Overview of Today's Lecture

- Bob Marley, *Acoustic Medley*
- Administrative Stuff
  - I went over the syllabus/website last time. See website for all details:
    - `http://fitelson.org/12A/`
  - If you weren't here last time, (a) please get a syllabus, and (b) fill-out an index card with your section preferences, among these 6 pairs:
    - (1) 10–11 MW, (2) 12–1 MW, (3) 10–11 TR (4) 11–12 TR (5) 1–2 MW, (6) 2–3 MW
  - ☞ **HW #1 is due next Thursday.** See Assignments & Handouts Page.
- Introduction to the Course & Chapter 1 of Forbes
  - Propositions — the basic building blocks of logical analysis
  - *Actual* truth and falisty *vs possible/necessary* truth and falsity
  - *Logical* necessity, *logical* truth, validity, and soundness

Background 1: Propositions and Sentences

- *Propositions* are the basic units of logical analysis. They are expressed by declarative sentences like "Snow is white."
- Not all sentences express propositions (*e.g.*, "What time is it?").
- Propositions are not identical to declarative sentences that express them. Consider: "Snow is white" and "Schnee ist weiß."
- Propositions are either true or false (not both). *True* and *False* are called *truth-values*. Propositions have exactly one truth-value. The truth-value of a proposition is *objective*.
- That is, whether a proposition is true or false (in a given situation) does not depend on what anyone thinks about *that* proposition or on how that proposition happens to be expressed.
- Even if a proposition is *about* something subjective, its truth-value remains objective (*e.g.*, Branden believes that the Yankees will win.)

Background 2: Actual, Possible, and Necessary Truth

- Some propositions are actually true (Snow is white), and some are not (Al Gore is President of the United States in 2007).
- Other propositions are not *actually* true, but still *possibly* true. Al Gore is not *actually* our President in 2007, but he *might have been*. As such, it is *possibly* true that Al Gore is President in 2007.
- Some propositions are not even *possibly* true. For instance:
  1. My car has traveled faster than the speed of light.
  2. $2 + 2 = 5$.
  3. Branden weighs 200 lbs and Branden does not weigh 200 lbs.
- (1) violates the laws of physics: it is *physically impossible*. (2) violates the laws of arithmetic: it is *arithmetically* impossible.
- (3) violates the laws of *logic*: it is *logically* impossible.

- This is the kind of impossibility that interests the logician. In slogan form, we might call this "the strongest possible kind of impossibility."
- Some propositions are not only *actually* true, but (logically) *necessarily* true. These *must* be true, on pain of *self-contradiction*:
  - Either Branden weighs 200lbs or he does not weigh 200lbs.
  - If Branden is a good man, then Branden is a man.
- Logical possibility and logical necessity are central concepts in this course. We will make extensive use of them.
- We will look at two precise, formal logical theories in which the notion of logical necessity will have a more precise meaning.
- But, before we get into our formal theorizing, we will look informally at the *following-from* relation between propositions.
- As we will see, understanding the following-from relation will require a grasp of the notions of logical necessity (and logical truth).

Bakckground 3: Arguments, Following-From, and Validity

- An *argument* is a collection of propositions, one of which (the *conclusion*) is supposed to *follow from* the rest (the *premises*).

  All men are mortal. [premise]
  Socrates is a man. [premise]
  Therefore, Socrates is mortal. [conclusion]

- If the conclusion of an argument *follows from* its premises, then the argument is said to be *valid* (otherwise, it's *in*valid).

☞ **Definition**. An argument $\mathscr{A}$ is *valid* if and only if:

  Rendition #1. It is (logically!) *necessary* that *if* all of the premises of $\mathscr{A}$ are true, *then* the conclusion of $\mathscr{A}$ is also true.

  Rendition #2. It is (logically!) *impossible* for both of the following to be true simultaneously: (1) all of the premises of $\mathscr{A}$ are true, *and* (2) the conclusion of $\mathscr{A}$ is false. [For us, this will be *equivalent* to #1.]

Background 4: Validity, Soundness, and "Good" Arguments

- A "good" argument is one in which the conclusion follows from the premises. But, intuitively, there is more to a "good" argument (all things considered) than mere validity.
- Ideally, arguments should also have (actually) *true premises*. If the premises of an argument are (actually) false, then (intuitively) the argument isn't very "good" — even if it is valid. *Why not*?

☞ **Definition**. An argument $\mathscr{A}$ is *sound* if and only if *both*: (*i*) $\mathscr{A}$ is valid, *and* (*ii*) all of $\mathscr{A}$'s premises are (actually) true.

- So, there are two components or aspects of "good" arguments:
  - Logical Component: Is the argument valid?
  - Non-Logical Component: Are the premises (actually) true?
- This course is only concerned with the *logical* component.

