Announcements & Such

- Administrative Stuff
  - **HW #4 resubmission due today.**
  - **HW #5 has been posted. First submission due next Thursday.**
  - **Handout on LMPL Interpretations has been posted (useful for part of HW #5). I will discuss this in class very soon.**
  - **From now on, my office hours will be 2–4pm Thursdays.**
- Today: Finishing-up Chapter 5 — LMPL Syntax
  - Lots of symbolizations with quantifiers and variables.
  - "Intuitive" meanings of and relationships between the quantifiers.
- Next: Chapter 6 — LMPL Semantics
  - Interpretations of LMPL sentences.
  - Validity in LMPL.

Symbolization in LMPL VI: More Examples with ∃

- Let's symbolize the following sentences. Whenever we symbolize in LMPL, we must state our dictionary of monadic predicates, and we must also say what the domain of discourse is over which we are quantifying.
  1. No smoggy city is unpolluted.
  2. Vampires do not exist.
  3. If ghosts and vampires do not exist, then nothing can be a ghost without being a vampire.
- If the dictionary is (where the domain is people in this classroom now):

  $S\_\_$: __ is standing up at the podium.
  $W\_\_$: __ is wealthy.
  b : Branden

  then what do the following two LMPL sentences assert (in English)?

  $\sim(\exists x)(Sx \,\&\, Wx)$ $\qquad$ $\sim Wb$

Symbolization in LMPL VII: Back to ① and ②

- Now, we are in a position to symbolize in LMPL the argument ① that we saw at the beginning of the previous lecture:

$$①_{\text{LMPL}} \quad \begin{array}{l} Ws \\ \therefore (\exists x)Wx \end{array}$$

- Since there are only finitely many people, we can see why this argument is valid, by representing its conclusion as a long (but finite!) *disjunction*, in which its only premise is a disjunct:

$$① \quad \begin{array}{l} Ws \\ \therefore Wa \vee \cdots \vee Ws \vee \cdots \end{array}$$

- We can use a similar trick for argument ②. The premise of ② ['$(\forall x)Hx$'] entails a *conjunction* ['$Ha \,\&\, \cdots \,\&\, Hp \,\&\, \cdots$'], and its conclusion ['Hp'] is one of the conjuncts of that conjunction.

Some Symbolizations Involving ∃

E__ : __ is an even number $\qquad$ a : the number 2
P__ : __ is a prime number $\qquad$ Domain : natural numbers (ℕ)
G__ : __ is greater than the number 2

(1) There exists a prime number and there exists an even number.

$(\exists x)Px \,\&\, (\exists x)Ex$

(2) There exists an even prime number. [$(\exists x)(Px \,\&\, Ex)$]

(3) 2 is an even prime number. [$Ea \,\&\, Pa$]

(4) If 2 is prime, then there are some even primes. [$Pa \to (\exists x)(Px \,\&\, Ex)$]

(5) No number is even if it is prime. [$\sim(\exists x)(Px \,\&\, Ex)$]

- Careful with this one! Why *isn't* this '$\sim(\exists x)(Px \to Ex)$'?
- Compare: No number is even if it is prime and greater than 2.
  - ∗ In LMPL, this is: '$\sim(\exists x)[(Px \,\&\, Gx) \,\&\, Ex]$', which is *true*. *Why?*
  - ∗ Note: '$\sim(\exists x)[(Px \,\&\, Gx) \to Ex]$' is *false*! *Why?*

The *Universal* Quantifier ∀

- To symbolize English sentences like 'Everyone is happy', we will need the *universal* quantifier '∀' (which means 'every' or 'all').
  - (*i*) We begin with the raw English sentence: 'Everyone is happy'.
  - (*ii*) Then, we move to the *Logish* form: 'For every x, x is happy'.
  - (*iii*) Finally, we have the full LMPL symbolization: '$(\forall x)Hx$'.
- As with the existential quantifier, we must be careful with the *scope* of '∀'. How would we symbolize the following two sentences?
  - (1) 'Everyone is happy and everyone is wise.'
  - (2) 'Everyone is happy and wise.'
- These sentences get symbolized differently, because they have different (syntactic) *structures*. But, do they have different *meanings*? In Chapter 6, we'll *prove* the answer to this question.

