Twenty-Five Properties of Conditionals

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In this handout, I will collect various (25, to be exact) properties of conditionals we have seen during the course of the semester (so far). As an exercise, it would be useful to figure out which conditionals we have studied (and which we still will study in the final chapter) have which of these properties. Also, it's worth thinking about *validity-versus-truth-preservtation* versions of the inferences below. I will assume that we are talking about some sentential logic $\mathcal{L}$ with the following operators: &, $\vee$, $\rightarrow$, $\sim$, $\Box$, and $\Diamond$, and with a meta-logical entailment relation $\models_{\mathcal{L}}$. If I want to talk about classical logic, then I'll just use the plain $\models$-sign for its meta-theoretic entailment relation, and $\supset$ for its conditional. All of the following 25 properties of conditionals are satisfied by $\supset$ in classical logic. Many of them are violated in other logics of $\rightarrow$.

1. $\models_{\mathcal{L}} p \rightarrow p$. [self-implication]
2. $p, p \rightarrow q \models_{\mathcal{L}} q$. [modus ponens]
3. $p \models_{\mathcal{L}} q \rightarrow p$. [paradox of material implication]
4. $\sim q \models_{\mathcal{L}} q \rightarrow p$. [paradox of material implication]
5. $(p \,\&\, q) \rightarrow s \models_{\mathcal{L}} (p \rightarrow s) \vee (q \rightarrow s)$. [one of Priest's objections to $\supset$]
6. $(p \rightarrow q) \,\&\, (s \rightarrow t) \models_{\mathcal{L}} (p \rightarrow t) \vee (s \rightarrow q)$. [one of Priest's objections to $\supset$]
7. $\sim(p \rightarrow q) \models_{\mathcal{L}} p$. [one of Priest's objections to $\supset$]
8. $\sim(p \rightarrow q) \models_{\mathcal{L}} \sim q$. [one of Priest's objections to $\supset$]
9. $p \rightarrow q \models_{\mathcal{L}} \sim q \rightarrow \sim p$. [contraposition]
10. $p \rightarrow q, q \rightarrow r \models_{\mathcal{L}} p \rightarrow r$. [transitivity]
11. $p \rightarrow s \models_{\mathcal{L}} (p \,\&\, q) \rightarrow s$. [antecedent strengthening]
12. $\models_{\mathcal{L}} (p \rightarrow q) \vee (p \rightarrow \sim q)$. [conditional excluded middle]
13. $\models_{\mathcal{L}} \sim[(p \rightarrow q) \,\&\, (p \rightarrow \sim q)]$. [conditional non-contradiction]
14. $\Box q \models_{\mathcal{L}} p \rightarrow q$. [paradox of strict implication]
15. $\Box \sim p \models_{\mathcal{L}} p \rightarrow q$. [paradox of strict implication]
16. $\models_{\mathcal{L}} (p \,\&\, \sim p) \rightarrow q$. [paradox of strict implication]
17. $\models_{\mathcal{L}} p \rightarrow (q \vee \sim q)$. [paradox of strict implication]
18. $\models_{\mathcal{L}} p \rightarrow (q \rightarrow q)$. [paradox of strict implication]
19. If $\models p \supset q$, then $\models_{\mathcal{L}} p \rightarrow q$. [relation to validity of material conditional]
20. If $\models_{\mathcal{L}} p \rightarrow q$, then $\models p \supset q$. [relation to validity of material conditional]
21. $\models_{\mathcal{L}} p \rightarrow (q \rightarrow p)$. [weakening]
22. $p \,\&\, q \models_{\mathcal{L}} p \rightarrow q$. [centering]
23. $p \rightarrow (q \rightarrow r) \models_{\mathcal{L}} (p \,\&\, q) \rightarrow r$. [importation]
24. $(p \,\&\, q) \rightarrow r \models_{\mathcal{L}} p \rightarrow (q \rightarrow r)$. [exportation]
25. $\models_{\mathcal{L}} ((p \rightarrow q) \rightarrow p) \rightarrow p$. [Peirce's Law]

How many logics of $\rightarrow$ have we seen? Many! Specifically, we have (1) the material conditional [$\supset$], (2) strict conditionals in normal modal logics [$\neg\Diamond_{K_x}$], (3) strict conditionals in non-normal modal logics [$\neg\Diamond_{N_x/L_x/E_x}$], (4) conditional (*viz.*, enthymematic/ceteris paribus/counterfactual) logics [$>$], and (5) the intuitionist conditional [$\sqsupset$]. In the final chapter of the course (chapter 7), we'll also see various *many-valued* conditionals (see, for instance, the table on page 126, which compares four many-valued conditionals on 10 of the above 25).