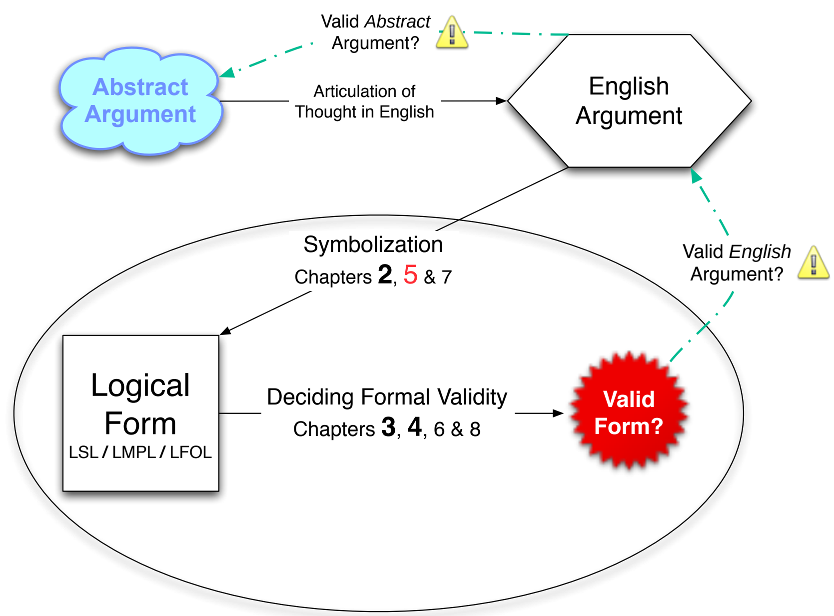
Announcements & Such

- Administrative Stuff
  - My take-home mid-term solutions have been posted. These are worth studying (*e.g.*, more examples of LSL natural deductions).
  - Grade Curve (so far). Take the average of:
    (1) your average HW score,
    (2) your in-class mid-term score (on a 100 point scale), and
    (3) your take-home mid-term score.
    The approximate "curve" for the course is as follows:
    A-ish ($\geq$ 90), B-ish (80–90), C-ish (60–80), D-ish (50-60).
  - **HW #4 is due today (first submission).**
  - **I am holding my office hours today from 2–4pm.**
  - Today: Intro. to Chapter 5 — Monadic Predicate Logic (LMPL)
    * The basic building blocks of LMPL.
  - But, first, I'll take requests of proof problems to go over by hand.



Chapter 5: Predication and Quantification

- Consider the following two arguments:

① Socrates is wise. ∴ Someone is wise. ② Everyone is happy. ∴ Plato is happy.

- Intuitively, both ① and ② are *valid* (*why*?). But, if we try to translate these into LSL, we get the *in*valid LSL forms:

①$_{\text{LSL}}$ S $\therefore W$ ②$_{\text{LSL}}$ H $\therefore P$

- In LSL, we are not able to capture the *logical structure* shared between premises and conclusions of these kinds of arguments.
- If it's not *atomic sentences* that the premises and conclusions of such arguments have in common (structurally), then what *is* it?
- This is what Chapter 5 is about...

Predication and Quantification: II

- We need a *richer language* than LSL — one which accurately captures the deeper *logical structure* of arguments like ① and ②. New Jargon:
- A **predicate** is something which *applies to* an object or *is true of* an object or which an object *satisfies*. *E.g.*, Socrates satisfies the predicate **(is) Wise**.
- A **proper name** is a word or a phrase which *stands for*, or *refers to*, or *denotes* a specific person, place, or thing. *E.g.*, 'Socrates' is a proper name.
- **Quantifier phrases** specify *quantities*. *E.g.*, 'someone' means *at least one* person and 'everyone' means *all* people. 'Some' and 'all' are **quantifiers**.
- The collection of objects to which the quantifiers in a statement are *relativized* is called the **domain of discourse** of the statement (*e.g.*, 'some*one*' quantifies only over *people*, 'some*time*' quantifies over *times*).
- Chapter 5 introduces the logical language LMPL (the Language of Monadic Predicate Logic) that contains these elements (and a few more tricks).

