Announcements & Such

- YES: *And You And I*. [I saw them in Chicago last week — amazing show.]
- Thanks to Julia and Tamar for filling-in for me last week!
- HW #2 solutions have been posted.
- **HW #3 (1st-submission) is due Thursday.**
- I have posted a handout with solutions to (some of) the lecture problems on logical truth, logical falsity, equivalence, consistency, etc.
- I have also posted a handout on the "short" truth-table method, which we will be going over in lecture today. **Please use that as a model.**
- Today: Chapter 3, Continued (Truth-Table methods for validity testing)
  - Review: The Exhaustive Truth-Table Method
  - The "Short" Truth-Table Method: going over the handout.

The Exhaustive Truth-Table Method for Testing Validity

- Remember, an argument is valid if it is *impossible* for its premises to be true while its conclusion is false. Let $p_1, \ldots, p_n$ be the premises of a LSL argument, and let q be the conclusion of the argument. Then, we have:

$$\begin{array}{c} p_1 \\ \vdots \\ p_n \\ \hline \therefore\ q \end{array}$$

is valid if and only if there is no row in the simultaneous truth-table of $p_1, \ldots, p_n$, and q which looks like the following:

atoms	premises			conclusion
$\cdots$	p_1	$\cdots$	p_n	q
$\cdots$	$\top$	$\top$	$\top$	$\bot$

- We will use simultaneous truth-tables to prove validities and invalidities. For example, consider the following valid argument:

A

$A \to B$

$\therefore B$

atoms		premises				conclusion
A	B	A	A	$\to$	B	B
$\top$	$\top$	$\top$	$\top$	$\top$	$\top$	$\top$
$\top$	$\bot$	$\top$	$\top$	$\bot$	$\bot$	$\bot$
$\bot$	$\top$	$\bot$	$\bot$	$\top$	$\top$	$\top$
$\bot$	$\bot$	$\bot$	$\bot$	$\top$	$\bot$	$\bot$

☞ VALID — there is no row in which A and $A \to B$ are both $\top$, but B is $\bot$.

- In general, we'll use the following procedure for evaluating arguments:
  1. Translate and symbolize the the argument (if given in English).
  2. Write out the symbolized argument (as above).
  3. Draw a simultaneous truth-table for the symbolized argument, outlining the columns representing the premises and conclusion.
  4. Is there a row of the table in which all premises are $\top$ but the conclusion is $\bot$? If so, the argument is invalid; if not, it's valid.
- We will practice this on examples. But, first, a "short-cut" method.

The "Short" Truth Table Method for Validity Testing I

- Consider the following LSL argument:

  $A \rightarrow (B \,\&\, E)$

  $D \rightarrow (A \vee F)$

  $\sim E$

  $\therefore D \rightarrow B$

- This argument has 3 premises and contains 5 atomic sentences. This would lead to a complete truth-table with 32 rows and 8 columns.
- So, the exhaustive method is *impractical* here. So, instead, let's try to construct or "reverse engineer" an invalidating interpretation.
- To do this, we "work backward" from the *assumption* that the conclusion is $\perp$ and all the premises are $\top$ on some row.
- I won't go over this problem again. Rather, I'll go over the hanodut.

The "Short" Method for Constructing Interpretations: Handout Problem #1

- Question: $A \to (C \vee E), B \to D \overset{?}{\vDash} (A \vee B) \to (C \to (D \vee E))$.
- Answer: $A \to (C \vee E), B \to D \nvDash (A \vee B) \to (C \to (D \vee E))$.
- Step 1: Assume there is an interpretation on which the premises is $\top$ but the conclusion is $\bot$. This leads to the following partial row:

A	B	C	D	E	$A \to (C \vee E)$	$B \to D$	$(A \vee B) \to (C \to (D \vee E))$
					$\top$	$\top$	$\bot$

- Step 2: There's only one way the conclusion can be $\bot$, which leads to:

A	B	C	D	E	$A \to (C \vee E)$	$B \to D$	$(A \vee B) \to (C \to (D \vee E))$
					$\top$	$\top$	$\top$ $\bot$ $\bot$

- Step 3: There's only one way $C \to (D \vee E)$ can be $\bot$, which leads to:

A	B	C	D	E	$A \to (C \vee E)$	$B \to D$	$(A \vee B) \to (C \to (D \vee E))$
		$\top$			$\top$	$\top$	$\top$ $\bot$ $\top$ $\bot$ $\bot$

