Announcements & Such

- Administrative Stuff
  - Grade Curve (so far). Take the average of:
    (1) your average HW score,
    (2) your in-class mid-term score (on a 100 point scale), and
    (3) your take-home mid-term score.
    * The approximate "curve" for the course is as follows: A-ish (≥ 90), B-ish (80–90), C-ish (60–80), D-ish (50-60).
  - **HW #4 is to be returned today. Resub due Thursday**
  - **From now on, my office hours will be 2–4pm Thursdays.**
- Today: back to Chapter 5 — Monadic Predicate Logic (LMPL)
  - The basic building blocks of LMPL.
  - The existential and universal quantifiers — and variables.
  - Symbolizations with quantifiers.

New Elements of LMPL

- Now, upper-case letters can be used as LSL sentences *or* **predicates**.
  - *E.g.*, the predicate 'is tall' can be symbolized using 'T'.
- Lower-case letters 'a'–'s' will be used as **individual constants** (**names**).
- This yields **new atomic sentences** with *subject-predicate structure*:
  - *E.g.*, 'Branden is tall' ↦ 'Tb'.
- We also have two **quantifier phrases**: **all** (∀) and **some** (∃).
- Lower-case letters 't'–'z' will be used as **variables**.
- We use **variables** + **quantifiers** to symbolize **general claims**.
  - *E.g.*, 'Someone is wise'
    ↦ 'There exists an x such that x is wise.'
    ↦ '$(\exists x)Wx$'
- Each general claim quantifies over a **domain/universe of discourse**.

Symbolization in LMPL III: The Existential Quantifier

- The quantifier 'some' is captured using the new symbol '∃' of LMPL.
- For instance, 'Someone is wise' gets symbolized as '$(\exists x)Wx$' in LMPL.
- One must be careful about the *scope* of the existential quantifier. For instance, consider the following two (*non*-equivalent!) sentences:

(1) Someone is happy and someone is wise.
(2) Someone is happy and wise.

- Sentence (1) is symbolized in LMPL as '$(\exists x)Hx \,\&\, (\exists x)Wx$'.
- Sentence (2) is symbolized in LMPL as '$(\exists x)(Hx \,\&\, Wx)$'.
- How would you symbolize the following sentence in LMPL

  'Some economists are wealthy and some are not.'

  using the following dictionary?

  $E\_\_$: __ is an economist. $L\_\_$: __ is wealthy.

Symbolization in LMPL IV: More on ∃

- What, exactly, does '∃' *mean*?
- When I say 'Someone in this room is wise', I'm asserting the *disjunction* 'Either Branden is wise, or Mike is wise, or ...' (*i.e.*, '$Wb \vee Wm \vee \ldots$').
- In *finite* domains of discourse, we can always express existentially quantified sentences as disjunctions. But, in *infinite* domains, we cannot, since our language does not permit infinitely long formulas.
- We can use negation, together with the existential quantifier, to express other kinds of quantifiers. For instance, we may symbolize

  'No unwise person is happy.'

  in LMPL as: '$\sim(\exists x)(\sim Wx \,\&\, Hx)$'.
- How would you symbolize the following sentence?

  'If a wealthy economist exists, then so does a famous mathematician.'

Symbolization in LMPL V: Free *vs* Bound and Open *vs* Closed

- In 'Wx', the variable 'x' is said to be *free*. But, in '$(\exists x)Wx$', the variable 'x' is said to be *bound* by the existential quantifier.
- Formulas like 'Wx' which contain free variables are called *open sentences*. Formulas with *no* free variables are called *closed sentences*.
- Only closed sentences assert things that can be either true or false of some particular individuals in the domain of discourse (or the domain).
- For instance, 'Wx' says 'x is wise'. But, since the 'x' in 'Wx' does not refer to any particular thing, 'Wx' can be neither true nor false.
- But, when we *existentially quantify* 'Wx', we end-up with '$(\exists x)Wx$', which clearly *does* make an assertion that is either true or false (depending on whether any *particular* person in the domain happens to be wise).
- Which of the following are open/closed? [NOTE: '$\exists$' binds like '$\sim$'!]

