broke if dropped.

On the account I am giving, understanding (36) as (38) depends on contextual prosuppositions that might have been absent. Suppose possibility (i) is taken seriously: both utterer and audience take it as something they might well come to believe, and there is vivid common awareness of the possibility. Then the presuppositions that make (37) the obvious basis for $d \rightarrow b$ have broken down. Thus on the account I am giving, (38) ceases to gloss (36), and indeed it becomes unclear how (36) is to be interpreted. That seems to me to be what indeed would happen in that case. The same goes for a case in which (ii) is taken seriously.

Perhaps, then, what is explainable about sentences with indicative conditional components is explainable in an *ad hoc* way, without the invocation of indicative conditional propositions. The alternative, propositional account may require at least as much *ad hocum,* while rendering strange and mysterious the central fact about indicative conditionals: that their assertability goes with high conditional credence.

10. CONCLUSION

None of the arguments I have given for denying that indicative conditionals express propositions apply to subjunctive conditionals. In the first place, the

Ramsey test does not apply to subjunctive conditionals, and so no problem of reconciling a propositional account of subjunctive conditionals with the Ramsey test arises. In the second, place, subjunctive conditionals embed. Take, for instance, (35), and render the antecedant subjunctive.

> If Kripke would have been there if Strawson had been, then Anscomb was there.

The result is Delphic but not incomprehensible; with ingenuity, we can imagine circumstances that would make this assertable. Finally, with subjunctive conditionals, it is often possible to give at least a rough account of their truth conditions. "If Pete had called, he would have won" is true if normal conditions of play prevailed and Pete had a winning hand, and false if normal conditions of play prevailed and Pete has a losing hand.

There is a clear need for both kinds of conditionals. Epistemic conditionals prepare us for the acquisition of new information, and nearness conditionals help us express our understanding of what depends on what in the world. It is not surprising, then, that we should have linguistic devices for both jobs. The surprise, once we realize how disparate the jobs are, should be that similar linguistic devices do both jobs. The devices are similar not, as far as I have been able to discover, because of any deep connection between the two jobs, but for these reasons. First, belief in a nearness conditional is often grounds for conditional belief: belief in a nearness conditional $a \,\Box\!\!\rightarrow b$ is grounds for belief in b given a in the frequent circumstances that the proposition $a \,\Box\!\!\rightarrow b$ is epistemically independent of its antecedant a. (See Lewis, 1976, p. 309) Thus it is easy to conflate the job a nearness conditional does with the expression of conditional belief.[19] Second, in their common domain, the two functions have the same logic. That surprising fact itself, though, as far as I have been able to discover, manifests no deep connnection between the two function, but only the possibility of tricks.

Indicative and subjunctive conditionals, I conclude, have distinct jobs, and do them in ways that have little important in common.

APPENDIX: EQUIVALENCE PROOFS

We note first that the relation of s-consequence is invariant when possible worlds are more closely subdivided. Let $\mathscr{F}$ be a field of sets with universal set t, let t^* be a set, and let $\mathscr{F}^*$ be a set of subsets of t^*. Where h is a function from t^* onto t, we define what it is for h to 'extend' $\mathscr{F}$ to $\mathscr{F}^*$. For any w, let

$$w^* = \{v \mid v \in t^* \& h(v) = w\},$$

and for any $a \in \mathscr{F}$, let

$$a^* = \{v \mid v \in t^* \& h(v) \in a\},$$

the *h-correspondant* of *a*. The function h *extends* field of sets $\mathscr{F}$ to $\mathscr{F}^*$ iff (i) h is a function whose domain is all of t^* and whose range is all of t, (ii) $\mathscr{F}^*$ is a field of sets, and (iii) for every $a \in \mathscr{F}$, $a^* \in \mathscr{F}^*$. Where $a \rightarrow b$ is a conditional A of $\mathscr{F}$, A^* is its h-correspondant $a^* \rightarrow b^*$.