The Universal Quantifier II

- How should one symbolize the following English sentence?
  - (3) 'Everyone who is happy is wise.' ['All happy people are wise.']
- Note: Unlike (1) and (2) above, (3) does *not* have the consequence that *everyone* is happy. So, what, exactly, *does* (3) say?
- (3) says that *if* a person is happy, *then* that person is wise. This suggests the following *Logish* form (*wrt* the domain of people):

  'For every x, if x is happy then x is wise.'
- Now, we are ready for the full LMPL symbolization:

$$(\forall x)(Hx \to Wx)$$

- We will use this same trick to symbolize sentences like 'Every happy person is a wise person' or 'If someone is happy then he/she is wise', which both assert the same thing as (3).

The Universal Quantifier III

- How should one symbolize the following English sentence?
  - (4) 'Only happy people are wise.'
- Note: (4) does *not* say that *all* happy people are wise. That is, unlike (3), (4) does *not* say that a person is wise *if* he/she is happy. Rather, (4) says that a person is wise *only if* he/she is happy.
- This suggests the following *Logish* form (domain of people):

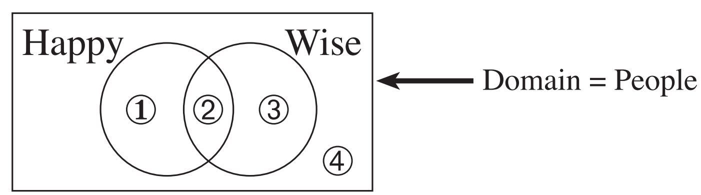
  'For every x, x is wise *only if* x is happy.'
- Now, we are ready for the full LMPL symbolization:

$$(\forall x)(Wx \to Hx)$$

- Here, we have the usual distinction between necessary and sufficient conditions. (3) says that happiness is *sufficient* for wisdom. But, (4) says that happiness is *necessary* for wisdom.

The Universal Quantifier IV, and Venn Diagrams

- Consider the following English sentence:
  - (5) 'No one who is unhappy is wise.'
- When trying to paraphrase or symbolize sentences like this in LMPL, it is useful to *picture* what they say using a *Venn Diagram*:



- (5) says that region ③ in the Venn Diagram is empty. So, (5) asserts the same thing as the following LMPL sentence:
  - (5.1) $(\forall x)(Wx \to Hx)$

The Intimate Relationship Between ∃ and ∀

- What we have just shown (informally) is:

  '$\sim(\exists x)(Wx \,\&\, \sim Hx)$' *is equivalent to* '$(\forall x)(Wx \to Hx)$'

- This is just a *special case* of the following *general equivalences*:

  $\ulcorner \sim(\exists v)\sim\phi v \urcorner$ *is equivalent to* $\ulcorner (\forall v)\phi v \urcorner$
  and
  $\ulcorner \sim(\forall v)\sim\phi v \urcorner$ *is equivalent to* $\ulcorner (\exists v)\phi v \urcorner$

- Here, 'ϕ' is a *metavariable* ranging over formulas of LMPL (thought of as functions of v), and 'v' ranges over variable symbols of LMPL.
- It follows from the second general equivalence above that $\sim(\exists x)(Wx \,\&\, \sim Hx)$ is equivalent to $\sim\sim(\forall x)\sim(Wx \,\&\, \sim Hx)$. But, this is equivalent to $(\forall x)\sim(Wx \,\&\, \sim Hx)$, hence $(\forall x)(Wx \to Hx)$.
- Our formal semantics will make these relationships more precise.

- Here's *why* (informally) $\ulcorner \sim(\exists v)\sim\phi v \urcorner$ and $\ulcorner (\forall v)\phi v \urcorner$ are equivalent.
- Start with the existential claim inside the negation $\ulcorner \sim(\exists v)\sim\phi v \urcorner$:

  $(\exists v)\sim\phi v$

- Next, note that, informally, $(\exists v)\sim\phi v$ asserts a *disjunction*:

  $\sim\phi a \vee \sim\phi b \vee \ldots$

- So, by DeMorgan, its negation '$\sim(\exists v)\sim\phi v$' asserts a *conjunction*:

  $\sim\sim\phi a \,\&\, \sim\sim\phi b \,\&\, \ldots$

- Then, by Double Negation (DN), we can see this is equivalent to:

  $\phi a \,\&\, \phi b \,\&\, \ldots$

- But, this just asserts that *every* individual has ϕ. In other words, this says the same thing that the universal claim $(\forall v)\phi v$ says!
- Therefore, $\ulcorner \sim(\exists v)\sim\phi v \urcorner$ *is equivalent to* $\ulcorner (\forall v)\phi v \urcorner$. *QED.*
- We can run a parallel argument for $\ulcorner \sim(\forall v)\sim\phi v \urcorner$ and $\ulcorner (\exists v)\phi v \urcorner$.

