Take-Home Mid-Term Solutions

Philosophy 12A
March 28, 2010

I.) The Premises (1) -- (7), in optimal LSL form for doing the requisite natural deduction proofs, are:

(1) (((G∨H) & (J∨G)) & (H∨J)) & (~J∨(~H∨~G))
(2) K → H
(3) (H & K) ∨ ~H
(4) H → (~G & ~I)
(5) J∨G
(6) J↔~K
(7) ((~J → K) ∨ H) → ~I

II.) The conclusion is the following conjunction: '(((G & ~H) & ~I) & J) & ~K'.

I figured this out by doing a full truth-table, and recognizing that there is only one interpretation in which all the premises (1) -- (7) are true. See my attached full truth-table (last page of this handout) for this alternative semantical solution.

So, we must prove each of the 5 conjuncts of the solution: G, ~H, ~I, J, and ~K, from (1) -- (7). I have done 5 natural deduction proofs, below. I've tried to give the simplest and most elegant natural deduction proofs possible from (1) -- (7) of each of the five conjuncts of the conclusion.

III.1) Problem #1 of 5: Derive 'G' from (1) -- (7). We only need (1), (3), and (6) for this proof. Specifically, we only need the following premises, which can – trivially – be derived from (1), (3), and (6) above:

G∨H (H&K)∨~H
J∨G J → ~K

Here's a MacLogic proof of the sequent: G∨H, (H&K)∨~H, J∨G, J → ~K ⊢ G

1	(1)	G∨H	Premise
2	(2)	(H&K)∨~H	Premise
3	(3)	J∨G	Premise
4	(4)	J→~K	Premise
5	(5)	G	Assumption (∨E)
6	(6)	H	Assumption (∨E)
7	(7)	J	Assumption (∨E)
8	(8)	H&K	Assumption (∨E)
9	(9)	~G	Assumption (~I)
4,7	(10)	~K	4,7 →E
8	(11)	K	8 &E
4,7,8	(12)	Λ	10,11 ~E
4,7,8	(13)	~~G	9,12 ~I
4,7,8	(14)	G	13 DN
15	(15)	~H	Assumption (∨E)
6,15	(16)	Λ	15,6 ~E
6,15	(17)	~~G	9,16 ~I
6,15	(18)	G	17 DN
2,4,6,7	(19)	G	2,8,14,15,18 ∨E
2,3,4,6	(20)	G	3,7,19,5,5 ∨E
1,2,3,4	(21)	G	1,5,5,6,20 ∨E

III.2) Problem #2 of 5: Derive '~H' from (1) -- (7). We only need (1), (3), (4), and (6) for this. Specifically, we only need the following four premises, which can – trivially – be derived from (1), (3), (4), and (6) above:

(H&K)∨~H	H → (~G & ~I)
J∨G	J → ~K

Here's a MacLogic proof of the sequent: (H&K)∨~H, H → (~G & ~I), J∨G, J → ~K ⊦ ~H

1	(1)	(H&K)∨~H	Premise
2	(2)	H→(~G&~I)	Premise
3	(3)	J∨G	Premise
4	(4)	J→~K	Premise
5	(5)	H	Assumption (~I)
6	(6)	H&K	Assumption (∨E)
7	(7)	J	Assumption (∨E)
4,7	(8)	~K	4,7 →E
6	(9)	K	6 &E
4,6,7	(10)	Λ	8,9 ~E
11	(11)	G	Assumption (∨E)
2,5	(12)	~G&~I	2,5 →E
2,5	(13)	~G	12 &E
2,5,11	(14)	Λ	13,11 ~E
2,3,4,5,6	(15)	Λ	3,7,10,11,14 ∨E
16	(16)	~H	Assumption (∨E)
5,16	(17)	Λ	16,5 ~E
1,2,3,4,5	(18)	Λ	1,6,15,16,17 ∨E
1,2,3,4	(19)	~H	5,18 ~I

III.3) Problem #3 of 5: Derive '~I' from (1) -- (7). We only need (6) and (7) for this proof. Specifically, we only need the following two premises, which can – trivially – be derived from (6) and (7):

~K → J
((~J → K) ∨ H) → ~I

Here's a MacLogic proof of the sequent: ~K → J, ((~J → K) ∨ H) → ~I ⊦ ~I

1	(1)	~K→J	Premise
2	(2)	(H∨(~J→K))→~I	Premise
3	(3)	~J	Assumption (→I)
4	(4)	~K	Assumption (~I)
1,4	(5)	J	1,4 →E
1,3,4	(6)	Λ	3,5 ~E
1,3	(7)	~~K	4,6 ~I
1,3	(8)	K	7 DN
1	(9)	~J→K	3,8 →I
1	(10)	H∨(~J→K)	9 ∨I
1,2	(11)	~I	2,10 →E

III.4) Problem #4 of 5: Derive 'J' from (1) -- (7). We only need (1) and (4) for this proof. Specifically, we only need the following three premises, which can – trivially – be derived from (1) and (4):

$H \vee J \qquad H \rightarrow (\sim G \,\&\, \sim I)$

$J \vee G$

Here's a MacLogic proof of the sequent: $H \vee J$, $H \rightarrow (\sim G \,\&\, \sim I)$, $J \vee G \vdash J$