THEOREM 1. *Let $\mathscr{F}$ be a field of sets, and let h extend $\mathscr{F}$ to $\mathscr{F}^*$. Let $A_1, \ldots, A_n$ be conditionals of $\mathscr{F}$, and let $A_1^*, \ldots, A_n^*$ be their h-correspondants. Then $\{A_1^*, \ldots, A_n^*\}$ is s-consistent iff $\{A_1, \ldots, A_n\}$ is s-consistent.*

THEOREM 2. *A finite set of conditionals is s-consistent in the strong sense iff it is s-consistent.*

Theorem 2 is proved in passing, since like Theorem 1, it follows from the three lemmas that are stated and proved below.

NOTATION: Let $N = \{1, \ldots, n\}$, and let $A_i = (a_i \rightarrow b_i)$ and $A_i^* = (a_i^* \rightarrow b_i^*)$ for each $i \in N$. Let $\mathscr{A} = \{A_1, \ldots, A_n\}$ and $\mathscr{A}^* = \{A_1^*, \ldots, A_n^*\}$.

LEMMA 1. Let $\mathscr{F}$ consist of all subsets of a finite set t, and suppose $\mathscr{A}$ is s-consistent. Then $\mathscr{A}^*$ is s-consistent in the strong sense.

Proof: We are given that for some s-function σ for $\mathscr{F}$ and world $u \in t$, $\sigma(a_i, u) \in b_i$ for all $i \in \{1, \ldots, n\}$. Construct an s-function τ for $\mathscr{F}^*$ as follows. Let $\prec$ be an arbitrary well-ordering of t^*, and for any non-empty subset x of t^*, let $\chi(x)$ be the $\prec$-first member of x. For any proposition $p \in \mathscr{F}^*$, let $p' = \{w \mid w \in t \& w^* \cap p \neq f\}$ and for any world $v \in t^*$, let $\tau(p, v) = v$ if $v \in p$, and otherwise, let

$$(39) \qquad \tau(p, v) = \chi(p \cap \sigma(p', h(v))^*).$$

Note that $p \cap \sigma(p', h(v))^*$ is non-empty, since by (S1), $\sigma(p', h(v)) \in p'$, and p' is the set of worlds w such that $p \cap w^* \neq f$.

Clearly τ satisfies (S1) and (S2). To check (S3), suppose $p, q \in t^*$, $p \subseteq q$, and $\tau(q, v) \in p$. Now if $v \in q$, then by (S2) $\tau(q, v) = v$; thus $v \in p$ and by (S2), $\tau(p, v) = v = \tau(q, v)$. Suppose $v \notin q$, so that $v \notin p$. Thus by (39),

$$\chi(q \cap \sigma(q', h(v))^*) \in p,$$

and so where $w = \sigma(q', h(v))$, w^* intersects p and $w \in p' \subseteq q'$. Therefore by (S3), $w = \sigma(p', h(v))$. Since $\chi(qw^*) \in pw^*$ and $pw^* \subseteq qw^*$, from the construction of χ we have $\chi(qw^*) = \chi(pw^*)$, and so (S3) is satisfied. Thus τ is an s-function.

τ is an internal s-function for $\mathscr{F}^*$. For let $p \rightarrow q$ be a conditional of $\mathscr{F}^*$. The proof is this, where the range of the variable v is t^*.

$$\begin{aligned} p \Box\!\!\rightarrow_\tau q &= \{v \mid \tau(p, v) \in q\} \\ &= pq \vee \{v \mid v \notin p \& \tau(p, w) \in q\} \\ &= pq \vee \{v \mid v \notin p \& \chi(p \cap \sigma(p', h(v))^*) \in q\} \\ &= pq \vee \bar{p}c \end{aligned}$$

where $c = \{v \mid \chi(p \cap \sigma(p', h(v))^*) \in q\}$. Now $\chi(p \cap \sigma(p', h(v))^*)$ depends on v only through its dependence on $h(v)$; thus for any $w \in t$, $\chi(p \cap \sigma(p', h(v))^*) \in q$ either throughout w^* or nowhere in w^*. Thus c is the union of a finite number of propositions of $\mathscr{F}^*$ of the form w^*, and is thus itself a proposition. Therefore $pq \vee \bar{p}c$ is a proposition, and τ is an internal s-function.

