Announcements & Such

- *Dire Straits*: *In the Gallery*
- Administrative Stuff
  - HW #2 solutions posted. [Don't sweat symbolizations too much.]
  - ☞ **Hints to controversial problems were on my "tips" handout.**
  - Two handouts: (1) solutions to problems from lecture on logical truth, equivalence, etc.; (2) 3 "short" truth-table method problems.
  - ☞ **Make sure you study my handouts. They tend to be useful.**
  - **HW #3 due Today**, usual drill (truth-table methods for validity).
  - Branden won't hold office hours today [there's a colloquium].
- Today: Chapter 3, Finalé — Some Final Topics on LSL Semantics
  - Expressive Completeness: re-cap + some additional remarks.
  - Some Chapter 2 Problems — in Light of Chapter 3
  - Knights & Knaves Puzzles, and an LSAT Problem
- Next: Chapter 4 — Natural Deduction Proofs in LSL

Expressive Completeness: Recap

- **Fact.** The set of 4 connectives $\langle \sim, \&, \vee, \rightarrow \rangle$ is expressively complete.

$$\ulcorner p \leftrightarrow q \urcorner \mapsto \ulcorner (p \rightarrow q) \,\&\, (q \rightarrow p) \urcorner$$

- **Fact.** The set of 3 connectives $\langle \sim, \&, \vee \rangle$ is expressively complete.

$$\ulcorner p \rightarrow q \urcorner \mapsto \ulcorner \sim p \vee q \urcorner$$

- **Fact.** The pairs $\langle \sim, \& \rangle$ and $\langle \sim, \vee \rangle$ are both expressively complete.

$$\ulcorner p \vee q \urcorner \mapsto \ulcorner \sim(\sim p \,\&\, \sim q) \urcorner$$

  - The $\langle \sim, \vee \rangle$ strategy is similar [$\ulcorner p \,\&\, q \urcorner \mapsto \ulcorner \sim(\sim p \vee \sim q) \urcorner$].

- Consider the binary connective '$|$' such that $\ulcorner p | q \urcorner \dashv\vDash \ulcorner \sim(p \,\&\, q) \urcorner$.

- **Fact.** '$|$' *alone* is expressively complete! How to express $\langle \sim, \& \rangle$ using '$|$':

$$\ulcorner \sim p \urcorner \mapsto \ulcorner p | p \urcorner, \text{ and } \ulcorner p \,\&\, q \urcorner \mapsto \ulcorner (p|q)|(p|q) \urcorner$$

  - I called '$|$' 'NAND' in a previous lecture. NOR is also expressively complete.

Expressive Completeness: Additional Remarks and Questions

- **Q.** How can we define $\leftrightarrow$ in terms of $|$? **A.** If you naïvely apply the schemes I described last time, then you get a *187 symbol monster*:

  $\ulcorner p \leftrightarrow q \urcorner \mapsto A|A$, where A is given by the following *93 symbol* expression:

  $$(((p|(q|q))|(p|(q|q)))|((p|(q|q))|(p|(q|q))))|(((q|(p|p))|(q|(p|p)))|((q|(p|p))|(q|(p|p))))$$

- There are *simpler* definitions of $\leftrightarrow$ using $|$. *E.g.*, this *43 symbol* answer:

  $$\ulcorner p \leftrightarrow q \urcorner \mapsto ((p|(q|q))|(q|(p|p)))|((p|(q|q))|(q|(p|p)))$$

- Can anyone give an *even simpler* definition of $\leftrightarrow$ using $|$? Extra-Credit!
- How could you show that the pair $\langle \rightarrow, \sim \rangle$ is expressively complete?
- **Fact.** No subset of $\langle \sim, \&, \vee, \rightarrow, \leftrightarrow \rangle$ that does *not* contain negation $\sim$ is expressively complete. [This is a 140A question, beyond our scope.]
- Let $\curlywedge$ denote the $\perp$ truth-function (*i.e.*, the trivial function that *always* returns $\perp$). How could you show that $\langle \rightarrow, \curlywedge \rangle$ is expressively complete?