(1) 'Ha'	(2) 'Wx'	(3) '$(\exists x)Hx$'	(4) '$(\exists x)Hx \,\&\, Wx$'	(5) '$(\exists x)Hx \,\&\, Wb$'

Symbolization in LMPL VI: More Examples with $\exists$

- Let's symbolize the following sentences. Whenever we symbolize in LMPL, we must state our dictionary of monadic predicates, and we must also say what the domain of discourse is over which we are quantifying.
  1. No smoggy city is unpolluted.
  2. Vampires do not exist.
  3. If ghosts and vampires do not exist, then nothing can be a ghost without being a vampire.
- If the dictionary is (where the domain is people in this classroom now):

  $S\_\_$: __ is standing up at the podium.
  $W\_\_$: __ is wealthy.
  b : Branden

  then what do the following two LMPL sentences assert (in English)?

  $\sim(\exists x)(Sx \,\&\, Wx)$ $\qquad$ $\sim Wb$

Symbolization in LMPL VII: Back to ① and ②

- Now, we are in a position to symbolize in LMPL the argument ① that we saw at the beginning of the previous lecture:

$$①_{\text{LMPL}} \quad \begin{array}{l} Ws \\ \therefore (\exists x)Wx \end{array}$$

- Since there are only finitely many people, we can see why this argument is valid, by representing its conclusion as a long (but finite!) *disjunction*, in which its only premise is a disjunct:

$$① \quad \begin{array}{l} Ws \\ \therefore Wa \vee \cdots \vee Ws \vee \cdots \end{array}$$

- We can use a similar trick for argument ②. The premise of ② ['$(\forall x)Hx$'] entails a *conjunction* ['$Ha \,\&\, \cdots \,\&\, Hp \,\&\, \cdots$'], and its conclusion ['Hp'] is one of the conjuncts of that conjunction.

Some Symbolizations Involving $\exists$

E__ : __ is an even number $\qquad$ a : the number 2
P__ : __ is a prime number $\qquad$ Domain : natural numbers ($\mathbb{N}$)
G__ : __ is greater than the number 2

(1) There exists a prime number and there exists an even number.

$(\exists x)Px \,\&\, (\exists x)Ex$

(2) There exists an even prime number. [$(\exists x)(Px \,\&\, Ex)$]

(3) 2 is an even prime number. [$Ea \,\&\, Pa$]

(4) If 2 is prime, then there are some even primes. [$Pa \to (\exists x)(Px \,\&\, Ex)$]

(5) No number is even if it is prime. [$\sim(\exists x)(Px \,\&\, Ex)$]

- Careful with this one! Why *isn't* this '$\sim(\exists x)(Px \to Ex)$'?
- Compare: No number is even if it is prime and greater than 2.
  - ∗ In LMPL, this is: '$\sim(\exists x)[(Px \,\&\, Gx) \,\&\, Ex]$', which is *true*. *Why?*
  - ∗ Note: '$\sim(\exists x)[(Px \,\&\, Gx) \to Ex]$' is *false*! *Why?*

The *Universal* Quantifier ∀

- To symbolize English sentences like ‘Everyone is happy’, we will need the *universal* quantifier ‘∀’ (which means ‘every’ or ‘all’).
  - (*i*) We begin with the raw English sentence: ‘Everyone is happy’.
  - (*ii*) Then, we move to the *Logish* form: ‘For every x, x is happy’.
  - (*iii*) Finally, we have the full LMPL symbolization: ‘$(\forall x)Hx$’.
- As with the existential quantifier, we must be careful with the *scope* of ‘∀’. How would we symbolize the following two sentences?
  - (1) ‘Everyone is happy and everyone is wise.’
  - (2) ‘Everyone is happy and wise.’
- These sentences get symbolized differently, because they have different (syntactic) *structures*. But, do they have different *meanings*? In Chapter 6, we’ll *prove* the answer to this question.