Finally, let $\hat{u} \in u^*$. Then for each conditional $a \rightarrow b \in \mathscr{A}$, $\hat{u} \in (a^* \Box\!\!\rightarrow_\tau b^*)$, and so $\mathscr{A}^*$ is s-consistent. For $u \in (a \Box\!\!\rightarrow_\sigma b)$; hence either (i) $u \in ab$ or (ii) $u \notin a$ and $\sigma(a, u) \in b$. In case (i), $\hat{u} \in a^*b^*$, and so $\hat{u} \in (a^* \Box\!\!\rightarrow_\tau b^*)$. In case (ii), $\hat{u} \notin a^*$, an so by (39),

$$\begin{aligned} \tau(a^*, \hat{u}) &= \chi(a^* \cap \sigma(a^{*\prime}, h(\hat{u}))^*) \\ &= \chi(a^* \cap \sigma(a, u)^*). \end{aligned}$$

Since $\sigma(a, u) \in ab$, $a^* \cap \sigma(a, u)^* \subseteq a^*b^*$, and so $\chi(a^* \cap \sigma(a, u)^*) \in b^*$. Thus $\tau(a^*, \hat{u}) \in b^*$, and $\hat{u} \in (a^* \Box\!\!\rightarrow_\tau b^*)$. That completes the proof of the Lemma.

LEMMA 2. If $\mathscr{A}^*$ is s-consistent, then $\mathscr{A}$ is s-consistent.

Proof: Suppose $\mathscr{A}^*$ is s-consistent. Then for some s-function τ for $\mathscr{F}^*$ and world $\hat{u} \in t^*$, $\tau(a_i^*, \hat{u}) \in b_i^*$ for each $i \in N$. Let $u = h(\hat{u})$, and for each $a \in \mathscr{F}$, let $\sigma(a, u) = h(\tau(a^*, \hat{u}))$. For worlds of $\mathscr{F}$ other than u, take an arbitrary s-function σ' for $\mathscr{F}$, and let $\sigma(a, w) = \sigma'(a, w)$. Then σ is an s-function. For from (S1), $\tau(a^*, \hat{u}) \in a^*$; therefore $h(\tau(a^*, \hat{u})) \in a$, satisfying (S1). From (S2), if $u \in a$, $\hat{u} \in a^*$; hence $\tau(a^*, \hat{u}) = \hat{u}$ and $h(\tau(a^*, \hat{u})) = u$, satisfying (S2). If $a \subseteq b$ and $\sigma(b, u) \in a$, i.e. $h(\tau(b^*, \hat{u}))$, then $\tau(b^*, \hat{u}) \in a^*$.

and $a^* \subseteq b^*$. Hence by (S3), $\tau(b^*, \hat{u}) = \tau(a^*, \hat{u})$ and $\sigma(b, u) = \sigma(a, u)$. For $w \neq u$, (S1)–(S3) follow directly from their satisfaction by σ'.

Now from the way b^* is defined, we have that for any conditional $a \to b$ of $\mathscr{F}$, $\tau(a^*, \hat{u}) \in b^*$ iff $h(\tau(a^*, \hat{u})) \in b$. This last is just $\sigma(a, u) \in b$, and so we see that for any conditionals A of $\mathscr{F}$, $u \in A^\sigma$ iff $\hat{u} \in A^{*\tau}$. Therefore $u \in A_1^\sigma, \ldots, u \in A_n^\sigma$, and $\mathscr{A}$ is s-consistent.

LEMMA 3. There exists a function g, a field of sets $\mathscr{F}''$ consisting of all subsets of a finite set t'', and conditionals $A_1'', \ldots, A_n''$ of $\mathscr{F}''$ such that (i) g extends $\mathscr{F}''$ to $\mathscr{F}$, and (ii) $A_1, \ldots, A_n$ are g-correspondants of $A_1'', \ldots, A_n''$ respectively.