- Step 4: There's only one way $D \vee E$ can be $\perp$, which leads to:

A	B	C	D	E	$A \to (C \vee E)$	$B \to D$	$(A \vee B) \to (C \to (D \vee E))$
		$\top$	$\perp$	$\perp$	$\top$	$\top\ \perp$	$\top\ \ \perp\ \top\ \perp\ \ \perp$

- Step 5: Since D is $\perp$, the only way $B \to D$ can be $\top$ is if B is $\perp$:

A	B	C	D	E	$A \to (C \vee E)$	$B \to D$	$(A \vee B) \to (C \to (D \vee E))$
	$\perp$	$\top$	$\perp$	$\perp$	$\top$	$\perp\ \top\ \perp$	$\top\ \ \perp\ \top\ \perp\ \ \perp$

- Step 6: Since C is $\top$, so is $C \vee E$, which makes $A \to (C \vee E)$ $\top$ *regardless of the truth-value of* A. So, I will just let A be $\top$, and then we're done.

A	B	C	D	E	$A \to (C \vee E)$	$B \to D$	$(A \vee B) \to (C \to (D \vee E))$
$\top$	$\perp$	$\top$	$\perp$	$\perp$	$\top\ \top\ \ \top$	$\perp\ \top\ \perp$	$\top\ \ \perp\ \top\ \perp\ \ \perp$

- When reporting your answer, all you need to do is give the single row that serves as a counterexample. Here, I recommend you include the quasi-columns that you used to calculate the truth-values in the row.

- Verbal explanations are optional. Here's the detailed handout solution.

Answer. $A \to (C \vee E), B \to D \nvDash (A \vee B) \to (C \to (D \vee E))$

Explanation.[a] Assume that '$A \to (C \vee E)$' is $\top$, '$B \to D$' is $\top$, and '$(A \vee B) \to (C \to (D \vee E))$' is $\bot$. In order for '$(A \vee B) \to (C \to (D \vee E))$' to be $\bot$, both '$A \vee B$' and 'C' must be $\top$, and both 'D' and 'E' must be $\bot$. This *guarantees* that the first premise is $\top$ (since '$A \to (C \vee E)$' *must*, at this point, have a $\top$ consequent). We can also make the second premise $\top$, simply by making 'B' $\bot$. So, as the following single-row truth-table shows, we have *succeeded* in finding an interpretation on which '$A \to (C \vee E)$' and '$B \to D$' are both $\top$, but '$(A \vee B) \to (C \to (D \vee E))$' is $\bot$. *QED*.

A	B	C	D	E	$A \to (C \vee E)$	$B \to D$	$(A \vee B) \to (C \to (D \vee E))$
$\top$	$\bot$	$\top$	$\bot$	$\bot$	$\top\ \mathbf{\top}\ \top\ \top\ \bot$	$\bot\ \mathbf{\top}\ \bot$	$\top\ \top\ \bot\ \mathbf{\bot}\ \top\ \bot\ \bot\ \bot\ \bot$

[a] You do *not* have to show *all* of your reasoning in cases like this one, where the argument is *in*valid (*i.e.*, where $\nvDash$). I am just showing you *all* of *my* reasoning to give you more information about how these kinds of problems are solved. All you *need* to do here is report an interpretation (*i.e.*, a single-row) which invalidates the inference. But, when you do so, I recommend filling-in all of the quasi-columns to make explicit all of the calculations required.

The "Short" Method for Constructing Interpretations: Handout Problem #2

- Question: $A \leftrightarrow (B \vee C), B \rightarrow D, D \leftrightarrow C \overset{?}{\vDash} A \leftrightarrow D$.
- Answer: $A \leftrightarrow (B \vee C), B \rightarrow D, D \leftrightarrow C \vDash A \leftrightarrow D$.
- Step 1: Assume there is an interpretation on which the premises is $\top$ but the conclusion is $\bot$. This leads to the following partial row:

A	B	C	D	$A \leftrightarrow (B \vee C)$	$B \rightarrow D$	$D \leftrightarrow C$	$A \leftrightarrow D$
				$\top$	$\top$	$\top$	$\bot$

- Already, we have to break this down into cases, since there are ($\geq$) two ways each premise can be $\top$ and also two ways the conclusion can be $\bot$.
  - Case 1: A is $\top$ and D is $\bot$.
  - Case 2: A is $\bot$ and D is $\top$.