Symbolization in LMPL I: New Atomic Sentences

- Among the atomic sentences of LMPL (*in addition to LSL sentence letters*) are (new) strings of the form ⌜Xn⌝, where 'X' is a (monadic) predicate, and 'n' is an individual constant (*i.e.*, a proper name).
- We will use the lower-case letters 'a'–'s' as *individual constants* ('t'–'z' are used as *variables* — much more on variables later).
- Some examples of these new kinds of atomic sentences:
  - 'Branden is tall.' $\mapsto$ 'Tb'.
  - 'Honda is an automobile manufacturer.' $\mapsto$ 'Ah'.
  - 'New York is a city.' $\mapsto$ 'Cn'.
- As in LSL, we can *combine* different LMPL atomic sentences using the sentential connectives to yield complex sentences. For instance:
  - 'Branden is tall, but Ruth is not tall.' $\mapsto$ '$Tb \& {\sim}Tr$'.

Symbolization in LMPL II: The Role of Variables

- So far, we can only symbolize sentences about *particular things*. We also want to be able to symbolize sentences like '*Someone* is wise.'
- 'Someone is wise' is called an *existentially quantified* sentence. This is because it asserts that *there exists at least one wise person.*
- Such statements are *not* about *particular* individuals. So, it would *not* be right to symbolize 'Someone is wise' as 'Ws', since 's' is an *individual constant.* This is where *variables* ('t'–'z') enter LMPL symbolizations.
- Intuitively, what want to be able to say is something like:

  There exists at least one person x such that x is wise.

- 'x' is a *variable* which ranges over *all* of the objects (*viz.*, *people*) in our domain of discourse. It does *not* denote any *particular* person.
- So far, so good. But, how do we deal with the *quantifier* 'some' *itself*?

Symbolization in LMPL III: The Existential Quantifier

- The quantifier 'some' is captured using the new symbol '$\exists$' of LMPL.
- For instance, 'Someone is wise' gets symbolized as '$(\exists x)Wx$' in LMPL.
- One must be careful about the *scope* of the existential quantifier. For instance, consider the following two (*non*-equivalent!) sentences:

(1) Someone is happy and someone is wise.

(2) Someone is happy and wise.

- Sentence (1) is symbolized in LMPL as '$(\exists x)Hx \& (\exists x)Wx$'.
- Sentence (2) is symbolized in LMPL as '$(\exists x)(Hx \& Wx)$'.
- How would you symbolize the following sentence in LMPL

  'Some economists are wealthy and some are not.'

  using the following dictionary?

  $E\_\_$: __ is an economist. $L\_\_$: __ is wealthy.

Symbolization in LMPL IV: More on ∃

- What, exactly, does '$\exists$' *mean*?
- When I say 'Someone in this room is wise', I'm asserting the *disjunction* 'Either Branden is wise, or Mike is wise, or ...' (*i.e.*, '$Wb \vee Wm \vee \ldots$').
- In *finite* domains of discourse, we can always express existentially quantified sentences as disjunctions. But, in *infinite* domains, we cannot, since our language does not permit infinitely long formulas.
- We can use negation, together with the existential quantifier, to express other kinds of quantifiers. For instance, we may symbolize

  'No unwise person is happy.'

  in LMPL as: '${\sim}(\exists x)({\sim}Wx \& Hx)$'.

- How would you symbolize the following sentence?

  'If a wealthy economist exists, then so does a famous mathematician.'

Symbolization in LMPL V: Free *vs* Bound and Open *vs* Closed

- In 'Wx', the variable 'x' is said to be *free*. But, in '$(\exists x)Wx$', the variable 'x' is said to be *bound* by the existential quantifier.
- Formulas like 'Wx' which contain free variables are called *open sentences*. Formulas with *no* free variables are called *closed sentences*.
- Only closed sentences assert things that can be either true or false of some particular individuals in the domain of discourse (or the domain).
- For instance, 'Wx' says 'x is wise'. But, since the 'x' in 'Wx' does not refer to any particular thing, 'Wx' can be neither true nor false.
- But, when we *existentially quantify* 'Wx', we end-up with '$(\exists x)Wx$', which clearly *does* make an assertion that is either true or false (depending on whether any *particular* person in the domain happens to be wise).
- Which of the following are open/closed? [NOTE: '$\exists$' binds like '$\sim$'!]