Proof: Take the set $\{a_1, \ldots, a_n, b_1, \ldots, b_n\}$, and let $\mathscr{F}'$ be its *Boolean closure*: its closure under the operations of union and complementation. $\mathscr{F}'$ is finite, and hence has a finite number of atoms: non-empty sets $\in \mathscr{F}'$ which partition t, none of which has a proper non-empty subset $\in \mathscr{F}'$. Let t'' be the set of atoms of $\mathscr{F}'$, and let $\mathscr{F}''$ be the set of all subsets of t''. Then where g takes each world of t into the atom of $\mathscr{F}'$ of which it is a member, g extends $\mathscr{F}''$ to $\mathscr{F}$. For each $i \in N$, A_i is the g-correspondant of a conditional A_i'' of $\mathscr{F}''$, for each a_i or b_i is the union of a finite number of atoms $w_1, \ldots, w_m$ of $\mathscr{F}'$, and hence is the g-correspondant of proposition $\{w_1, \ldots, w_m\}$ of $\mathscr{F}''$.

Proof of Theorem 1: The composition of g and h extends $\mathscr{F}''$ to $\mathscr{F}^*$. From Lemmas 1 and 2, we have that $\mathscr{A}$ is s-consistent iff $\mathscr{A}''$ is s-consistent, and $\mathscr{A}^*$ is s-consistent iff $\mathscr{A}''$ is s-consistent. Therefore $\mathscr{A}^*$ is s-consistent iff $\mathscr{A}$ is s-consistent.

Proof of Theorem 2: Suppose $\mathscr{A}$ is s-consistent. Since g extends $\mathscr{F}''$ to $\mathscr{F}$, $\mathscr{A}''$ is s-consistent by Lemma 3. Thus by Lemma 1, since $\mathscr{A}''$ is s-consistent, $\mathscr{A}$ is s-consistent in the strong sense. Therefore if $\mathscr{A}$ is s-consistent, then it is s-consistent in the strong sense. The converse is trivial.

Here is the part of the Van Fraassen result needed for the equivalence proof.

THEOREM 3 (Van Fraassen). *Let $\mathscr{F}''$ be a finite field of sets. Then there are h and $\mathscr{F}^*$ such that h extends $\mathscr{F}''$ to $\mathscr{F}^*$, and for every probability measure ρ'' on $\mathscr{F}''$, there are an internal s-function τ for $\mathscr{F}^*$ and a probability measure ρ^* on $\mathscr{F}^*$ such that*

(i) *For every $a'' \in \mathscr{F}''$, $\rho^*(a^*) = \rho''(a'')$*

(ii) *For every $a'', b'' \in \mathscr{F}''$ with $\rho''(a'') \neq 0$, $\rho^*(a^* \,\Box\!\!\rightarrow_\tau b^*) = \rho''(b''/a'')$.*

THEOREM 4. *Let $A_1, \ldots, A_n$, C be conditionals of a field of sets $\mathscr{F}$, and let $\mathscr{A} = \{A_1, \ldots, A_n\}$. Then if C is an s-consequence of $\mathscr{A}$, then C is a p-consequence of $\mathscr{A}$.*

Proof: Let $A_i = (a_i \to b_i)$ for each $i \in N = \{1, \ldots, n\}$, and let $C = (c \to d)$. Let C be an s-consequence of $\mathscr{A}$. Given $\epsilon > 0$, let $\delta = \epsilon/n$. Suppose ρ is a probability measure on $\mathscr{F}$ with $\rho(a_i) > 0$ for each $i \in N$, $\rho(c) > 0$, and $\rho(b_i/a_i) \geq 1 - \delta$ for all $i \in N$; we are to prove that $\rho(d/c) \geq 1 - \epsilon$. Let $\mathscr{F}''$ be constructed from the conditionals $A_1, \ldots, A_n$, C as in Lemma 3, and for each proposition a'' of $\mathscr{F}''$, let $\rho''(a'') = \rho(a)$ where a is its g-correspondant in $\mathscr{F}$. Let h extend $\mathscr{F}''$ to $\mathscr{F}^*$ as in Theorem 3. Then by Theorem 1, C^* is an s-consequence of $\{A_1^*, \ldots, A_n^*\}$, and so $\{A_1^{*\tau}, \ldots, A_n^{*\tau}\}$ entails $C^{*\tau}$. Therefore since for each $i \in N$,

$$\rho^*(A_i^{*\tau}) = \rho''(b_i''/a_i'') = \rho(b_i/a_i) > 1 - \delta,$$

we have $1 - \epsilon < \rho^*(C^{*\tau}) = \rho''(d''/c'') = \rho(d/c)$, and the proof is complete.