- Step 2 (Case 1): If A is $\top$ and D is $\bot$, then we have the following:

A	B	C	D	$A \leftrightarrow (B \vee C)$	$B \to D$	$D \leftrightarrow C$	$A \leftrightarrow D$
$\top$			$\bot$	$\top\ \top$	$\top\ \bot$	$\bot\ \top$	$\top\ \bot\ \bot$

- Step 3 (Case 1): Now, the only way for $B \to D$ to be $\top$ is for B to be $\bot$. And, the only way for $D \leftrightarrow C$ to be $\top$ is for C to be $\bot$, which yields:

A	B	C	D	$A \leftrightarrow (B \vee C)$	$B \to D$	$D \leftrightarrow C$	$A \leftrightarrow D$
$\top$	$\bot$	$\bot$	$\bot$	$\top\ \top\ \bot\ \ \ \bot$	$\bot\ \top\ \bot$	$\bot\ \top\ \bot$	$\top\ \bot\ \bot$

- Step 4 (Case 1): But, we need $A \leftrightarrow (B \vee C)$ to be $\top$, which means we need $B \vee C$ to be $\top$. However, this contradicts our assumptions — dead end!

A	B	C	D	$A \leftrightarrow (B \vee C)$	$B \to D$	$D \leftrightarrow C$	$A \leftrightarrow D$
$\top$	$\bot$	$\bot$	$\bot$	$\top\ \top\ \bot\ \top/\bot!!\ \bot$	$\bot\ \top\ \bot$	$\bot\ \top\ \bot$	$\top\ \bot\ \bot$

- As usual, we cannot infer — yet — that this argument is valid.
- We must continue on with an examination of Case 2 . . .

- Step 2 (Case 2): If A is $\perp$ and D is $\top$, then we have the following:

A	B	C	D	$A \leftrightarrow (B \vee C)$	$B \to D$	$D \leftrightarrow C$	$A \leftrightarrow D$
$\perp$			$\top$	$\perp\ \top$	$\top\ \top$	$\top\ \top$	$\perp\ \perp\ \top$

- Step 3 (Case 2): Now, the only way for $D \leftrightarrow C$ to be $\top$ is for C to be $\top$, which forces $B \vee C$ to be $\top$, contradicting our assumptions — dead end!

A	B	C	D	$A \leftrightarrow (B \vee C)$	$B \to D$	$D \leftrightarrow C$	$A \leftrightarrow D$
$\perp$		$\top$	$\top$	$\perp\ \top/\perp!!\quad \top\ \top$	$\top\ \top$	$\top\ \top\ \top$	$\perp\ \perp\ \top$

- Since *both* of the two possible cases lead to a dead-end (*i.e.*, a contradiction), we may (finally) infer that this argument is *valid*.
- For valid arguments, you must give a verbal explanation of your "short" method answers. The handout contains two model solutions.
- Here's what the model solution on the handout looks like for this problem. Note: there are no "partial rows" included in the solution. You *may* include these (as in the lecture notes above), but you *need not*.

Answer. $A \leftrightarrow (B \vee C), B \to D, D \leftrightarrow C \vDash A \leftrightarrow D$.

Explanation. Assume '$A \leftrightarrow (B \vee C)$' is $\top$, '$B \to D$' is $\top$, '$D \leftrightarrow C$' is $\top$, and '$A \leftrightarrow D$' is $\bot$. There are *exactly two* ways in which '$A \leftrightarrow D$' can be $\bot$:

1. 'A' is $\top$, and 'D' is $\bot$. In this case, in order for '$D \leftrightarrow C$' to be $\top$, 'C' must be $\bot$. And, in order for '$B \to D$' to be $\top$, 'B' must be $\bot$. This means that the *disjunction* '$B \vee C$' must be $\bot$. So, in order for '$A \leftrightarrow (B \vee C)$' to be $\top$, we must have 'A' $\bot$ as well, which contradicts our assumption. So, in this first case, we have been forced into a *contradiction*.