Two Chapter 2 Examples — In Light of Chapter 3

If Yossarian flies his missions then he is putting himself in danger, and it is irrational to put oneself in danger. If Yossarian is rational he will ask to be grounded, and he will be grounded only if he asks. But only irrational people are grounded, and a request to be grounded is proof of rationality. So, Yossarian will fly his missions whether he is rational or irrational.

- Basic Sentences: Yossarian flies his missions (F), Yossarian puts himself in danger (D), Yossarian is rational (R), Yosarian asks to be grounded (A).
- We reconstructed this argument as having the following form:

$$\begin{array}{ll} & (F \to D) \,\&\, (D \to {\sim}R) \\ & (R \to A) \,\&\, ({\sim}F \to A) \\ (1) & ({\sim}F \to {\sim}R) \,\&\, (A \to R) \\ & \therefore\ (R \to F) \,\&\, ({\sim}R \to F) \end{array}$$

- (1) is valid. This can be verified using various truth-table techniques.

$$(1)\quad \begin{array}{l} (F \to D)\;\&\;(D \to {\sim}R) \\ (R \to A)\;\&\;({\sim}F \to A) \\ ({\sim}F \to {\sim}R)\;\&\;(A \to R) \\ \therefore\; (R \to F)\;\&\;({\sim}R \to F) \end{array}$$

is valid, since its corresponding conditional is a tautology.

A	D	F	R	(((F→D)	&	(D→	~R))	&	(((R→A)	&	(~F	→A))	&	((~F	→	~R)	&	(A→R))))	→	((R→F)	&	(~R	→F))
⊤	⊤	⊤	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊤	⊤	**⊤**	⊤	⊤	⊥	⊤
⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊥	**⊤**	⊤	⊤	⊤	⊤
⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	**⊤**	⊥	⊥	⊥	⊤
⊤	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊥	**⊤**	⊤	⊥	⊤	⊥
⊤	⊥	⊤	⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊤	⊤	**⊤**	⊤	⊤	⊥	⊤
⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊥	**⊤**	⊤	⊤	⊤	⊤
⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	**⊤**	⊥	⊥	⊥	⊤
⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊥	**⊤**	⊤	⊥	⊤	⊥
⊥	⊤	⊤	⊤	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊤	⊤	**⊤**	⊤	⊤	⊥	⊤
⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊤	**⊤**	⊤	⊤	⊤	⊤
⊥	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊤	**⊤**	⊥	⊥	⊥	⊤
⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	**⊤**	⊤	⊥	⊤	⊥
⊥	⊥	⊤	⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊤	⊤	**⊤**	⊤	⊤	⊥	⊤
⊥	⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊤	**⊤**	⊤	⊤	⊤	⊤
⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊤	**⊤**	⊥	⊥	⊥	⊤
⊥	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	**⊤**	⊤	⊥	⊤	⊥

- If we replace '$\sim F$' with 'G' throughout the argument, and then add the additional premise '$G \to \sim F$', then the resulting argument is *not* valid.

$(F \to D)\ \&\ (D \to \sim R)$

$(R \to A)\ \&\ (G \to A)$

(2) $(G \to \sim R)\ \&\ (A \to R)$ is *not* valid — see the truth-table on the following slide.

$G \to \sim F$ [implicit]

$\therefore\ (R \to F)\ \&\ (\sim R \to F)$

- What is needed is the other direction of '$G \to \sim F$', as in the following:

$(F \to D)\ \&\ (D \to \sim R)$

$(R \to A)\ \&\ (G \to A)$

(3) $(G \to \sim R)\ \&\ (A \to R)$

$\sim F \to G$ [implicit]

$\therefore\ (R \to F)\ \&\ (\sim R \to F)$

- As an exercise, use truth-table methods to show that (3) is valid. [Of course, the argument is also valid if we use the biconditional '$G \leftrightarrow \sim F$'.]

