Naive Set Theory: Basic Concepts

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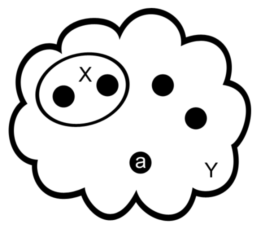

notation: If a is a member of set Y, $a \in Y$. If all the elements of set X are included in set Y, $X \subseteq Y$.
extension axiom: $(X = Y) \leftrightarrow (\forall a)(a \in X \leftrightarrow a \in Y) \leftrightarrow (X \subseteq Y \wedge Y \subseteq X)$

some simple sets:
$\emptyset = \{a|a \neq a\}$
$X \cup Y = \{a|a \in X \vee a \in Y\}$
$X \cap Y = \{a|a \in X \wedge a \in Y\}$
$X - Y = \{a|a \in X \wedge a \notin Y\}$

more complicated sets:
$\langle a, b\rangle = \{\{a\}, \{a, b\}\}$
$X_1 \times ... \times X_n = \{\langle x_1, ..., x_n\rangle | x_1 \in X_1 \wedge ... \wedge x_n \in X_n\}$
binary *relation* R on X is a subset of $X \times X$
function f from X to Y is a binary relation between X and Y s.t. $(\forall x \in X)(\exists$ unique $y \in Y)(xfy)$

equivalence relations:
(i) R is *reflexive* if $(\forall a \in X)(Raa)$
(ii) R is *symmetric* if $(\forall a, b \in X)(Rab \rightarrow Rba)$
(iii) R is *transitive* if $(\forall a, b, c \in X)((Rab \wedge Rbc) \rightarrow Rac)$
- an R that satisfies (i), (ii), and (iii) is an *equivalence relation*
- if $a \in X$, its *equivalence class* is defined as $[a] = \{b|b \in X \wedge Rab\}$