The Universal Quantifier II

- How should one symbolize the following English sentence?
  - (3) ‘Everyone who is happy is wise.’
- Note: Unlike (1) and (2) above, (3) does *not* have the consequence that *everyone* is happy. So, what, exactly, *does* (3) say?
- (3) says that *if* a person is happy, *then* that person is wise. This suggests the following *Logish* form (*wrt* the domain of people):

  ‘For every x, if x is happy then x is wise.’
- Now, we are ready for the full LMPL symbolization:

$$(\forall x)(Hx \to Wx)$$
- We will use this same trick to symbolize sentences like ‘Every happy person is a wise person’ or ‘If someone is happy then he/she is wise’, which both assert the same thing as (3).

The Universal Quantifier III

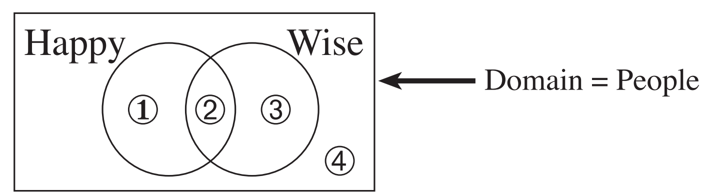
- How should one symbolize the following English sentence?
  - (4) ‘Only happy people are wise.’
- Note: (4) does *not* say that *all* happy people are wise. That is, unlike (3), (4) does *not* say that a person is wise *if* he/she is happy. Rather, (4) says that a person is wise *only if* he/she is happy.
- This suggests the following *Logish* form (domain of people):

  ‘For every x, x is wise *only if* x is happy.’
- Now, we are ready for the full LMPL symbolization:

$$(\forall x)(Wx \to Hx)$$
- Here, we have the usual distinction between necessary and sufficient conditions. (3) says that happiness is *sufficient* for wisdom. But, (4) says that happiness is *necessary* for wisdom.

The Universal Quantifier IV, and Venn Diagrams

- Consider the following English sentence:
  - (5) ‘No one who is unhappy is wise.’
- When trying to paraphrase or symbolize sentences like this in LMPL, it is useful to *picture* what they say using a *Venn Diagram*:



- (5) says that region ③ in the Venn Diagram is empty. So, (5) asserts the same thing as the following LMPL sentence:
  - (5.1) $(\forall x)(Wx \to Hx)$

The Intimate Relationship Between ∃ and ∀

- What we have just shown (informally) is:

  '$\sim(\exists x)(Wx \,\&\, \sim Hx)$' *is equivalent to* '$(\forall x)(Wx \to Hx)$'

- This is just a *special case* of the following *general equivalences*:

  ⌜$\sim(\exists v)\sim\phi v$⌝ *is equivalent to* ⌜$(\forall v)\phi v$⌝
  and
  ⌜$\sim(\forall v)\sim\phi v$⌝ *is equivalent to* ⌜$(\exists v)\phi v$⌝

- Here, 'ϕ' is a *metavariable* ranging over formulas of LMPL (thought of as functions of v), and 'v' ranges over variable symbols of LMPL.
- It follows from the second general equivalence above that $\sim(\exists x)(Wx \,\&\, \sim Hx)$ is equivalent to $\sim\sim(\forall x)\sim(Wx \,\&\, \sim Hx)$. But, this is equivalent to $(\forall x)\sim(Wx \,\&\, \sim Hx)$, hence $(\forall x)(Wx \to Hx)$.
- Our formal semantics will make these relationships more precise.