2. 'A' is $\bot$, and 'D' is $\top$. In this case, in order for '$D \leftrightarrow C$' to be $\top$, 'C' must be $\top$. But, if 'C' is $\top$, then so is '$B \vee C$'. Hence, if '$A \leftrightarrow (B \vee C)$' is going to be $\top$, then 'A' must be $\top$, which contradicts our assumption. So, in this second (and *last*) case, we have been forced into a *contradiction*.

$\therefore$ There are no interpretations on which '$A \leftrightarrow (B \vee C)$', '$B \to D$', and '$D \leftrightarrow C$' are all $\top$ and '$A \leftrightarrow D$' is $\bot$. So, $A \leftrightarrow (B \vee C), B \to D, D \leftrightarrow C \vDash A \leftrightarrow D$. $\square$

Presenting Your "Short-Method" Truth-Table Tests

- In any application of the "short" method, there are two possibilities:

1. You find an interpretation (*i.e.*, a row of the truth-table) on which all the premises $p_1, \ldots, p_n$ of an argument are true and the conclusion q is false. *All you need to do here* is (*i*) write down the relevant row of the truth-table, and (*ii*) say "Here is an interpretation on which $p_1, \ldots, p_n$ are all true and q is false. So, $p_1, \ldots, p_n \therefore q$ is *in*valid."
2. You discover that there is *no possible way* of making $p_1, \ldots, p_n$ true and q false. Here, you need to *explain all of your reasoning* (as I do in lecture, or as Forbes does, or as I do in my handout). It must be clear that you have *exhausted all possible cases*, before concluding that $p_1, \ldots, p_n \therefore q$ is *valid*. This can be rather involved, and should be spelled out in a step-by-step fashion. Each salient case has to be examined.

- Consult my handout and lecture notes for model answers of both kinds.

Properties of the Semantic Consequence Relation: $\vDash$

- The following four metalinguistic statements are *synonymous:*
  - The argument $\begin{array}{l} p_1, p_2, \ldots, p_n \\ \therefore q \end{array}$ is *valid*.
  - q *follows from* $p_1, p_2, \ldots, p_n$.
  - $p_1, p_2, \ldots, p_n$ (jointly) *entail* q.
  - $p_1, p_2, \ldots, p_n \vDash q$
- Here are some important properties of $\vDash$ with explanations:
  - $p \vDash p$
    * Every interpretation on which p is true is an interpretation on which p is true. That is, all p-interpretations are p-interpretations.
  - If $p \vDash q$ and $q \vDash r$, then $p \vDash r$.
    * If all p-interpretations are q-interpretations and all q-interpretations are r-interpretations, then all p-interpretations are r-interpretations.

* Remember: the following argument is valid (but not sententially!).

  All Ps are Qs.

  All Qs are Rs.

  $\therefore$ all Ps are Rs.

* More on arguments like this in the second half of the course ...

- If $p \vDash r$, then $p \,\&\, q \vDash r$.

  * If all p-interpretations are r-interpretations, then all $(p \,\&\, q)$-interps are r-interpretations [since all $(p \,\&\, q)$-interpretations *are* p-interpretations!].

- $(p \,\&\, q) \vDash r$ *if and only if* $p, q \vDash r$

  * If all $p \,\&\, q$-interpretations are r-interpretations, then all $\{p, q\}$-interpretations are r-interpretations (pretty obviously).

- $p \vDash q$ *if and only if* $\vDash p \to q$

  * If all p-interpretations are q-interpretations, then *all* interpretations (whatsoever) are $(p \to q)$-interpretations.

  * $p \to q$ is a tautology [$\vDash p \to q$] iff there is no interpretation on which p is true and q is false, which is just the definition of $p \vDash q$!

Expressive Completeness

- In LSL, we have five connectives: $\langle \sim, \&, \vee, \rightarrow, \leftrightarrow \rangle$. But, we don't "need" all five. We can express all the same propositions with fewer connectives.
- If a set of connectives is sufficient to express all the propositions expressible in LSL, then we say that set is *expressively complete*.
- To show that a set is expressively complete, all we need to do is show that we can emulate all five LSL connectives using just that set.
- **Fact**. The set of 4 connectives $\langle \sim, \&, \vee, \rightarrow \rangle$ is expressively complete.
  - All we need to do is explain how $\langle \sim, \&, \vee, \rightarrow \rangle$ allows us to express all statements that involve '$\leftrightarrow$' — *i.e.* — to *define* '$\leftrightarrow$' using $\langle \sim, \&, \vee, \rightarrow \rangle$.
  - There are many ways we could do this. Here's one:

$$\ulcorner p \leftrightarrow q \urcorner \mapsto \ulcorner (p \rightarrow q) \,\&\, (q \rightarrow p) \urcorner$$

  - This works because: $\ulcorner p \leftrightarrow q \urcorner \;=\!\mid\models\; \ulcorner (p \rightarrow q) \,\&\, (q \rightarrow p) \urcorner$.