(1) 'Ha'	(2) 'Wx'	(3) '$(\exists x)Hx$'	(4) '$(\exists x)Hx \,\&\, Wx$'	(5) '$(\exists x)Hx \,\&\, Wb$'

Symbolization in LMPL VI: More Examples with $\exists$

- Let's symbolize the following sentences. Whenever we symbolize in LMPL, we must state our dictionary of monadic predicates, and we must also say what the domain of discourse is over which we are quantifying.
  1. No smoggy city is unpolluted.
  2. Vampires do not exist.
  3. If ghosts and vampires do not exist, then nothing can be a ghost without being a vampire.
- If the dictionary is (where the domain is people in this classroom now):

  $S\_\_$: __ is standing up at the podium.
  $W\_\_$: __ is wealthy.
  b : Branden

  then what do the following two LMPL sentences assert (in English)?

  $\sim(\exists x)(Sx \,\&\, Wx)$ $\sim Wb$

Symbolization in LMPL VII: Back to ① and ②

- Now, we are in a position to symbolize in LMPL the argument ① that we saw at the beginning of this lecture:

$$①_{\text{LMPL}} \quad \begin{array}{l} Ws \\ \therefore (\exists x)Wx \end{array}$$

- Since there are only finitely many people, we can see why this argument is valid, by representing its conclusion as a long (but finite!) disjunction, in which its only premise is a disjunct:

$$① \quad \begin{array}{l} Ws \\ \therefore Wa \vee \cdots \vee Ws \vee \cdots \end{array}$$

- We can use a similar trick for argument ②. In that case, it's premise ['$(\forall x)Hx$'] entails a conjunction ['$Ha \,\&\, \cdots \,\&\, Hp \,\&\, \cdots$'], and its conclusion ['Hp'] is one of the conjuncts of that conjunction.

Some Symbolizations Involving $\exists$

E__ : __ is an even number a : the number 2
P__ : __ is a prime number   Domain : natural numbers ($\mathbb{N}$)
G__ : __ is greater than the number 2

(1) There exists a prime number and there exists an even number.

$(\exists x)Px \,\&\, (\exists x)Ex$

(2) There exists an even prime number. [$(\exists x)(Px \,\&\, Ex)$]

(3) 2 is an even prime number. [$Ea \,\&\, Pa$]

(4) If 2 is prime, then there are some even primes. [$Pa \to (\exists x)(Px \,\&\, Ex)$]

(5) No number is even if it is prime. [$\sim(\exists x)(Px \,\&\, Ex)$]

- Careful with this one! Why *isn't* this '$\sim(\exists x)(Px \to Ex)$'?
- Compare: No number is even if it is prime and greater than 2.
  - ∗ In LMPL, this is: '$\sim(\exists x)[(Px \,\&\, Gx) \,\&\, Ex]$', which is *true*. *Why?*
  - ∗ Note: '$\sim(\exists x)[(Px \,\&\, Gx) \to Ex]$' is *false*! *Why?*

The *Universal* Quantifier ∀

- To symbolize English sentences like 'Everyone is happy', we will need the *universal* quantifier '∀' (which means 'every' or 'all').
  - (*i*) We begin with the raw English sentence: 'Everyone is happy'.
  - (*ii*) Then, we move to the *Logish* form: 'For every x, x is happy'.
  - (*iii*) Finally, we have the full LMPL symbolization: '$(\forall x)Hx$'.
- As with the existential quantifier, we must be careful with the *scope* of '∀'. How would we symbolize the following two sentences?
  - (1) 'Everyone is happy and everyone is wise.'
  - (2) 'Everyone is happy and wise.'
- These sentences get symbolized differently, because they have different (syntactic) *structures*. But, do they have different *meanings*? In Chapter 6, we'll *prove* the answer to this question.

The Universal Quantifier II

- How should one symbolize the following English sentence?

  (3) 'Everyone who is happy is wise.'
- Note: Unlike (1) and (2) above, (3) does *not* have the consequence that *everyone* is happy. So, what, exactly, *does* (3) say?
- (3) says that *if* a person is happy, *then* that person is wise. This suggests the following *Logish* form (*wrt* the domain of people):

  'For every x, if x is happy then x is wise.'
- Now, we are ready for the full LMPL symbolization:

$$(\forall x)(Hx \rightarrow Wx)$$

- We will use this same trick to symbolize sentences like 'Every happy person is a wise person' or 'If someone is happy then he/she is wise', which both assert the same thing as (3).

