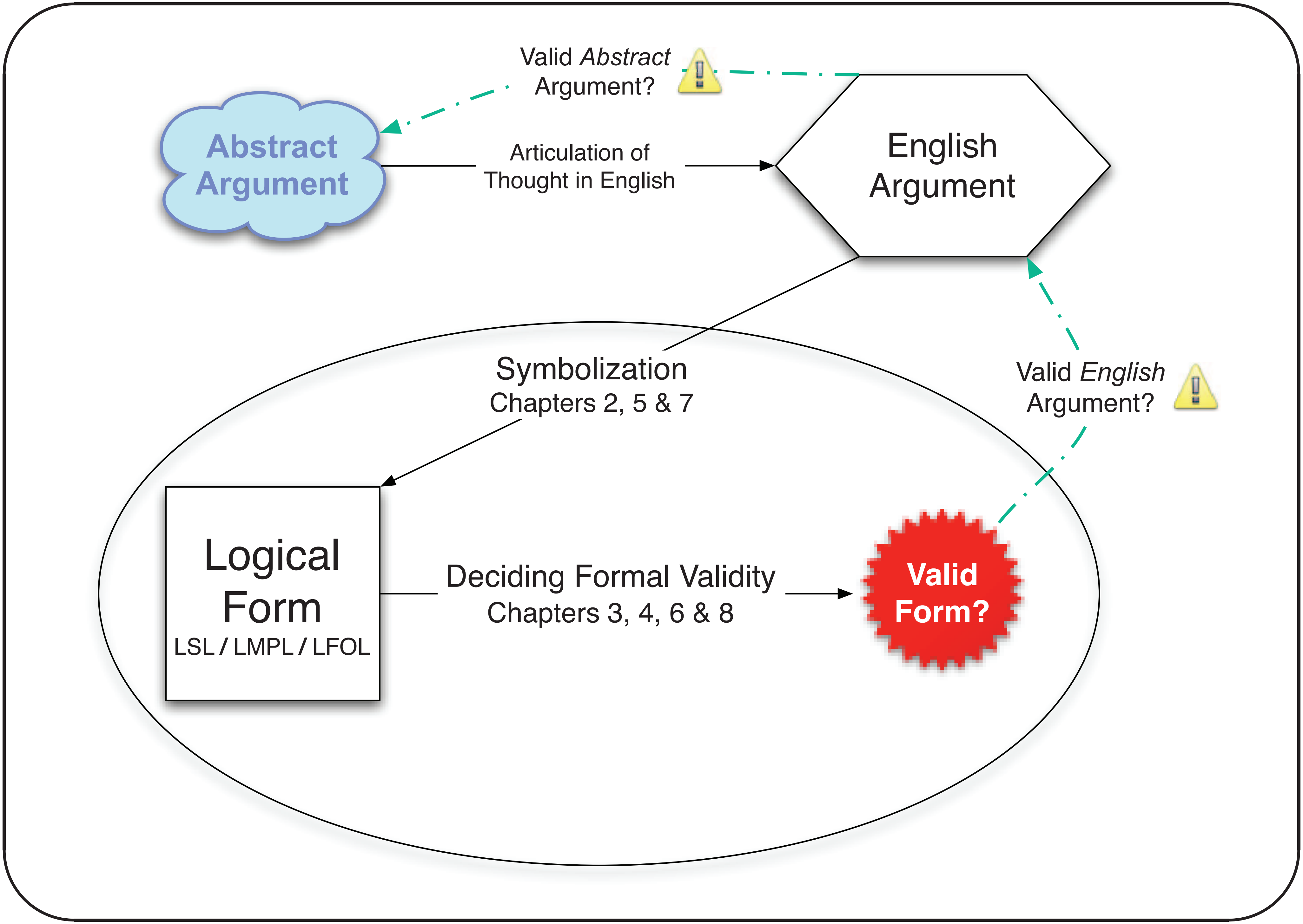
Announcements & Such

- *Bob Dylan*: *Desolation Row*
- Administrative Stuff
  - If you did not receive an email telling you which section you are in, then you are not going to be enrolled in the course this semester (unless we made an error, in which case, see us at end of class).
  - Section rosters have been set (see website for times and locations). Sections with (as yet) undetermined permanent locations will meet temporarily in 301 Moses. Stay tuned for permanent locations.
  - I will not be holding office hours on Thursday this week.
  - **HW #1 (1^{st} sub) due Thursday @ 4pm @ 12A drop box (301 Moses).**
- Introduction to the Course & Chapter 1 of Forbes
  - A "Big Picture" perspective on the course.
  - Are our two "renditions"/"definitions" of *validity* really equivalent?
  - Working with our official (idealized) definition of validity.
  - Next: Chapter 2 — The Language of Sentential Logic (LSL)