THEOREM 5. *Any finite s-consistent set of conditionals is p-consistent.*

Proof: Let $\{a_1 \to b_1, \ldots, a_n \to b_n\}$ be s-consistent. Then for some s-function σ, the set

$$\{a_1 \,\Box\!\!\rightarrow_\sigma b_1, \ldots, a_n \,\Box\!\!\rightarrow_\sigma b_n\}$$

is consistent. Hence we may let w^* be a world, fixed for the rest of the proof, such that all of $a_1 \,\Box\!\!\rightarrow_\sigma b_1, \ldots, a_n \,\Box\!\!\rightarrow_\sigma b_n$ hold at w^*.

Define $\sigma^*(a_i) = \sigma(w^*, a_i)$. We weakly order the set $\{a_1, \ldots, a_n\}$ of antecedants by 'distance' from w^* as follows: $a_i \lesssim a_j$ iff $\sigma^*(a_i \vee a_j) = \sigma^*(a_i)$. Let $a_i \approx a_j$ iff both $a_i \lesssim a_j$ and $a_j \lesssim a_i$. The relation $\lesssim$ is connected and transitive, and $a_i \approx a_j$ iff $\sigma^*(a_i) = \sigma^*(a_j)$.

(i) $a_i \lesssim a_j$ or $a_j \lesssim a_i$, and if $a_i \approx a_j$, then $\sigma^*(a_i) = \sigma^*(a_j)$.

Proof: Let $v = \sigma^*(a_i \vee a_j)$. Then by definition, $a_i \lesssim a_j$ iff $\sigma^*(a_i) = v$ and $a_j \lesssim a_i$ iff $\sigma^*(a_j) = v$. It follows that if $a_i \approx a_j$, then $\sigma^*(a_i) = \sigma^*(a_j)$. Now by (S1), $v \in a_i \vee a_j$. If $v \in a_i$, then by (S3), $\sigma^*(a_i) = v$, and hence $a_i \lesssim a_j$. Similarly, if $v \in a_j$, then $\sigma^*(a_j) = v$ and $a_j \lesssim a_i$. We have seen that either $\sigma^*(a_i) = v$ or $\sigma^*(a_j) = v$. Hence if $\sigma^*(a_i) = \sigma^*(a_j)$, then both are v, and $a_i \approx a_j$.

(ii) Let $a_i \lesssim a_j$ and $a_j \lesssim a_k$. Then $a_i \lesssim a_k$.

Proof: Let $u = \sigma^*(a_i \vee a_j \vee a_k)$. *Case* 1. $u \in a_i$. Then by (S3), $\sigma^*(a_i \vee a_k) = u$ and $\sigma^*(a_i) = u$; thus $a_i \lesssim a_k$. *Case* 2. $u \in a_j$. Then by (S3), $\sigma^*(a_j) = u$, $\sigma^*(a_i \vee a_j) = u$, and hence $a_j \lesssim a_i$. Since by hypothesis $a_i \lesssim a_j$, we have $a_i \approx a_j$, and by (i), $\sigma^*(a_i) = \sigma^*(a_j) = u$, and so $u \in a_i$, and this reduces to Case 1. *Case* 3. $u \in a_k$. Then by similar reasoning, since $a_j \precsim a_k$, we have $u \in a_j$, and this reduces to Case 2. That proves (ii).

Let $W = \{\sigma^*(a_1), \ldots, \sigma^*(a_n)\}$. Then the relation $\precsim$ induces a linear ordering on w, again by 'distance' from w^*. Where $w, w' \in W$, let $w \prec w'$ iff for some a_i and a_j, $w = \sigma^*(a_i)$, $w' = \sigma^*(a_j)$, and $a_i \precsim a_j$ but not $a_j \precsim a_i$. Since, as we have seen, for any $w \in W$, the set $\{a \mid \sigma^*(a) = w\}$ is an equivalence class under $\approx$, the relation $\prec$ is a linear ordering. Let $w_1, \ldots, w_m$ be the members of W, with $w_1 \prec \ldots \prec w_m$.