- Here's *why* (informally) ⌜$\sim(\exists v)\sim\phi v$⌝ and ⌜$(\forall v)\phi v$⌝ are equivalent.
- Start with the existential claim inside the negation ⌜$\sim(\exists v)\sim\phi v$⌝:

  $(\exists v)\sim\phi v$

- Next, note that, informally, $(\exists v)\sim\phi v$ asserts a *disjunction*:

  $\sim\phi a \vee \sim\phi b \vee \ldots$

- So, by DeMorgan, its negation '$\sim(\exists v)\sim\phi v$' asserts a *conjunction*:

  $\sim\sim\phi a \,\&\, \sim\sim\phi b \,\&\, \ldots$

- Then, by Double Negation (DN), we can see this is equivalent to:

  $\phi a \,\&\, \phi b \,\&\, \ldots$

- But, this just asserts that *every* individual has ϕ. In other words, this says the same thing that the universal claim $(\forall v)\phi v$ says!
- Therefore, ⌜$\sim(\exists v)\sim\phi v$⌝ *is equivalent to* ⌜$(\forall v)\phi v$⌝. *QED.*
- We can run a parallel argument for ⌜$\sim(\forall v)\sim\phi v$⌝ and ⌜$(\exists v)\phi v$⌝.

Further Symbolization Problems

- If someone says "all athletes are not superstars" (another example: "all that glitters is not gold"), they are not to be symbolized exactly as read.
  - Sounds like $(\forall x)(Ax \to \sim Sx)$, but it's really $\sim(\forall x)(Ax \to Sx)$.
  - Note: this is equivalent to $(\exists x)(Ax \,\&\, \sim Sx)$.
- "The only" gets symbolized like "All". Example:
  - "The only animals in this canyon are skunks" is $(\forall x)((Ax \,\&\, Cx) \to Sx)$. Where Ax: x is an animal, Cx: x is in this canyon, and Sx: x is a skunk.
  - Clearly, $(\forall x)(Sx \to (Ax \,\&\, Cx))$ is *not* what's intended. Why?
- "None but", "none except" and "no ... except" are like "Only". Examples:
  - "None but the brave deserve a Purple Heart" is $(\forall x)(Px \to Bx)$. Where Bx: x is brave, Px: x deserves a Purple Heart.
  - "No birds except peacocks are proud of their tails" is equivalent to "Only peacocks are birds that are proud of their tails".

LMPL Symbolizations: Summary and Tips

- Some general symbolization forms we've seen so far:
  - All Fs are Gs. LMPL: $(\forall x)(Fx \to Gx)$.
  - An F is a G. LMPL: $(\forall x)(Fx \to Gx)$.
  - Fs are Gs. LMPL: $(\forall x)(Fx \to Gx)$.
  - Only Fs are Gs. LMPL: $(\forall x)(Gx \to Fx)$.
  - The only Fs are Gs. LMPL: $(\forall x)(Fx \to Gx)$.
  - Some Fs are Gs. LMPL: $(\exists x)(Fx \,\&\, Gx)$.
  - No Fs are Gs. LMPL: $\sim(\exists x)(Fx \,\&\, Gx)$.
  - Nothing is an F if it's G. $\sim(\exists x)(Gx \,\&\, Fx)$. [NOT $\sim(\exists x)(Gx \to Fx)$!]
  - If anything is an F, then Gs are. LMPL: $(\exists x)Fx \to (\forall x)(Gx \to Fx)$.
  - 'All Fs are not Gs' can sometimes *really* be $\sim(\forall x)(Fx \to Gx)$.
  - None but Fs are Gs (or None except Fs are Gs). $(\forall x)(Gx \to Fx)$.
- Remember: ⌜$\sim(\exists v)\sim\phi v$⌝ *is equivalent to* ⌜$(\forall v)\phi v$⌝ and ⌜$\sim(\forall v)\sim\phi v$⌝ *is equivalent to* ⌜$(\exists v)\phi v$⌝. You should be able to use these proficiently.