Further Symbolization Problems

- If someone says "all athletes are not superstars" (another example: "all that glitters is not gold"), they are not to be symbolized exactly as read.
  - Sounds like $(\forall x)(Ax \to \sim Sx)$, but it's really $\sim(\forall x)(Ax \to Sx)$.
  - Note: this is equivalent to $(\exists x)(Ax \,\&\, \sim Sx)$.
- "The only" gets symbolized like "All". Example:
  - "The only animals in this canyon are skunks" is $(\forall x)((Ax \,\&\, Cx) \to Sx)$. Where Ax: x is an animal, Cx: x is in this canyon, and Sx: x is a skunk.
  - Clearly, $(\forall x)(Sx \to (Ax \,\&\, Cx))$ is *not* what's intended. Why?
- "None but", "none except" and "no ... except" are like "Only". Examples:
  - "None but the brave deserve a Purple Heart" is $(\forall x)(Px \to Bx)$. Where Bx: x is brave, Px: x deserves a Purple Heart.
  - "No birds except peacocks are proud of their tails" is equivalent to "Only peacocks are birds that are proud of their tails".

LMPL Symbolizations: Summary and Tips

- Some general symbolization forms we've seen so far:
  - All Fs are Gs. LMPL: $(\forall x)(Fx \to Gx)$.
  - An F is a G. LMPL: $(\forall x)(Fx \to Gx)$.
  - Fs are Gs. LMPL: $(\forall x)(Fx \to Gx)$.
  - Only Fs are Gs. LMPL: $(\forall x)(Gx \to Fx)$.
  - The only Fs are Gs. LMPL: $(\forall x)(Fx \to Gx)$.
  - Some Fs are Gs. LMPL: $(\exists x)(Fx \,\&\, Gx)$.
  - No Fs are Gs. LMPL: $\sim(\exists x)(Fx \,\&\, Gx)$.
  - Nothing is an F if it's G. $\sim(\exists x)(Gx \,\&\, Fx)$. [NOT $\sim(\exists x)(Gx \to Fx)$!]
  - If anything is an F, then Gs are. LMPL: $(\exists x)Fx \to (\forall x)(Gx \to Fx)$.
  - 'All Fs are not Gs' can sometimes *really* be $\sim(\forall x)(Fx \to Gx)$.
  - None but Fs are Gs (or None except Fs are Gs). $(\forall x)(Gx \to Fx)$.
- Remember: $\ulcorner \sim(\exists v)\sim\phi v \urcorner$ *is equivalent to* $\ulcorner (\forall v)\phi v \urcorner$ and $\ulcorner \sim(\forall v)\sim\phi v \urcorner$ *is equivalent to* $\ulcorner (\exists v)\phi v \urcorner$. You should be able to use these proficiently.

- Some equivalences:
  - 'All Fs are Gs' is equivalent to 'No Fs are non-Gs'.
    * $(\forall x)(Fx \to Gx)$ is equivalent to $\sim(\exists x)(Fx \& \sim Gx)$.
  - 'All Fs are Gs' is equivalent to 'All non-Gs are non-Fs'.
    * $(\forall x)(Fx \to Gx)$ is equivalent to $(\forall x)(\sim Gx \to \sim Fx)$.
  - 'Some Fs are Gs' is equivalent to 'Some Gs are Fs'.
    * $(\exists x)(Fx \& Gx)$ is equivalent to $(\exists x)(Gx \& Fx)$.
  - 'No Fs are Gs' is equivalent to 'No Gs are Fs'.
    * $\sim(\exists x)(Fx \& Gx)$ is equivalent to $\sim(\exists x)(Gx \& Fx)$.
- Some *non*-equivalences:
  - 'All Fs are Gs' is *not* equivalent to 'All Gs are Fs'.
    * $(\forall x)(Fx \to Gx)$ is *not* equivalent to $(\forall x)(Gx \to Fx)$.
  - 'Some Fs are non-Gs' is *not* equivalent to 'Some Gs are non-Fs'.
    * $(\exists x)(Fx \& \sim Gx)$ is *not* equivalent to $(\exists x)(Gx \& \sim Fx)$.
- The LSL equivalences + the general quantifier equivalences yield all.