(iii) If $\sigma^*(a_k) = w_j$ and $i < j$, then $w_i \in a_k$.

Proof: Since $i < j$, by the way the w_i's are indexed, $w_i \neq w_j$, and for some a_l, $w_i = \sigma^*(a_l)$, $a_l \precsim a_k$, and not $a_k \precsim a_l$. By the definition of '$\precsim$', this means $w_i = \sigma^*(a_l \vee a_k)$. Suppose $w_i \in a_k$. Then by (S3), $w_i = \sigma(a_k)$, and $a_k \precsim a_l$, contradicting what was said earlier. Thus $w_i \notin a_k$.

The proof of the Theorem is this. Let $\rho(w_m) = \delta^m$, and for each $i < m$, let $\rho(w_i) = \delta^i(1-\delta)$. For any proposition x, then, we let $\rho(x) = \Sigma_{i=0}^m \rho_x(w_i)$, where $\rho_x(w_i) = \rho(w_i)$ if $w_i \in x$ and $\rho_x(w_i) = 0$ if $w_i \notin x$. Define $r_i = w_i \vee w_{i+1} \vee \ldots \vee w_m$; then $\rho(r_i) = \delta^i$, so that in particular $\rho(W) = \rho(r_0) = 1$.

Now let $w_j = \sigma^*(a_k)$. Since by (iii), for $i < j$, $w_i \notin a_k$, we have $\rho(a_k) \leqslant \rho(r_j) = \delta^j$. Now we assumed at the outset that $a_k \,\Box\!\!\rightarrow_\sigma b_k$ holds at w^*, and that is to say that $\sigma^*(a_k) \in b_k$; thus $w_j \in a_k b_k$. Thus $\rho(a_k b_k) \geqslant \rho(w_j) = \delta^j(1-\delta)$. Therefore $\rho(b_k/a_k) = \rho(a_k b_k)/\rho(a_k) \geqslant 1-\delta$. For arbitrary δ, we have shown how to construct a probability measure ρ such that $\rho(b_k/a_k) > 1-\delta$ for all $k = 1, \ldots, n$. Thus $\{a_1 \to b_1, \ldots, a_n \to b_n\}$ is p-consistent.

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NOTES

[1] Adams himself argues, as I will later, that indicative conditionals are not propositions in the sense in which I am using the term (1975, pp. 5–6).

[2] See Lewis (1976, p. 311) for more discussion of the contrast between these two kinds of minimal revision.

[3] I get this use of Venn diagrams from Adams (1975).

[4] Stalnaker proposes this requirement (1970, p. 75), but later rejects it (see Stalnaker, 1976).

[5] Lewis (1976, pp. 300–303). Stalnaker has offered an interpretation of this possible divergence, which is developed in Gibbard and Harper (1978). Examples in which one's credence in a subjunctive conditional is not equal to one's corresponding conditional credence are given in Section 5 of this paper.

[6] Stalnaker's result, unlike Lewis's, depends on the conditional's having the Stalnaker logic; Van Fraassen (1976, pp. 286–291) has shown that a weaker logic of conditionals is compatible with the supposition that $\to$ is a propositional function such that, for some fixed ρ, $\rho(a \to b) = \rho(b/a)$ whenever $\rho(a) \neq 0$.

[7] I refer here to what Van Fraassen calls 'Stalnaker–Bernoulli models' (1976, pp. 279–282, 293–295). Here I state only those aspects of Van Fraassen's results that are needed in this paper; Van Fraassen's results are not, for instance, restricted to finite fields of sets.

[9] The modal auxilliaries are 'will', 'shall', 'can', 'may', and 'must'; I treat the first four as having the respective past tenses 'would', 'should', 'could', and 'might'. (See Chomsky, 1957, p. 40.) The past tense of 'must' is lacking, and 'should' as a synonym for 'ought to' does not act as the past tense of 'shall'. Only examples with 'will' and its past tense 'would' are given here.

[8] These rules, indeed, are sound for Lewis's semantics (1973). It follows that on the domain to which the Adams logic applies, the Lewis and Stalnaker logics are equivalent. That should not be surprising: The axiom by which the Lewis and Stalnaker logics differ, conditional excluded middle, says in effect that $-(a \to b)$ is equivalent to $a \to \bar{b}$. The constraint it imposes, then, bears solely on negations of conditionals and constructions that involve them, all of which include embedded conditionals.