Valid *Abstract* Argument?
Abstract Argument
Articulation of Thought in English
English Argument
Symbolization
Chapters 2, 5 & 7
Valid *English* Argument?
Logical Form
LSL / LMPL / LFOL
Deciding Formal Validity
Chapters 3, 4, 6 & 8
Valid Form?

Our Two "Renditions" of *Validity* — Are They Equivalent?

- Informally, if the conclusion of an argument *follows from* its premises, then the argument is said to be *valid* (otherwise, it's *in*valid).
- It is this *informal* (Forbes calls it "absolute") *validity* concept that we're interested in. Plausibly, this can be made more precise, as follows:

☞ **Definition #1.** An argument $\mathscr{A}$ is *valid* if and only if:

> It is (logically!) *necessary* that *if* all of the premises of $\mathscr{A}$ are true, *then* the conclusion of $\mathscr{A}$ is also true.

☞ **Definition #2.** An argument $\mathscr{A}$ is *valid* if and only if:

> It is (logically!) *impossible* that *both*: (1) all of the premises of $\mathscr{A}$ are true, *and* (2) the conclusion of $\mathscr{A}$ is false.

- I mentioned las time that, for us, these two definitions are *equivalent*, and that we'll take #2 as our *official* definition. Let's think about a pair of "limiting" cases of #2, which will reveal a fundamental *idealization*.

Two "Strange" Validities — Under Our Official Definition #2

(†) p. Therefore, either q or not q.

- (†) is valid because it is (logically) *impossible* that *both*:

  (i) p is true, *and*
  (ii) "either q or not q" is false.

  This is impossible because *(ii) alone* is impossible.

(‡) p and not p. Therefore, q.

- (‡) is valid because it is (logically) *impossible* that *both*:

  (iii) "p and not p" is true, *and*
  (iv) q is false.

  This is impossible because *(iii) alone* is impossible.

- These are valid under #2, but perhaps not (intuitively) under #1. This reveals a *fundamental idealization* about "if … then statements". I will say a lot more about this key idealization when we get to chapter 3.

Logical Form I

- We will adopt the following "conservative" heuristic in this course:
  - ☞ We will conclude that an argument $\mathscr{A}$ is valid *only if* we have identified $\mathscr{A}$ as having a *valid* ***logical form*** — *according to one of our theories of valid logical forms*.
- Thus, our strategy will be to develop *formal* theories/methods for *modeling* intuitive/absolute validity of arguments expressed in English.
- We will begin with a very simple (and highly idealized) formal theory of validity: the theory of Sentential Logic (SL).
- Later in the course, we will refine this theory in several ways, which will lead to formal theories of validity that are more encompassing.
- We won't be able to capture *all* intuitively/absolutely valid arguments with *any* of our theories, but this is OK. [Analogy: theoretical physics.]

Logical Form II — Sentential Form

☞ **Definition**. The *sentential form* of an argument (or, the sentences faithfully expressing an argument) is obtained by replacing each basic (or, atomic) sentence in the argument with a single (lower-case) letter.

- What's a "basic" sentence? A basic sentence is a sentence that doesn't contain any (complete) sentence as a proper part. How about these?

  (a) Branden is a philosopher and Branden is a man. [p and q]

  (b) It is not the case that Branden is 6 feet tall. [not-p]

  (c) Snow is white. [p]

  (d) Either it will rain today or it will be sunny today. [p or q]

- Sentences (a), (b), and (d) are *not* basic (we'll call them "complex" or "compound"). Only (c) is basic. We'll also use "atomic" for basic.