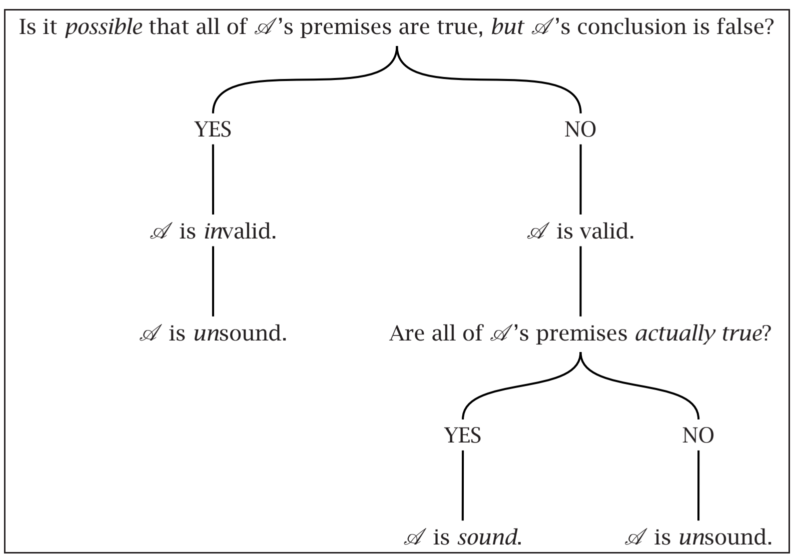


Figure 1: Testing an argument $\mathscr{A}$ for validity and soundness.

Why study logic *formally* or *symbolically*?

- Ultimately, we want to decide whether arguments expressible in *natural* languages are valid. But, in this course, we will only study arguments expressible in *formal* languages. And, we will use *formal* tools. *Why?*
- Analogous question: What we want from natural science is explanations and predictions about *natural* systems. But, our theories (strictly) apply only to systems faithfully describable in *formal*, *mathematical* terms.
- Although formal models are *idealizations* which abstract away some aspects of natural systems, they are *useful idealizations* that help us understand *many* natural relationships and regularities.
- Similarly, studying arguments expressible in formal languages allows us to develop powerful tools for testing validity. We won't be able to capture *all* valid arguments this way. But, we can grasp many.

A Subtle Argument, and the Notion of Logical Form

(i) John is a bachelor.
∴ John is unmarried.

- Is (i) valid? Well, this is tricky. Intuitively, being unmarried is part of the *meaning* of "bachelor". So, it *seems* like it is (intuitively) logically impossible for the premise of (i) to be true while its conclusion is false
- This suggests that (i) is (intuitively/absolutely) valid.
- On the other hand, consider the following argument:

If John is a bachelor, then John is unmarried.
(ii) John is a bachelor.
∴ John is unmarried.

- The correct judgment about (ii) seems *clearly* to be that it is valid – *even if we don't know the meaning of "bachelor" (or "unmarried").*
- This is clear because the logical form of (ii) is *obvious* [(i)'s form is not].

Logical Form II

- This suggests the following additional "conservative" heuristic:
  - ☞ We should conclude that an argument $\mathscr{A}$ is valid only if we can see that $\mathscr{A}$'s conclusion follows from $\mathscr{A}$'s premises *without appealing to the meanings of the predicates involved in* $\mathscr{A}$.
- But, if validity does not depend on the meanings of predicates, then what *does* it depend on? This is a deep question about logic. We will not answer it here. That's for more advanced philosophical logic courses.
- What we will do instead is adopt a conservative methodology that only classifies *some* "intuitively/absolutely valid" arguments as valid.
- The strategy will be to develop some *formal* methods for *modeling* intuitive/abolsute validity of arguments expressed in English.
- We won't be able to capture *all* intuitively/absolutely valid arguments with our methods, but this is OK. [Analogy: mathematical physics.]

Logical Form III

- We will begin with *sentential logic*. This will involve providing a characterization of valid *sentential forms*. Here's a paradigm example:

Dr. Ruth is a man.
(1) If Dr. Ruth is a man, then Dr. Ruth is 10 feet tall.
∴ Dr. Ruth is 10 feet tall.

- (1) is a set of sentences with a valid sentential form. So, whatever argument it expresses is a valid argument. What's its *form*?

p.
(1_f) If p, then q.
∴ q.

- (1)'s valid *sentential form* (1_f) is so famous it has a name: *Modus Ponens*. [Usually, latin names are used for the *valid* forms.]

☞ **Definition**. The *sentential form* of an argument (or, the sentences faithfully expressing an argument) is obtained by replacing each basic (or, atomic) sentence in the argument with a single (lower-case) letter.