[10] Harper (1976a, p. 97; 1976b) uses the term this way.

[11] This is the interpretation adopted in Gibbard and Harper (1978, p. 127). Lewis (1979) offers a more general account of truth conditions for such conditionals, and tries to show that the account given here is a consequence of his account and some deep contingent facts that, at least in part, constitute the direction of time.

[12] For a contrasting treatment of acceptance, see Harper (1976a, especially pp. 77–81).

[13] Adams (1976) proposes an alternative to the account of subjunctive conditionals I have given. On that account, they act much like epistemic conditionals, but the relevant conditional probability is not one's conditional credence but a 'prior' conditional probability. In his book (1975, pp. 129–133) he gives a counterexample to that theory, and proposes a more complex probabilistic account of subjunctive conditionals. Skyrms (this volume) proposes an account of subjunctive conditionals which I think correct: that a subjunctive conditional is accepted iff the subjectively expected value of the corresponding *propensity* is sufficiently high. On this view as I would express it, subjunctive conditionals involve conditional propositions, but those propositions involve objective chance: they take the form "If a had obtained at t, the chance, as of t, of b would have been α." Sobel (1978) calls the subjectively expected value of this propensity α the 'probable chance' of b given a. Here I endorse the nearness account only as an approximation to Skyrms' view that a subjunctive conditional is accepted iff the corresponding expectation is sufficiently high. Roughly, the expectation is high iff one puts high credence in a proposition: that the chance, as of t, with which b would obtain if a did is high. Thus to adopt the Skyrms account is roughly to treat subjunctive conditionals as propositions.

[14] Additional factors may be at work in determining what the audience can conclude from the fact that I uttered S: my audience may suppose that even if I believed a, I would not have said S unless certain other conditions also obtained – say, that I do not accept another proposition c. Thus we have conversation implicatures; see Grice (c. 1969).

[15] The classical account along these lines is in Grice (1957).
[16] Stalnaker (1975) treats indicative conditionals as context dependent propositions, and says "The most important element of a context, I suggest, is the common knowledge, or presumed common knowledge and common assumption of the participants in the discourse." He calls this presumed common ground the 'presuppositions' of the speaker. The example shows that if an indicative conditional utterance of the form 'If a then b' expresses a proposition, what proposition it expresses depends on more than a, b, and the speaker's presuppositions. Indeed it seems that the crucial aspect of the context that makes the utterance express the proposition it does is the speaker's conditional credence in b on a, and if that were presupposed, the utterance of the conditional would be pointless.
[17] Adams (1975, p. 33) regards the equivalence of $a \rightarrow (b \rightarrow c)$ with $ab \rightarrow c$ as problematical, since with *modus ponens*, it allows us to infer $b \rightarrow a$ from a. His argument is this: If the equivalence claim is correct, then $a \rightarrow (b \rightarrow a)$ is equivalent to $ab \rightarrow a$, which is a logical truth. Hence from a and a logical truth, we get $b \rightarrow a$ by *modus ponens.* My intuition are that sentences of the form $a \rightarrow (b \rightarrow a)$ indeed are logical truths, and are accepted even by someone for whom a is assertable and $b \rightarrow a$ is not. I am prepared to assert

> Andrew Jackson was President in 1836,

and I am not prepared to assert

> Even if Andrew Jackson died in 1835, he was President in 1836.

Nevertheless, the following strikes me as something which I accept as a logical truth.

> If Andrew Jackson was President in 1836, then even if he died in 1835, he was president in 1836.

[18] Adams (1975, pp. 31–37) gives a number of examples in which the truth-functional theory seems to fail for embedded 'will' conditionals. Here is one, adapted to the past tense.

> If switches A and B were both on, the motor was running. Therefore, either if switch A was on the motor was running or if switch B was on the motor was running.