- Some equivalences:
  - 'All Fs are Gs' is equivalent to 'No Fs are non-Gs'.
    * $(\forall x)(Fx \to Gx)$ is equivalent to $\sim(\exists x)(Fx \,\&\, {\sim}Gx)$.
  - 'All Fs are Gs' is equivalent to 'All non-Gs are non-Fs'.
    * $(\forall x)(Fx \to Gx)$ is equivalent to $(\forall x)({\sim}Gx \to {\sim}Fx)$.
  - 'Some Fs are Gs' is equivalent to 'Some Gs are Fs'.
    * $(\exists x)(Fx \,\&\, Gx)$ is equivalent to $(\exists x)(Gx \,\&\, Fx)$.
  - 'No Fs are Gs' is equivalent to 'No Gs are Fs'.
    * $\sim(\exists x)(Fx \,\&\, Gx)$ is equivalent to $\sim(\exists x)(Gx \,\&\, Fx)$.
- Some *non*-equivalences:
  - 'All Fs are Gs' is *not* equivalent to 'All Gs are Fs'.
    * $(\forall x)(Fx \to Gx)$ is *not* equivalent to $(\forall x)(Gx \to Fx)$.
  - 'Some Fs are non-Gs' is *not* equivalent to 'Some Gs are non-Fs'.
    * $(\exists x)(Fx \,\&\, {\sim}Gx)$ is *not* equivalent to $(\exists x)(Gx \,\&\, {\sim}Fx)$.
- The LSL equivalences + the general quantifier equivalences yield all.

Further Symbolizations Involving ∀ and ∃

- How should we paraphrase and/or symbolize the following sentence?

  (6) If anyone is wealthy, then economists are.
- At first blush, we might try to paraphrase (6) as follows:

  (6.1) If everyone is wealthy, then all economists are wealthy (which gives the LMPL symbolization: '$(\forall x)Wx \to (\forall x)(Ex \to Wx)$').
- But, (6.1) *cannot* be right. If the antecedent of (6.1) is true, then *everybody* is wealthy (not just the economists!). In this sense, (6.1) is analogous to an LSL *tautology* — it's true *in all possible worlds*. Is *that* all (6) asserts?
- In fact, (6) asserts something *much stronger* than (6.1). What (6) says is that all it takes for every economist to be wealthy is for there to exist *one* wealthy person. This leads to the following alternative paraphrase of (6):

  (6.2) If *someone* is wealthy, then all economists are wealthy (which gives the LMPL symbolization: '$(\exists x)Wx \to (\forall x)(Ex \to Wx)$').

Still More Symbolizations Involving ∀ and ∃

- How should we paraphrase and/or symbolize the following sentence?

  (7) Every wealthy logician is happy.
- It helps to do a *Logish*, intermediate form first:

  (7.1) For every x, if x is wealthy and x is a logician, then x is happy.
- This leads to the following LMPL symbolization:

  (7.2) $(\forall x)((Wx \,\&\, Lx) \to Hx)$
- OK, but what about the following sentence?

  (8) No wealthy economists are happy.
- This time, the *Logish*, intermediate form is:

  (8.1) Not: there is at least one x such that x is wealthy, and x is an economist, and x is happy.
- Which leads to the following LMPL symbolization:

  (8.2) $\sim(\exists x)((Wx \,\&\, Ex) \,\&\, Hx)$

One Last Symbolization Involving ∀

(9) A fetus is a person, but an embryo is not.

- In this case, the domain of discourse must be *wider* than the domain of people (since we need to be able to say that some things are *not* persons). And, 'is a person' must then be included as a *predicate* in our dictionary.

P__ : __ is a person	F__ : __ is a fetus
E__ : __ is an embryo	Domain of Discourse : *all things*

- Now, it helps to do a *Logish*, intermediate form first:

  (9.1) For every x, if x is a fetus then x is a person, and for every x, if x is an embryo then x not a person.
- This leads to the following LMPL symbolization:

  (9.2) $(\forall x)(Fx \to Px) \,\&\, (\forall x)(Ex \to {\sim}Px)$

  which is *semantically equivalent* (as we will *prove* in Chapter 6) to:

  (9.3) $(\forall x)((Fx \to Px) \,\&\, (Ex \to {\sim}Px))$

  But, (9.2) is *preferred* over (9.3), since (9.2) is closer to the *structure* of (9).