- Here are some important sentential forms…

Some Valid and Invalid Sentential Forms

Sentential Argument Form	Name	Valid/Invalid
p If p, then q ——— $\therefore q$	*Modus Ponens*	Valid
q If p, then q ——— $\therefore p$	Affirming the Consequent	Invalid
It is not the case that q If p, then q ——— $\therefore$ It is not the case that p	*Modus Tollens*	Valid
It is not the case that p If p, then q ——— $\therefore$ It is not the case that q	Denying the Antecedent	Invalid
If p, then q If q, then r ——— $\therefore$ If p, then r	Hypothetical Syllogism	Valid
It is not the case that p Either p or q ——— $\therefore q$	Disjunctive Syllogism	Valid

Logical Form III — Beyond Sentential Form

- The first half of the course involves developing a precise *theory* of *sentential* validity, and several rigorous techniques for *deciding* whether a sentential form is (or is not) valid. This only takes us so far.
- Not all (absolutely) valid arguments have valid *sentential* forms, *e.g.*:

All men are mortal.

(2) Socrates is a man.

∴ Socrates is mortal.

- The argument expressed by (2) seems clearly valid. But, the sentential form of (2) is not a valid form. Its sentential form is:

p.

(2_f) q.

$\therefore r$.

- In the second half of the course, we'll see a more general theory of logical forms which will encompass both (2) and (1) as valid forms.
- In this more general theory, we will be able to see that (2) has something like the following (non-sentential!) logical form:

$$
\begin{array}{ll}
 & \text{All } X\text{s are } Y\text{s.} \\
(2_f*) & a \text{ is an } X. \\
 & \therefore\ a \text{ is a } Y.
\end{array}
$$

- But, we won't need to worry about such non-sentential forms until chapter 7. Meanwhile, we will focus on *sentential logic*.
- This will involve learning a (simple) purely formal language for talking about sentential forms, and then developing rigorous methods for determining whether sentential forms are valid.
- As we will see, the fit between our simple formal sentential language and English (or other natural languages) is not perfect.

Three Important Things to Remember About Validity

- The premises and the conclusion of an invalid argument can all be true.

  All men are mortal.

  Socrates is mortal.

  ∴ Socrates is a man.

- The premises and the conclusion of a valid argument can all be false.

  All men are bald.

  Jane Fonda is a man.

  ∴ Jane Fonda is bald.

- A valid argument with false premises can still have a true conclusion.

  All men are actors.

  Jane Fonda is a man.

  ∴ Jane Fonda is an actor.

Validity and Soundness of Arguments — Some Non-Sentential Examples

- Can we classify the following according to validity/soundness?

1)	All wines are beverages. Chardonnay is a wine. Therefore, chardonnay is a beverage.	5)	All wines are beverages. Chardonnay is a beverage. Therefore, chardonnay is a wine.
2)	All wines are whiskeys. Chardonnay is a wine. Therefore, chardonnay is a whiskey.	6)	All wines are beverages. Ginger ale is a beverage. Therefore, ginger ale is a wine.
3)	All wines are soft drinks. Ginger ale is a wine. Therefore, ginger ale is a soft drink.	7)	All wines are whiskeys. Chardonnay is a whiskey. Therefore, chardonnay is a wine.
4)	All wines are whiskeys. Ginger ale is a wine. Therefore, ginger ale is a whiskey.	8)	All wines are whiskeys. Ginger ale is a whiskey. Therefore, ginger ale is a wine.

	Valid	Invalid
True premises True conclusion	All wines are beverages. Chardonnay is a wine. Therefore, chardonnay is a beverage. [sound]	All wines are beverages. Chardonnay is a beverage. Therefore, chardonnay is a wine. [unsound]
True premises False conclusion	**Impossible** None exist	All wines are beverages. Ginger ale is a beverage. Therefore, ginger ale is a wine. [unsound]
False premises True conclusion	All wines are soft drinks. Ginger ale is a wine. Therefore, ginger ale is a soft drink. [unsound]	All wines are whiskeys. Chardonnay is a whiskey. Therefore, chardonnay is a wine. [unsound]
False premises False conclusion	All wines are whiskeys. Ginger ale is a wine. Therefore, ginger ale is a whiskey. [unsound]	All wines are whiskeys. Ginger ale is a whiskey. Therefore, ginger ale is a wine. [unsound]

- For more Ch. 1 practice, see our *validity and soundness handout* ...

Some Brain Teasers Involving Validity and Soundness

- Here are two very puzzling arguments:

($\mathscr{A}_1$)
Either $\mathscr{A}_1$ is valid or $\mathscr{A}_1$ is invalid.
$\therefore$ $\mathscr{A}_1$ is invalid.