((((F→D) & (D→∼R)) & (((R→A) & (G→A)) & ((G→∼R) & (A→R)))) & (G→∼F)) → ((R→F) & (∼R→F))

A	D	F	G	R	F→D	&	D→∼R	∼R	&	R→A	&	G→A	&	G→∼R	∼R	&	A→R	&	G→∼F	∼F	→	R→F	&	∼R	∼R→F
⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊥	⊤
⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊤
⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊥	⊤
⊤	⊤	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊤
⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊤	⊤	⊥	⊥	⊥	⊤
⊤	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥
⊤	⊤	⊥	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊥	⊥	⊤
⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥
⊤	⊥	⊤	⊤	⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊥	⊤
⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊤
⊤	⊥	⊤	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊥	⊤
⊤	⊥	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊤
⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊥	⊥	⊤	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊤	⊤	⊥	⊥	⊥	⊤
⊤	⊥	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥
⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊥	⊥	⊥	⊥	⊤
⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥
⊥	⊤	⊤	⊤	⊤	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊥	⊤
⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊤
⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊥	⊤
⊥	⊤	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊤
⊥	⊤	⊥	⊤	⊤	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊤	⊤	⊥	⊥	⊥	⊤
⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥
⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊥	⊥	⊤
⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊤	⊥
⊥	⊥	⊤	⊤	⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊥	⊤
⊥	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊤
⊥	⊥	⊤	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊥	⊤
⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊤
⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊤	⊤	⊥	⊥	⊥	⊤
⊥	⊥	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥
⊥	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊥	⊥	⊤
⊥	⊥	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊤	⊥

Chapter 2 in Light of Chapter 3: Example #2

Suppose no two contestants enter; then there will be no contest. No contest means no winner. Suppose all contestants perform equally well. Still no winner. There won't be a winner unless there's a loser. And conversely. Therefore, there will be a loser only if at least two contestants enter and not all contestants perform equally well.

- Here are the atomic sentences:

  T: At least two contestants enter. C: There is a contest.

  P: All contestants perform equally well.

  W: There is a winner. L: There is a loser.

- The resulting sentential form of the argument is as follows:

  $\sim T \to \sim C$. $\sim C \to \sim W$. $P \to \sim W$. $\sim L \leftrightarrow \sim W$. Therefore, $L \to (T \,\&\, \sim P)$.

- This is a valid form, as can be seen via the following truth-table, which shows that its corresponding conditional is tautologous:

C	L	P	T	W	((~T	→	~C)	&	(~C	→	~W)	&	(P →	~W)	&	(~L	↔	~W))	→	(L →	(T &	~P))
⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊥	⊥
⊤	⊤	⊤	⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊥	⊥	⊥
⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊥	⊥
⊤	⊤	⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊥	⊥	⊥
⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊤
⊤	⊤	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊤
⊤	⊤	⊥	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊥	⊤
⊤	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊥	⊥	⊤
⊤	⊥	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊥
⊤	⊥	⊤	⊤	⊥	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊥
⊤	⊥	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊥
⊤	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊥	⊥
⊤	⊥	⊥	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊤
⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤
⊤	⊥	⊥	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊤
⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊥	⊤
⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊥	⊥
⊥	⊤	⊤	⊤	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊥	⊥	⊥
⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊥	⊥
⊥	⊤	⊤	⊥	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊥	⊥	⊥
⊥	⊤	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊤	⊤	⊤	⊤
⊥	⊤	⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊤
⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊥	⊤
⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊥	⊥	⊤
⊥	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊥
⊥	⊥	⊤	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊥
⊥	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊥
⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊥
⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊤
⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤
⊥	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊤
⊥	⊥	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤

- We saw the following premise our last argument: 'There won't be a winner unless there's a loser. And conversely.' I symbolized it as:
  - "Logish": If not L, then not W, *and conversely*. [*i.e.*, not L *iff* not W.]
  - LSL: '$\sim L \leftrightarrow \sim W$', *equivalently*: '$(\sim L \to \sim W)\ \&\ (\sim W \to \sim L)$'.
- Why not interpret the "and conversely" to be operating on the *unless* operator itself? This yields the following *different* symbolization:
  - "Logish": not W unless L, and L unless not W.
  - LSL: '$(\sim L \to \sim W)\ \&\ (\sim\sim W \to L)$', *equivalently*: '$(\sim L \to \sim W)\ \&\ (W \to L)$'.
- Answer: This is a *redundant* symbolization in LSL, since '$\sim L \to \sim W$' *is equivalent to* '$W \to L$'. Moreover, the resulting argument *isn't* valid.
- If we replace '$\sim L \leftrightarrow \sim W$' with '$\sim L \to \sim W$', then the resulting sentential form is not valid — see the truth-table on the following slide.
- **Principle of Charity**. If an argument $\mathscr{A}$ has two *plausible but semantically distinct* LSL symbolizations (where neither is *obviously* preferable) — and *only one of them is valid — choose the valid one*.