- What's a "basic" sentence? A basic sentence is a sentence that doesn't contain any sentence as a proper part. How about these?

(a) Branden is a philosopher and Branden is a man.
(b) It is not the case that Branden is 6 feet tall.
(c) Snow is white.
(d) Either it will rain today or it will be sunny today.

- Sentences (a), (b), and (d) are *not* basic (we'll call them "complex" or "compound"). Only (c) is basic. We'll also use "atomic" for basic.
- What's the sentential form of the following argument (is it valid?):

If Tom is at his Fremont home, then he's in California.
Tom is in California.
∴ Tom is at his Fremont home.

Two "Strange" Valid Sentential Forms

(†) p. Therefore, either q or not q.

- (†) is valid because it is (logically) *impossible* that *both*:
  - (i) p is true, *and*
  - (ii) "either q or not q" is false.

  This is impossible because *(ii) alone* is impossible.

(‡) p and not p. Therefore, q.

- (‡) is valid because it is (logically) *impossible* that *both*:
  - (iii) "p and not p" is true, *and*
  - (iv) q is false.

  This is impossible because *(iii) alone* is impossible.
- We'll soon see why we have these "oddities". They stem from our semantics for "If … then" statements (and our first def. of validity).

Some Valid and Invalid Sentential Forms

Sentential Argument Form	Name	Valid/Invalid
p If p, then q ∴ q	*Modus Ponens*	Valid
q If p, then q ∴ p	Affirming the Consequent	Invalid
It is not the case that q If p, then q ∴ It is not the case that p	*Modus Tollens*	Valid
It is not the case that p If p, then q ∴ It is not the case that q	Denying the Antecedent	Invalid
If p, then q If q, then r ∴ If p, then r	Hypothetical Syllogism	Valid
It is not the case that p Either p or q ∴ q	Disjunctive Syllogism	Valid

Logical Form IV — Beyond Sentential Form

- The first half of the course involves developing a precise *theory* of *sentential* validity, and several rigorous techniques for *deciding* whether a sentential form is (or is not) valid. This only takes us so far.
- Not all (absolutely) valid arguments have valid *sentential* forms, *e.g.*:

All men are mortal.
(2) Socrates is a man.
∴ Socrates is mortal.

- The argument expressed by (2) seems clearly valid. But, the sentential form of (2) is not a valid form. Its sentential form is:

p.
(2_f) q.
∴ r.

- In the second half of the course, we'll see a more general theory of logical forms which will encompass both (2) and (1) as valid forms.
- In this more general theory, we will be able to see that (2) has something like the following (non-sentential!) logical form:

All Xs are Ys.
(2_f*) a is an X.
∴ a is a Y.

- But, we won't need to worry about such non-sentential forms until chapter 7. Meanwhile, we will focus on *sentential logic*.
- This will involve learning a (simple) purely formal language for talking about sentential forms, and then developing rigorous methods for determining whether sentential forms are valid.
- As we will see, the fit between our simple formal sentential language and English (or other natural languages) is not perfect.

Validity and Soundness of Arguments — Some Non-Sentential Examples

- Can we classify the following according to validity/soundness?

1) All wines are beverages. Chardonnay is a wine. Therefore, chardonnay is a beverage.	**5)** All wines are beverages. Chardonnay is a beverage. Therefore, chardonnay is a wine.
2) All wines are whiskeys. Chardonnay is a wine. Therefore, chardonnay is a whiskey.	**6)** All wines are beverages. Ginger ale is a beverage. Therefore, ginger ale is a wine.
3) All wines are soft drinks. Ginger ale is a wine. Therefore, ginger ale is a soft drink.	**7)** All wines are whiskeys. Chardonnay is a whiskey. Therefore, chardonnay is a wine.
4) All wines are whiskeys. Ginger ale is a wine. Therefore, ginger ale is a whiskey.	**8)** All wines are whiskeys. Ginger ale is a whiskey. Therefore, ginger ale is a wine.

	Valid	**Invalid**
True premises True conclusion	All wines are beverages. Chardonnay is a wine. Therefore, chardonnay is a beverage. [sound]	All wines are beverages. Chardonnay is a beverage. Therefore, chardonnay is a wine. [unsound]
True premises False conclusion	**Impossible** None exist	All wines are beverages. Ginger ale is a beverage. Therefore, ginger ale is a wine. [unsound]
False premises True conclusion	All wines are soft drinks. Ginger ale is a wine. Therefore, ginger ale is a soft drink. [unsound]	All wines are whiskeys. Chardonnay is a whiskey. Therefore, chardonnay is a wine. [unsound]
False premises False conclusion	All wines are whiskeys. Ginger ale is a wine. Therefore, ginger ale is a whiskey. [unsound]	All wines are whiskeys. Ginger ale is a whiskey. Therefore, ginger ale is a wine. [unsound]

- See, also, our validity and soundness handout …

Some Brain Teasers Involving Validity and Soundness

- Here are two very puzzling arguments:

($\mathscr{A}_1$) Either $\mathscr{A}_1$ is valid or $\mathscr{A}_1$ is invalid.
$\therefore$ $\mathscr{A}_1$ is invalid.