Further Symbolizations Involving $\forall$ and $\exists$

- How should we paraphrase and/or symbolize the following sentence?

  (6) If anyone is wealthy, then economists are.
- At first blush, we might try to paraphrase (6) as follows:

  (6.1) If everyone is wealthy, then all economists are wealthy (which gives the LMPL symbolization: '$(\forall x)Wx \to (\forall x)(Ex \to Wx)$').
- But, (6.1) *cannot* be right. If the antecedent of (6.1) is true, then *everybody* is wealthy (not just the economists!). In this sense, (6.1) is analogous to an LSL *tautology* — it's true *in all possible worlds*. Is *that* all (6) asserts?
- In fact, (6) asserts something *much stronger* than (6.1). What (6) says is that all it takes for every economist to be wealthy is for there to exist *one* wealthy person. This leads to the following alternative paraphrase of (6):

  (6.2) If *someone* is wealthy, then all economists are wealthy (which gives the LMPL symbolization: '$(\exists x)Wx \to (\forall x)(Ex \to Wx)$').

Still More Symbolizations Involving $\forall$ and $\exists$

- How should we paraphrase and/or symbolize the following sentence?

  (7) Every wealthy logician is happy.
- It helps to do a *Logish*, intermediate form first:

  (7.1) For every x, if x is wealthy and x is a logician, then x is happy.
- This leads to the following LMPL symbolization:

  (7.2) $(\forall x)((Wx \& Lx) \to Hx)$
- OK, but what about the following sentence?

  (8) No wealthy economists are happy.
- This time, the *Logish*, intermediate form is:

  (8.1) Not: there is at least one x such that x is wealthy, and x is an economist, and x is happy.
- Which leads to the following LMPL symbolization:

  (8.2) $\sim(\exists x)((Wx \& Ex) \& Hx)$

One Last Symbolization Involving $\forall$

(9) A fetus is a person, but an embryo is not.

- In this case, the domain of discourse must be *wider* than the domain of people (since we need to be able to say that some things are *not* persons). And, 'is a person' must then be included as a *predicate* in our dictionary.

P__ : __ is a person	F__ : __ is a fetus
E__ : __ is an embryo	Domain of Discourse : *all things*

- Now, it helps to do a *Logish*, intermediate form first:

  (9.1) For every x, if x is a fetus then x is a person, and for every x, if x is an embryo then x not a person.
- This leads to the following LMPL symbolization:

  (9.2) $(\forall x)(Fx \to Px) \& (\forall x)(Ex \to \sim Px)$

  which is *semantically equivalent* (as we will *prove* in Chapter 6) to:

  (9.3) $(\forall x)((Fx \to Px) \& (Ex \to \sim Px))$

  But, (9.2) is *preferred* over (9.3), since (9.2) is closer to the *structure* of (9).

Chapter 6 — Formal Semantics for LMPL

- Venn diagrams can be useful to help us figure out and visualize the conditions under which some *simple* LMPL sentences are true or false.
- But, this technique only works for sentences that have three predicates or less. If a sentence has four predicates or more, then Venn diagrams become quite difficult to draw or comprehend. [Explain this.]
- Chapter 6 provides us with a *general* semantics for LMPL. This will allow us to understand, more generally, the conditions under which *any* (*closed*!) LMPL sentence will be true or false. [Like truth-tables for LSL.]
- In Chapter 6, we will also see a precise definition of the *semantic consequence relation* ($\vDash$) for our new theory LMPL. This will allow us to determine whether LMPL *arguments* are valid or invalid (in general).
- We begin with some new terminology ...