1	(1)	$H \vee J$	Premise
2	(2)	$H \rightarrow (\sim G \& \sim I)$	Premise
3	(3)	$J \vee G$	Premise
4	(4)	H	Assumption ($\vee$E)
5	(5)	$\sim J$	Assumption (~I)
6	(6)	J	Assumption ($\vee$E)
5,6	(7)	Λ	5,6 ~E
8	(8)	G	Assumption ($\vee$E)
2,4	(9)	$\sim G \& \sim I$	2,4 $\rightarrow$E
2,4	(10)	$\sim G$	9 &E
2,4,8	(11)	Λ	10,8 ~E
2,3,4,5	(12)	Λ	3,6,7,8,11 $\vee$E
2,3,4	(13)	$\sim\sim J$	5,12 ~I
2,3,4	(14)	J	13 DN
1,2,3	(15)	J	1,4,14,6,6 $\vee$E

III.5) Problem #5 of 5: Derive '~K' from (1) -- (7). We only need (1), (4), and (6) for this proof. Specifically, we only need the following four premises, which can – trivially – be derived from (1), (4), and (6):

$H \vee J \qquad H \rightarrow (\sim G \,\&\, \sim I)$

$J \vee G \qquad J \rightarrow \sim K$

Here's a MacLogic proof of the sequent: $H \vee J$, $H \rightarrow (\sim G \,\&\, \sim I)$, $J \vee G$, $J \rightarrow \sim K \vdash \sim K$

1	(1)	$H \vee J$	Premise
2	(2)	$H \rightarrow (\sim G \& \sim I)$	Premise
3	(3)	$J \vee G$	Premise
4	(4)	$J \rightarrow \sim K$	Premise
5	(5)	K	Assumption (~I)
6	(6)	J	Assumption ($\vee$E)
4,6	(7)	$\sim K$	4,6 $\rightarrow$E
4,5,6	(8)	Λ	7,5 ~E
9	(9)	G	Assumption ($\vee$E)
10	(10)	H	Assumption ($\vee$E)
2,10	(11)	$\sim G \& \sim I$	2,10 $\rightarrow$E
2,10	(12)	$\sim G$	11 &E
2,9,10	(13)	Λ	12,9 ~E
1,2,4,5,9	(14)	Λ	1,10,13,6,8 $\vee$E
1,2,3,4,5	(15)	Λ	3,6,8,9,14 $\vee$E
1,2,3,4	(16)	$\sim K$	5,15 ~I

Some Notes on this Problem: You might have noticed that we never made any use of either premise (2) or premise (5) in our natural deduction proofs. Indeed, each of these premises follows from the remaining five premises. And, so, they are entirely redundant.

Here's a MacLogic proof of (2) from (1) and (6) (i.e., of the sequent $H \vee J$, $J \to \sim K \vdash K \to H$).

1	(1)	$H \vee J$	Premise
2	(2)	$J \to \sim K$	Premise
3	(3)	K	Assumption ($\to$I)
4	(4)	H	Assumption ($\vee$E)
5	(5)	J	Assumption ($\vee$E)
6	(6)	$\sim H$	Assumption ($\sim$I)
2,5	(7)	$\sim K$	2,5 $\to$E
2,3,5	(8)	Λ	7,3 $\sim$E
2,3,5	(9)	$\sim\sim H$	6,8 $\sim$I
2,3,5	(10)	H	9 DN
1,2,3	(11)	H	1,4,4,5,10 $\vee$E
1,2	(12)	$K \to H$	3,11 $\to$I

It is easy to see that (5) follows from (1). Indeed, this is trivial, both semantically and syntactically, because of the way in which I symbolized premise (1). Here, a judicious choice of symbolization for premise (1) has saved us a lot of work.

I used the automated theorem-proving program **`Otter`** to help me figure out which premises were required to complete the proofs reported above, and to help me figure out the optimal LSL form for premise (1). This greatly reduced the complexity of the problem. Other LSL symbolizations of premise (1) would have made my natural deduction proofs significantly more complicated. For instance, if we had used the following alternative symbolization of (1) instead, we would have made things much harder for ourselves!

(1*) $(((G \,\&\, H) \,\&\, \sim J) \vee ((G \,\&\, \sim H) \,\&\, J)) \vee ((\sim G \,\&\, H) \,\&\, J)$

For example, try to prove 'G' from (1*), (3), and (6) instead of (1), (3), and (6). The shortest proof I was able to find of this alternative sequent was 29 steps, with 4 applications of $\vee$E --- as opposed to the 21-step proof with 3 applications of $\vee$E that I have reported above (III.1), above. In general, the proofs involving (1*) are significantly more complicated than the corresponding proofs involving (1).

The remaining five premises: (1), (3), (4), (6), and (7) are mutually logically independent (i.e., none of them follows from any conjunction of the others). Hence, (1), (3), (4), (6), and (7) constitute a complete and independent "axiomatization" of this problem. This can be established using truth-tables.

On the next page, you'll see the (minimal) full truth-table which I used to find the conclusion.

(Minimal) **Full Truth-Table For The Take-Home Problem**

G	H	I	J	K	((((G∨H)	&	(J∨G))	&	(H∨J))	&	(~J	∨	(~H	∨	~G)))	&	(((H&K)	∨	~H)	&	((H→	(~G	&	~I))	&	((J↔	~K)	&	(((~J	→K)	∨H)	→	~I))))
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The highlighted row is the only row in which all of the premises are true (i.e., the conjunction of (1), (3), (4), (6), and (7) is true). And, as you can see from the table, this row is the row in which G, ~H, ~I, J, and ~K are true. Those are the conclusions that are entailed by the premises (1)--(7), since there is no possible world in which all the premises (1)--(7) are true, and any of the conclusions G, ~H, ~I, J, or ~K is false.