((~T→~C) & (~C→~W) & (P→~W) & (~L→~W)) → (L→(T&~P))

C	L	P	T	W	~T	→	~C	&	~C	→	~W	&	→	~W	&	~L	→	~W	→	→	&	~P
⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊥	⊥
⊤	⊤	⊤	⊤	⊥	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊥
⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊥	⊥
⊤	⊤	⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊤	⊤	⊤	⊥	⊥	⊥
⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊤
⊤	⊤	⊥	⊤	⊥	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊤
⊤	⊤	⊥	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊥	⊤
⊤	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊤	⊤	⊤	⊥	⊥	⊤
⊤	⊥	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊥
⊤	⊥	⊤	⊤	⊥	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊥
⊤	⊥	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊥
⊤	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊥	⊥
⊤	⊥	⊥	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊤
⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤
⊤	⊥	⊥	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊤
⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊥	⊤
⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊥	⊥
⊥	⊤	⊤	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊥
⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊥	⊥
⊥	⊤	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊥
⊥	⊤	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊤	⊤	⊤	⊤
⊥	⊤	⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊤
⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊥	⊤
⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤
⊥	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊥
⊥	⊥	⊤	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊥
⊥	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊥
⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊥
⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊤
⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤
⊥	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊤
⊥	⊥	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤

Some Fun with LSL Translation and Semantics: Knights & Knaves

- The island of Knights and Knaves has two types of inhabitants, Knights who always tell the truth, and Knaves who always lie.
- Suppose A is the proposition person a is a knight and suppose a makes a statement S. Then, A is semantically equivalent to S [$A \mathrel{=\!\!|\!\!\models} S$]. Why?
- So, whenever an inhabitant x makes a claim S, we can infer that $X \leftrightarrow S$. That is, we can infer that x is a knight iff S is true. Simple applications:
- If a says "I am a Knight" then we can infer that $A \leftrightarrow A$. But, since this is always true (a *tautology*), we get no information from this statement.
- Conversely, it *cannot* be the case that a native says "I am a Knave" because we could then conclude $A \leftrightarrow \sim A$, which is self-contradictory.
- If a says "I am the same type as b," then we can infer $A \leftrightarrow (A \leftrightarrow B)$ which is equivalent to B [$B \mathrel{=\!\!|\!\!\models} A \leftrightarrow (A \leftrightarrow B)$] (we proved this above). So, this statement allows us to infer that person b is a Knight!

- Given this set-up, use truth-tables (complete or "shortened") to justify your answers to the following questions about Knights and Knaves. These are good exercises that combine LSL translation, LSL semantics, and truth-table methods (*i.e.*, chapters 2 and 3 of our textbook).
  1. It is rumored that there is gold buried on the island (G). You ask one of the natives, a, whether there is gold on the island. He makes the following response: "There is gold on this island if and only if I am a Knight." Is there gold buried on the island? [Answer: Yes.]
  2. Inhabitant a says "Either I am a Knave or b is a Knight." What can we infer about a and b? [Answer: a and b are both Knights.]
  3. Three of the inhabitants — a, b and c — were standing together in the garden. A stranger passed by and asked a, "Are you a Knight or a Knave?". a answered, but rather indistinctly, so the stranger could not make out what he said. The stranger then asked b, "What did a say?". b replied, "a said that he is a Knave." At this point the third man, c, said "Don't believe b; he's lying!". What are b and c? [Answer: b is a Knave and c is a Knight.]

12A and The LSAT: A Sample Question

A university library budget committee must reduce exactly five of eight areas of expenditure--G, L, M, N, P, R, S, and W--in accordance with the following conditions:

If both G and S are reduced, W is also reduced.
If N is reduced, neither R nor S is reduced.
If P is reduced, L is not reduced.
Of the three areas L, M, and R, exactly two are reduced.

Which one of the following could be a complete and accurate list of the areas of expenditure reduced by the committee?