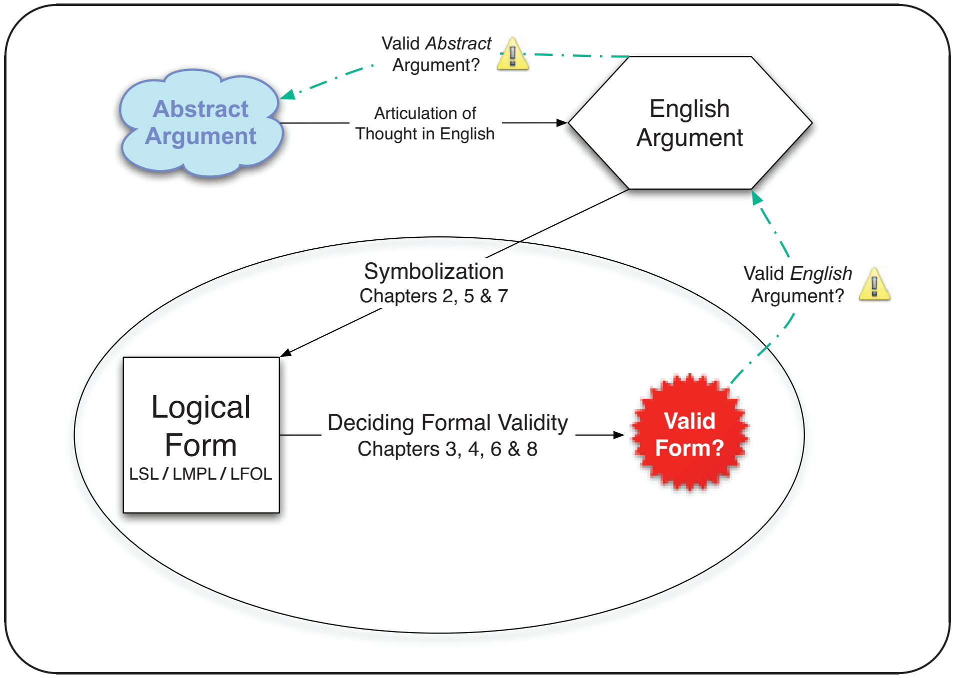
($\mathscr{A}_2$) $\mathscr{A}_2$ is valid.
$\therefore$ $\mathscr{A}_2$ is invalid.

- I'll discuss $\mathscr{A}_2$ ($\mathscr{A}_1$ is left as an exercise).
  - If $\mathscr{A}_2$ is valid, then it has a true premise and a false conclusion. But, this means that if $\mathscr{A}_2$ is valid, then $\mathscr{A}_2$ invalid!
  - If $\mathscr{A}_2$ is invalid, then its conclusion must be true (as a matter of logic). But, this means that if $\mathscr{A}_2$ is invalid then $\mathscr{A}_2$ is valid!
  - This *seems* to imply that $\mathscr{A}_2$ is *both valid and invalid*. But, remember our conservative validity-principle. What is the *logical form* of $\mathscr{A}_2$?

Absolute Validity *vs* Formal Validity

- Forbes calls the general, informal notion of validity "absolute validity".
- Our notion is a bit more conservative than his, since we'll only call an argument valid if one of our *formal theories* captures it as falling under a valid *form*. Our first formal theory (LSL) is about *sentential* validity.
- An argument is *sententially* valid if it has a valid *sentential form*.
- Sentential form is obtained by replacing each basic or atomic sentence in an argument with a corresponding lower-case letter.
- Once we know the sentential form of an argument (chapter 2), we will be able to apply purely formal, mechanical methods (chapters 3 and 4) for determining whether that sentential form is valid.

☞ Even if an argument fails to be sententially valid, it could still be valid according to a richer logical theory than LSL. We'll see two or three theories like this later in the course (LMPL, L2PL, and LFOL).


Valid *Abstract* Argument?
Abstract Argument
Articulation of Thought in English
English Argument
Symbolization
Chapters 2, 5 & 7
Valid *English* Argument?
Logical Form
LSL / LMPL / LFOL
Deciding Formal Validity
Chapters 3, 4, 6 & 8
Valid Form?