($\mathscr{A}_2$)
$\mathscr{A}_2$ is valid.
$\therefore$ $\mathscr{A}_2$ is invalid.

- I'll discuss $\mathscr{A}_2$ ($\mathscr{A}_1$ is left as an exercise).
  - If $\mathscr{A}_2$ is valid, then it has a true premise and a false conclusion. But, this means that if $\mathscr{A}_2$ is valid, then $\mathscr{A}_2$ invalid!
  - If $\mathscr{A}_2$ is invalid, then its conclusion must be true (as a matter of logic). But, this means that if $\mathscr{A}_2$ is invalid then $\mathscr{A}_2$ is valid!
  - This *seems* to imply that $\mathscr{A}_2$ is *both valid and invalid*. But, remember our conservative validity-principle. What is the *logical form* of $\mathscr{A}_2$? [This is another reason we banish *self-reference* from our discussion.]

Absolute Validity *vs* Formal Validity

- Forbes calls the general, informal notion of validity "absolute validity".
- Our notion is a bit more conservative than his, since we'll only call an argument valid if one of our *formal theories* captures it as falling under a valid *form*. Our first formal theory (LSL) is about *sentential* validity.
- An argument is *sententially* valid if it has a valid *sentential form*.
- Sentential form is obtained by replacing each basic or atomic sentence in an argument with a corresponding lower-case letter.
- Once we know the sentential form of an argument (chapter 2), we will be able to apply purely formal, mechanical methods (chapters 3 and 4) for determining whether that sentential form is valid.

☞ Even if an argument fails to be sententially valid, it could still be valid according to a richer logical theory than LSL. We'll see two or three theories like this later in the course (LMPL, L2PL, and LFOL).

Introduction to the Syntax of the LSL: The Lexicon

- The syntax of LSL is quite simple. Its lexicon has the following symbols:
  - Upper-case letters 'A', 'B', ... which stand for *basic sentences*.
  - Five *sentential connectives/operators* (one *unary*, four *binary*):

Operator	**Name**	**Logical Function**	**Used to translate**
'$\sim$'	tilde	negation	not, it is not the case that
'&'	ampersand	conjunction	and, also, moreover, but
'$\vee$'	vee	disjunction	or, either ... or ...
'$\rightarrow$'	arrow	conditional	if ... then ..., only if
'$\leftrightarrow$'	double arrow	biconditional	if and only if

  - Parentheses '(', ')', brackets '['. ']', and braces '{', '}' for grouping.
- If a string of symbols contains anything else, then it's not a sentence of LSL. And, only *certain* strings of these symbols are LSL sentences.
- Some LSL symbol strings aren't *well-formed*: '$(A \,\&\, B$', '$A \,\&\, B \vee C$', *etc*.

Digression: The Use/Mention Distinction

- Consider the following two sentences:

  (1) California has more than nine residents.

  (2) ‘California’ has more than nine letters.

- In (1), we are *using* the word ‘California’ to talk about the State of California. But, in (2), we are merely *mentioning* the word ‘California’ (*i.e.*, we’re talking about *the word itself*).

- If Jeremiah = ‘California’, which of these sentences are true?

  (3) Jeremiah has (exactly) eight letters [false].

  (4) Jeremiah has (exactly) ten letters [true].

  (5) ‘Jeremiah’ has eight letters [true].

  (6) ‘Jeremiah’ is the name of a state [false].

Digression: More on Use/Mention and ‘ ’ *versus* ⌜ ⌝

- Consider the following two statements about LSL sentences

  (*i*) If p and q are both sentences of LSL, then so is $\ulcorner(p \,\&\, q)\urcorner$.

  (*ii*) If p and q are both sentences of LSL, then so is ‘$(p \,\&\, q)$’.

- As it turns out, (*i*) is true, but (*ii*) is *false*. The string of symbols ‘$(p \,\&\, q)$’ *cannot* be a sentence of LSL, since ‘p’ and ‘q’ are *not* part of the lexicon of LSL. They allow us to talk about LSL *forms*.

- The trick is that $\ulcorner(p \,\&\, q)\urcorner$ abbreviates the long-winded phrase:

  – The symbol-string which results from writing ‘(’ followed by p followed by ‘&’ followed by q followed by ‘)’.