(A) G, L, M, N, W
(B) G, L, M, P, W
(C) G, M, N, R, W
(D) G, M, P, R, S
(E) L, M, R, S, W

- Formalization of given information in LSL:
  - $(G \,\&\, S) \to W$
  - $N \to ({\sim}R \,\&\, {\sim}S)$
  - $P \to {\sim}L$
  - $((((L \,\&\, M) \vee (L \,\&\, R)) \vee (M \,\&\, R)) \,\&\, {\sim}(L \,\&\, (M \,\&\, R)))$
- Ruling-out answers:

(A) G, L, M, N, W

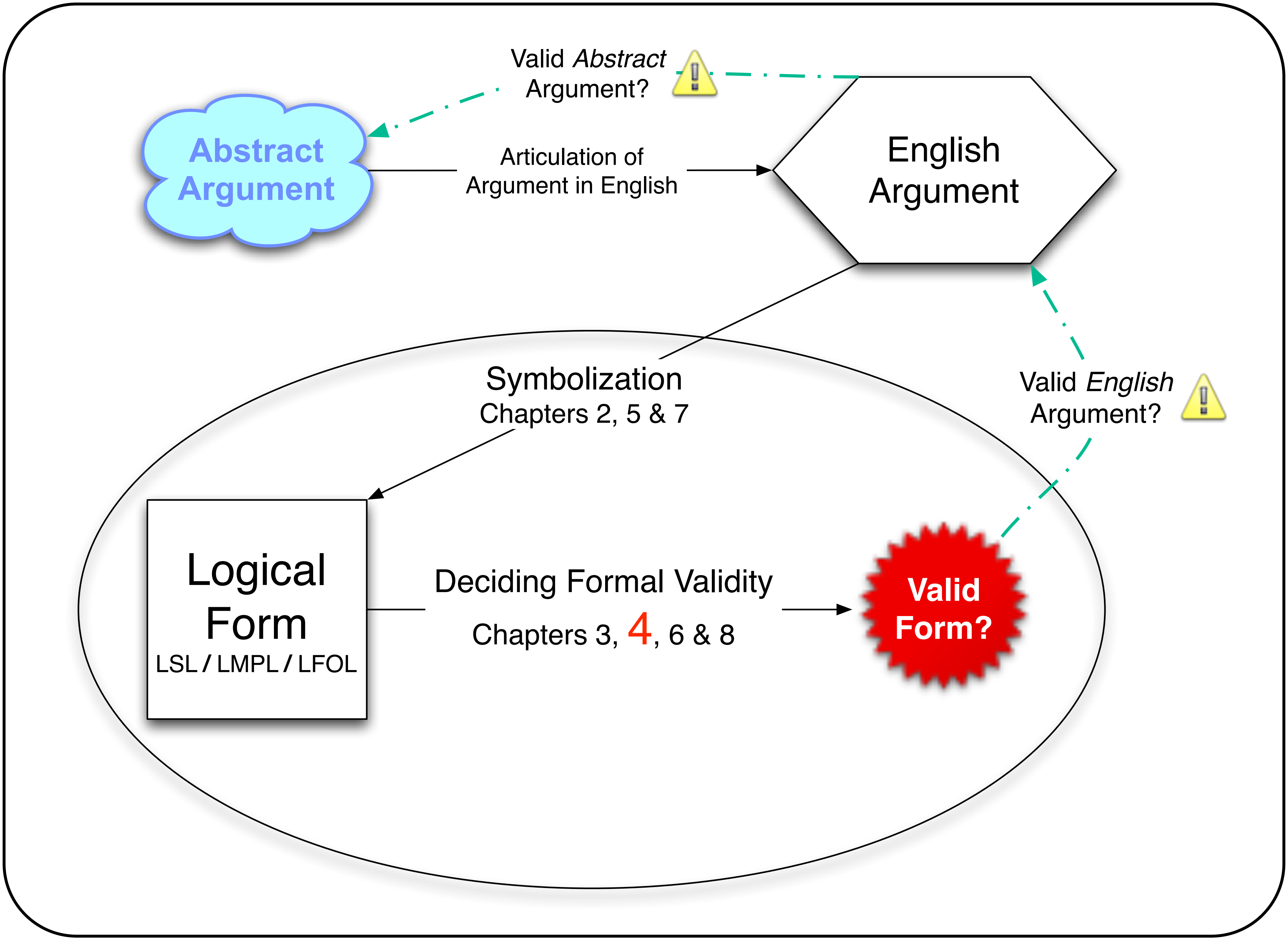
(B) G, L, M, P, W [impossible, since $P \to {\sim}L$]