Formal Semantics for LMPL I: Some Terminology

- A **domain** ($\mathcal{D}$) is a nonempty (finite) set of individuals.
- The **reference of an individual constant** τ [Ref(τ)] is the object in the domain $\mathcal{D}$ to which τ refers (*e.g.*, $\ulcorner \text{Ref}(\tau) = x \urcorner$ abbreviates $\ulcorner \tau$ denotes $x \urcorner$).
- The **extension of a predicate P** [Ext(**P**)] is the set of all objects in the domain which satisfy **P** (*e.g.*, if $P\_\_$: __ is at the podium, and Ref(b) = Branden, then Ext(P) = $\{b\}$). Note: extensions are always subsets of the domain $\mathcal{D}$.
- The **instances of a** (*closed*!) **quantified sentence** $\ulcorner (\mathsf{Q}v)\phi v \urcorner$ **in a domain** $\mathcal{D}$ are the sentences one gets by replacing all occurrences of v in $\ulcorner \phi v \urcorner$ with the name of each element of $\mathcal{D}$ (*e.g.*, instances of '$(\forall x)Px$' in $\mathcal{D}$ are 'Pa', 'Pb', ..., for each individual in $\mathcal{D}$. $\therefore$ there are $|\mathcal{D}|$ instances of $\ulcorner (\mathsf{Q}v)\phi v \urcorner$ in $\mathcal{D}$).
- An **interpretation** ($\mathcal{I}$) **of an** (*closed*!) **LMPL sentence** p (**or argument** $\mathscr{A}$) is:
  (*i*) a *domain* $\mathcal{D}$,
  (*ii*) an assignment of *extensions* to any *predicate letters* in p ($\mathscr{A}$),
  (*iii*) an assignment of *references* to any *individual constants* in p ($\mathscr{A}$), and
  (*iv*) an assignment of *truth-values* to any *sentence letters* in p ($\mathscr{A}$).

Formal Semantics for LMPL II: $\top$ and $\bot$ in LMPL

- We're now in a position to give precise *truth-conditions* for each kind of (*closed*!) LMPL sentence (augmenting the truth-table definitions of LSL).
- First, the truth conditions for the (*closed*!) *atomic* sentences of LMPL:
  - An atomic sentence $\mathbf{P}\tau$ is *true* ($\top$) on an interpretation $\mathcal{I}$ if the object referred to by the individual constant τ belongs to the extension of the predicate **P** (*i.e.*, if $\tau \in \text{Ext}(\mathbf{P})$). If τ does *not* belong to the extension of the predicate **P** — that is, if $\tau \notin \text{Ext}(\mathbf{P})$ — then $\mathbf{P}\tau$ is *false* ($\bot$).
- Next, the truth conditions for the (*closed*!) *quantified* sentences of LMPL:
  - A universal sentence $\ulcorner (\forall v)\phi v \urcorner$ is *true* ($\top$) *in* $\mathcal{I}$ if *all* its instances in $\mathcal{I}$ are true. If some of its instances are false (in $\mathcal{I}$), then $\ulcorner (\forall v)\phi v \urcorner$ is *false* ($\bot$).
  - An existential sentence $\ulcorner (\exists v)\phi v \urcorner$ is *true* ($\top$) *in* $\mathcal{I}$ if *some* of its instances are true in $\mathcal{I}$. If *all* its instances are false (in $\mathcal{I}$), then it's *false* ($\bot$).
- NOTE: the usual *truth-tables* for $\&, \vee, \rightarrow, \leftrightarrow, \sim$ are still in force in LMPL!

An Example of an LMPL Interpretation

Matrix Representation: ($\mathcal{I}$) [Ignoring sentence letters.]

	F	G
α	+	–
β	–	+

- Greek letters 'α'–'σ' (*viz.*, the objects named by the *constants* 'a'–'s') are placed in the left column, alphabetically. All of the predicates in the interpretation $\mathcal{I}$ are placed across the top row, alphabetically. '+' means 'satisfies the predicate', and '–' means 'does *not* satisfy the predicate'.
- This matrix says (in addition to Ref(a) = α, and Ref(b) = β):
  (*i*) The *domain* $\mathcal{D}$ of $\mathcal{I}$ consists of the two objects α, β (*i.e.*, $\mathcal{D} = \{\alpha, \beta\}$).
  (*ii*) The *extension* of 'F' consists of the object α (*i.e.*, Ext(F) = $\{\alpha\}$), and the *extension* of 'G' consists of the object β (*i.e.*, Ext(G) = $\{\beta\}$).
- **Quiz**: What are the truth-values — in $\mathcal{I}$ — of the following 4 sentences?

(1) $(\exists x)Fx \,\&\, (\exists x)Gx$, (2) $(\exists x)(Fx \,\&\, Gx)$, (3) $(\forall x)(Fx \vee Gx)$, (4) $(\forall x)Fx \vee (\forall x)Gx$