I do not see how the apparent fallaciousness of this inference could be explained away with conversational implicatures.
[19] Adams (1976) argues that the logics of indicative and subjunctive conditionals are isomorphic (pp. 6–16), and notes that that is what would be expected on his 'prior probability' representation, according to which indicative and subjunctive conditionals have similar semantics. The equivalence of the Adams and restricted Stalnaker logics shown in Section 3 shows that there is an alternative explanation for a logical isomorphism of indicative and subjunctive conditionals.

BIBLIOGRAPHY

Adams, E. W., *The Logic of Conditionals,* D. Reidel, Dordrecht, Holland, 1975.
Adams, E. W., 'Prior Probabilities and Counterfactual Conditionals', in W. L Harper and

C. A. Hooker (eds.), *Foundations of Probability Theory, Statistical Inference, and Statistical Theories of Science,* Volume I. D. Reidel, Dordrecht, Holland, 1976, pp. 1–21.
Chomsky, N., *Syntactic Structures,* Mouton, The Hague, 1957.
Gibbard, A. and Harper, W. L., 'Counterfactuals and Two Kinds of Expected Utility', C. A. Hooker, J. J. Leach, and E. F. McClennen (eds.), *Foundations and Applications of Decision Theory,* Volume I, D. Reidel, Dordrecht, Holland, 1978, pp. 125–162.
Gibbard, A., 'Chance Conditionals and Beliefs about Influence', Duplicated typescript, University of Michigan, 1978.
Grice, H. P., 'Meaning', *Pilosophical Review* **67** (1957), 377–388.
Grice, H. P., *William James Lectures,* Duplicated typescript (c. 1969).
Harper, W. L., 1976a, 'Rational Belief Change, Popper Functions, and Counterfactuals', in W. L. Harper and C. A. Hooker (eds.), *Foundations of Probability Theory, Statistical Inference, and Statistical Theories of Science,* Volume I, D. Reidel, Dordrecht, Holland, 1976, pp. 73–112.
Harper, W. L., 1976b, 'Ramsey Test Conditionals and Iterated Belief Change (A Response to Stalnaker)', in W. L. Harper and C. A. Hooker (eds.), *Foundations of Probability Theory, Statistical Inference, and Statistical Theories of Science,* Volume I, D. Reidel, Dordrecht, Holland, 1976, p. 117.
Kyburg, H. E., *Probability and Inductive Logic,* Macmillan, London, 1970.
Lewis, D., *Counterfactuals,* Harvard Univeristy Press, Cambridge, Mass., 1973.
Lewis, D., 'Probabilities of Conditionals and Conditional Probabilities', *Philosophical Review* **85** (1976), 297–315; also this volume, pp. 129–147.
Lewis, D., 'Counterfactual Dependence and Time's Arrows', *Noûs* **13** (1979), 455–476.
Ramsey, F. P., 'General Propositions and Causality', *The Foundations of Mathematics and Other Logical Essays,* Routledge and Kegan Paul, London, 1931.
Skyrms, B., 'The Prior Propensity Account of Subjunctive Conditionals', this volume, pp. 259–265.
Sobel, J. H., *Choice, Chance, and Action: Newcomb's Problem Resolved,* Duplicated typescript, University of Toronto, 1978.
Stalnaker, R., 'A Theory of Conditionals', *Studies in Logical Theory, American Philosophical Quarterly Monograph Series,* No. 2, Blackwell, Oxford, 1968, pp. 98–112.
Stalnaker, R., 'Probability and Conditionals', *Philosophy of Science* 37 (1970), 64–80; also this volume, pp. 107–128.
Stalnaker, R., 'Indicative Conditionals', *Philosophia* 5 (1975), 269–286; also this volume, pp. 193–210.
Stalnaker, R., 'Letter to Van Fraassen', in W. L. Harper and C. A. Hooker (eds.), *Foundations of Probability Theory, Statistical Inference, and Statistical Theories of Science,* Volume I, D. Reidel, Dordrecht, Holland, 1976, pp. 302–306.
Van Fraassen, B. C., 'Probabilities of Conditonals', in W. L. Harper and C. A. Hooker (eds.), *Foundations of Probability Theory, Statistical Inference, and Statistical Theories of Science,* Volume I, D. Reidel, Dordrecht, Holland, 1976, pp. 261–308.