(C) G, M, N, R, W [impossible, since $N \to ({\sim}R \,\&\, {\sim}S)$]

(D) G, M, P, R, S [impossible, since $(G \,\&\, S) \to W$]

(E) L, M, R, S, W [impossible, since ${\sim}(L \,\&\, (M \,\&\, R))$]

- The question is asking: which of (A)–(E) is *consistent* (in the LSL sense!) with the given information. Hint: (B)–(E) can be *ruled-out* quickly (shortcuts!).
- So, there is no need to *prove* (A) is consistent with the given information. To do that, one would produce a truth-table *row* in which G, L, M, N, W all come out $\top$, and such that all four given sentences also come out $\top$.


Valid *Abstract* Argument?
Abstract Argument
Articulation of Argument in English
English Argument
Symbolization
Chapters 2, 5 & 7
Valid *English* Argument?
Logical Form
LSL / LMPL / LFOL
Deciding Formal Validity
Chapters 3, 4, 6 & 8
Valid Form?

Chapter 4 Introduction: Truth *vs* Proof ($\vDash$ *vs* $\vdash$)

- Recall: $p \vDash q$ iff it is impossible for p to be true while q is false.
- We have methods (truth-tables) for establishing $\vDash$ and $\nvDash$ claims. These methods are especially good for $\nvDash$ claims, but they get very complex for $\vDash$ claims. Is there another more "natural" way to prove $\vDash$'s? Yes!
- In Chapter 4, we will learn a *natural deduction system* for LSL. This is a system of *rules of inference* that will allow us to prove all valid LSL arguments in a purely syntactical way (no appeal to semantics).
- The notation $p \vdash q$ means that *there exists a natural deduction proof of q from p* in our natural deduction system for sentential logic.
- $\ulcorner p \vdash q \urcorner$ is short for $\ulcorner p$ *deductively* entails $q\urcorner$.
- While $\vDash$ has to do with *truth*, $\vdash$ does *not*. $\vdash$ has only to do with what can be *deduced*, using a *fixed set* of formal, natural deduction rules.

- Happily, our system of natural deduction rules is *sound* and *complete*:
  - **Soundness**. If $p \vdash q$, then $p \vDash q$. [no proofs of *in*validities!]
  - **Completeness**. If $p \vDash q$, then $p \vdash q$. [proofs of *all* validities!]
- We will not prove the soundness and completeness of our system of natural deduction rules. I will say a few things about soundness as we go along, but completeness is much harder to establish (140A!).
- We'll have rules that permit the *elimination* or *introduction* of each of the connectives &, $\rightarrow$, $\vee$, $\sim$, $\leftrightarrow$ within natural deductions. These rules will make sense, from the point of view of the semantics.
- A *proof* of q from p is a sequence of LSL formulas, beginning with p and ending with q, where each formula in the sequence is *deduced* from previous lines, *via* a correct application of one of the *rules*.
- Generally, we will be talking about deductions of formulas q from sets of premises $p_1, \ldots, p_n$. We call these $\ulcorner p_1, \ldots, p_n \vdash q \urcorner$'s *sequents*.

An Example of a Natural Deduction Involving & and →

- The following is a valid LSL argument form:

$$A \,\&\, B$$
$$C \,\&\, D$$
$$(A \,\&\, D) \to H$$
$$\therefore H$$

- Here's a (7-line) natural deduction proof of the sequent corresponding to this argument: $A \,\&\, B, C \,\&\, D, (A \,\&\, D) \to H \vdash H$.