- In (*ii*), we are merely *mentioning* ‘p’ and ‘q’ (in ‘$(p \,\&\, q)$’). But, in (*i*), we are *using* ‘p’ and ‘q’ (in $\ulcorner(p \,\&\, q)\urcorner$) to talk about (forms of) sentences in LSL. In (*i*), ‘p’ and ‘q’ are *used* as *metavariables*.

Digression: Object language, Metalanguage, *etc.* . . .

- LSL is the *object language* of our current studies. The symbol string '$(A \& B) \vee C$' is a sentence of LSL. But, the symbol string '$(p \& q) \vee r$' is *not* a sentence of LSL. Why?
- We use a *metalanguage* to talk about the object language LSL. This metalanguage is not formalized. It's mainly English, plus *metavariables* like 'p', 'q', 'r', and *selective quotes* '$\ulcorner$' and '$\urcorner$'.
- If $p =$ '$(A \vee B)$', and $q =$ '$(C \to D)$', then what are the following?
  - $\ulcorner p \& q \urcorner$ [$(A \vee B) \& (C \to D)$], '$p \& q$' [$p \& q$], 'p' [p], 'q' [q]
- And, which of the following are true?
  - p has five symbols [true]. 'p' has five symbols [false].
  - $\ulcorner p \& q \urcorner$ is a sentence of LSL [true]. So is '$\ulcorner p \& q \urcorner$' [false].

The Five Kinds (Forms) of *Non-Basic* LSL Sentences

- Sentences of the form $\ulcorner p \,\&\, q \urcorner$ are called *conjunctions*, and their constituents (p, q) are called *conjuncts*.
- Sentences of the form $\ulcorner p \vee q \urcorner$ are called *disjunctions*, and their constituents (p, q) are called *disjuncts*.
- Sentences of the form $\ulcorner p \rightarrow q \urcorner$ are called *conditionals*. p is called the *antecedent* of $\ulcorner p \rightarrow q \urcorner$, and q is called its *consequent*.
- Sentences of the form $\ulcorner p \leftrightarrow q \urcorner$ are called *biconditionals*. p is called the *left-hand side* of $\ulcorner p \leftrightarrow q \urcorner$, and q is its *right-hand side*.
- Sentences of the form $\ulcorner \sim p \urcorner$ are called *negations*. The sentence p is called the *negated sentence*.
- These 5 kinds of sentences (+ *atoms*) are the *only* kinds in LSL.
- Next, we begin to think about "translation" from English into LSL.

English $\mapsto$ LSL I: Basic Steps Toward Symbolization

- Sentences with *no* connectives are *trivial* to “translate” or symbolize:
  - ‘It is cold.’ $\mapsto$ ‘C’.
  - ‘It is rainy.’ $\mapsto$ ‘R’.
  - ‘It is sunny.’ $\mapsto$ ‘S’.
- Sentences with just one sentential connective are also pretty easy:
  - ‘It is cold and rainy.’ $\mapsto$ ‘$C \& R$’. [why two atomic letters?]

☞ Try to give the most *precise* (fine-grained) LSL rendition you can, and try to come as close as possible to capturing the meaning of the original.

- Sentences with two connectives can be trickier:
  - ‘Either it is sunny or it is cold and rainy.’ $\mapsto$ ‘$S \vee (C \& R)$’.
- Q: Why is ‘$(S \vee C) \& R$’ incorrect? A: The English is *not* a conjunction.

English ↦ LSL II: Symbolizing in Two Stages

☞ When symbolizing English sentences in LSL (especially complex ones), it is useful to perform the symbolization in (at least) *two stages*.

Stage 1: Replace all basic sentences (explicit or implicit) with atomic letters. This yields a sentence in "Logish" (neither English nor LSL).

Stage 2: Eliminate remaining English by replacing English connectives with LSL connectives, and properly grouping the resulting symbolic expression (w/parens, *etc.*) to yield pure LSL.

- Here are some simple examples involving only single connectives:

English:	**"Logish":**	**LSL:**
Either it's raining or it's snowing.	Either R or S.	$R \vee S$
If Dell introduces a new line, then Apple will also.	If D, then A.	$D \rightarrow A$
Snow is white and the sky is blue.	W and B.	$W \,\&\, B$
It is not the case that Emily Bronte wrote *Jane Eyre*.	It is not the case that E.	$\sim E$
John is a bachelor if and only if he is unmarried.	J if and only if not M.	$J \leftrightarrow \sim M$