The Universal Quantifier III

- How should one symbolize the following English sentence?

  (4) 'Only happy people are wise.'
- Note: (4) does *not* say that *all* happy people are wise. That is, unlike (3), (4) does *not* say that a person is wise *if* he/she is happy. Rather, (4) says that a person is wise *only if* he/she is happy.
- This suggests the following *Logish* form (domain of people):

  'For every x, x is wise *only if* x is happy.'
- Now, we are ready for the full LMPL symbolization:

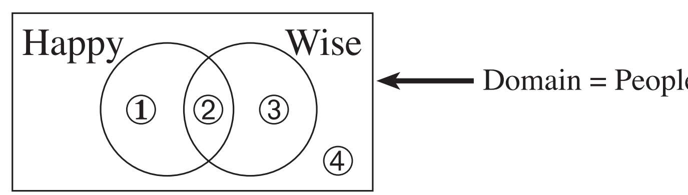
$$(\forall x)(Wx \rightarrow Hx)$$

- Here, we have the usual distinction between necessary and sufficient conditions. (3) says that happiness is *sufficient* for wisdom. But, (4) says that happiness is *necessary* for wisdom.

The Universal Quantifier IV, and Venn Diagrams

- Consider the following English sentence:

  (5) 'No one who is unhappy is wise.'
- When trying to paraphrase or symbolize sentences like this in LMPL, it is useful to *picture* what they say using a *Venn Diagram*:



- (5) says that region ③ in the Venn Diagram is empty. So, (5) asserts the same thing as the following LMPL sentence:

  (5.1) $(\forall x)(Wx \rightarrow Hx)$

The Intimate Relationship Between $\exists$ and $\forall$

- What we have just shown (informally) is:

  '$\sim(\exists x)(Wx \,\&\, \sim Hx)$' *is equivalent to* '$(\forall x)(Wx \to Hx)$'

- This is just a *special case* of the following *general equivalences*:

  $\ulcorner \sim(\exists \nu)\sim\phi\nu \urcorner$ *is equivalent to* $\ulcorner (\forall \nu)\phi\nu \urcorner$

  and

  $\ulcorner \sim(\forall \nu)\sim\phi\nu \urcorner$ *is equivalent to* $\ulcorner (\exists \nu)\phi\nu \urcorner$

- Here, 'ϕ' is a *metavariable* ranging over formulas of LMPL (thought of as functions of ν), and 'ν' ranges over variable symbols of LMPL.
- It follows from the second general equivalence above that $\sim(\exists x)(Wx \,\&\, \sim Hx)$ is equivalent to $\sim\sim(\forall x)\sim(Wx \,\&\, \sim Hx)$. But, this is equivalent to $(\forall x)\sim(Wx \,\&\, \sim Hx)$, hence $(\forall x)(Wx \to Hx)$.
- Our formal semantics will make these relationships more precise.

- Here's *why* (informally) $\ulcorner \sim(\exists \nu)\sim\phi\nu \urcorner$ and $\ulcorner (\forall \nu)\phi\nu \urcorner$ are equivalent.
- Start with the existential claim inside the negation $\ulcorner \sim(\exists \nu)\sim\phi\nu \urcorner$:

  $$(\exists \nu)\sim\phi\nu$$

- Next, note that, informally, $(\exists \nu)\sim\phi\nu$ asserts a *disjunction*:

  $$\sim\phi a \vee \sim\phi b \vee \ldots$$

- So, by DeMorgan, its negation '$\sim(\exists \nu)\sim\phi\nu$' asserts a *conjunction*:

  $$\sim\sim\phi a \,\&\, \sim\sim\phi b \,\&\, \ldots$$

- Then, by Double Negation (DN), we can see this is equivalent to:

  $$\phi a \,\&\, \phi b \,\&\, \ldots$$

- But, this just asserts that *every* individual has ϕ. In other words, this says the same thing that the universal claim $(\forall \nu)\phi\nu$ says!
- Therefore, $\ulcorner \sim(\exists \nu)\sim\phi\nu \urcorner$ *is equivalent to* $\ulcorner (\forall \nu)\phi\nu \urcorner$. *QED*.
- We can run a parallel argument for $\ulcorner \sim(\forall \nu)\sim\phi\nu \urcorner$ and $\ulcorner (\exists \nu)\phi\nu \urcorner$.