1	(1)	$A \,\&\, B$	Premise
2	(2)	$C \,\&\, D$	Premise
3	(3)	$(A \,\&\, D) \to H$	Premise
1	(4)	A	1 &E
2	(5)	D	2 &E
1, 2	(6)	$A \,\&\, D$	4, 5 &I
1, 2, 3	(7)	H	3, 6 →E ✦

The Rule of Assumptions (Preliminary Version)

- **Rule of Assumptions** (preliminary version): The premises of an argument-form are listed at the start of a proof in the order in which they are given, each labeled 'Premise' on the right and numbered with its own line number on the left. Schematically:

j	(j)	p	Premise

- We can see that our example proof begins, as it should, with the three premises of the argument-form, written as follows:

1	(1)	$A \& B$	Premise
2	(2)	$C \& D$	Premise
3	(3)	$(A \& D) \to H$	Premise

The Rule of &-Elimination (&E)

- **Rule of &-Elimination**: If a conjunction $\ulcorner p \,\&\, q \urcorner$ occurs at line j, then at any *later* line k one may infer either conjunct, labeling the line 'j &E' and writing on the left all the numbers which appear on the left of line j.

  Schematically:

$a_1, \ldots, a_n$	(j)	$p \,\&\, q$			$a_1, \ldots, a_n$	(j)	$p \,\&\, q$	
	$\vdots$			OR		$\vdots$		
$a_1, \ldots, a_n$	(k)	p	j &E		$a_1, \ldots, a_n$	(k)	q	j &E

- We can see that our example deduction continues, in lines (4) and (5), with two correct applications of the &-Elimination Rule:

1	(4)	A	1 &E
2	(5)	D	2 &E

The Rule of &-Introduction (&I)

- **Rule of &-Introduction**: For any formulae p and q, if p occurs at line j and q occurs at line k then the formula $\ulcorner p \,\&\, q \urcorner$ may be inferred at line m, labeling the line 'j, k &I' and writing on the left all numbers which appear on the left of line j *and* all which appear on the left of line k. [Note: we may have j < k, j > k, *or* j = k. *Why*?]

$$
\begin{array}{rccl}
a_1, \ldots, a_n & (j) & p & \\
 & \vdots & & \\
b_1, \ldots, b_u & (k) & q & \\
 & \vdots & & \\
a_1, \ldots, a_n, b_1, \ldots, b_u & (m) & p \,\&\, q & \text{j, k \&I}
\end{array}
$$

- We can see that our example deduction continues, in lines (6), with a correct application of the &-Introduction Rule:

$$
\begin{array}{cccc}
1, 2 & (6) & A \,\&\, D & 4, 5 \text{ \&I}
\end{array}
$$

The Rule of →-Elimination (→E)

- **Rule of →-Elimination**: For any formulae p and q, if ⌜$p \to q$⌝ occurs at a line j and p occurs at a line k, then q may be inferred at line m, labeling the line 'j, k →E' and writing on the left all numbers which appear on the left of line j *and* all numbers which appear on the left of line k.
  [Note: We may have either j < k or j > k.]

$$\begin{array}{rcll} a_1,\ldots,a_n & (j) & p \to q & \\ & \vdots & & \\ b_1,\ldots,b_u & (k) & p & \\ & \vdots & & \\ a_1,\ldots,a_n,b_1,\ldots,b_u & (m) & q & \text{j, k} \to\text{E} \end{array}$$

- Our example deduction *concludes* (we indicate the end of a proof with a '✦'), in line (7), with a correct application of the →-Elimination Rule:

$$\begin{array}{cccc} 1,2,3 & (7) & H & 3,6 \to\text{E} ✦ \end{array}$$

Deduction #2 Using the Rules &E and &I

- Consider the valid LSL argument form:
  $A \,\&\, (B \,\&\, C)$
  $\therefore\ C \,\&\, (B \,\&\, A)$

- Let's do a deduction of this argument form:

1	(1)	$A \,\&\, (B \,\&\, C)$	Premise
1	(2)	A	1 &E
1	(3)	$B \,\&\, C$	1 &E
1	(4)	B	3 &E
1	(5)	C	3 &E
1	(6)	$B \,\&\, A$	4, 2 &I
1	(7)	$C \,\&\, (B \,\&\, A)$	5, 6 &I ✦

- NOTE: &E can *only* be applied to formulas whose *main* connective is '&', and &E *must* be *applied to that particular* connective.