- **Fact**. The set of 3 connectives $\langle \sim, \&, \vee \rangle$ is expressively complete.
  - Since we already know that $\langle \sim, \&, \vee, \rightarrow \rangle$ is expressively complete, all we need to do is explain how $\langle \sim, \&, \vee \rangle$ allows us to emulate '$\rightarrow$'.
  - Again, there are many ways to do this. The most obvious is:

$$\ulcorner p \rightarrow q \urcorner \mapsto \ulcorner \sim p \vee q \urcorner$$

- **Fact**. The pairs $\langle \sim, \& \rangle$ and $\langle \sim, \vee \rangle$ are both expressively complete.
  - For $\langle \sim, \& \rangle$, we just need to show how to express '$\vee$':

$$\ulcorner p \vee q \urcorner \mapsto \ulcorner \sim(\sim p \,\&\, \sim q) \urcorner$$

  - The $\langle \sim, \vee \rangle$ strategy is similar [$\ulcorner p \,\&\, q \urcorner \mapsto \ulcorner \sim(\sim p \vee \sim q) \urcorner$].
- Consider the binary connective '$|$' such that $\ulcorner p | q \urcorner \dashv\vDash \ulcorner \sim(p \,\&\, q) \urcorner$.
- **Fact**. '$|$' *alone* is expressively complete! How to express $\langle \sim, \& \rangle$ using '$|$':

$$\ulcorner \sim p \urcorner \mapsto \ulcorner p | p \urcorner, \text{ and } \ulcorner p \,\&\, q \urcorner \mapsto \ulcorner (p|q)|(p|q) \urcorner$$

  - I called '$|$' 'NAND' in a previous lecture. NOR is also expressively complete.

- How would you show that the pair $\langle \rightarrow, \sim \rangle$ is expressively complete?
  - Can you define & in terms of $\langle \rightarrow, \sim \rangle$?
  - Can you define $\vee$ in terms of $\langle \rightarrow, \sim \rangle$?
  - Can you define $\leftrightarrow$ in terms of $\langle \rightarrow, \sim \rangle$?
- It turns out that no subset of $\langle \sim, \&, \vee, \rightarrow, \leftrightarrow \rangle$ that does not contain negation $\sim$ is not expressively complete.
- You won't be required know how to show that a set of connectives is *not* expressively complete. That's something we do in 140A.
- Let $\curlywedge$ denote an (arbitrary) self-contradictory statement of LSL. How would you show that $\langle \rightarrow, \curlywedge \rangle$ is expressively complete?
  - Can you define $\sim$ in terms of $\langle \rightarrow, \curlywedge \rangle$?
  - Can you define & in terms of $\langle \rightarrow, \curlywedge \rangle$?
  - Can you define $\vee$ in terms of $\langle \rightarrow, \curlywedge \rangle$?
  - Can you define $\leftrightarrow$ in terms of $\langle \rightarrow, \curlywedge \rangle$?

Two Chapter 2 Examples — In Light of Chapter 3

If Yossarian flies his missions then he is putting himself in danger, and it is irrational to put oneself in danger. If Yossarian is rational he will ask to be grounded, and he will be grounded only if he asks. But only irrational people are grounded, and a request to be grounded is proof of rationality. So, Yossarian will fly his missions whether he is rational or irrational.

- Basic Sentences: Yossarian flies his missions (F), Yossarian puts himself in danger (D), Yossarian is rational (R), Yosarian asks to be grounded (A).
- We reconstructed this argument as having the following form:

$$
(1) \qquad
\begin{array}{l}
(F \to D) \,\&\, (D \to {\sim}R) \\
(R \to A) \,\&\, ({\sim}F \to A) \\
({\sim}F \to {\sim}R) \,\&\, (A \to R) \\
\therefore\ (R \to F) \,\&\, ({\sim}R \to F)
\end{array}
$$

- (1) is valid. This can be verified using various truth-table techniques.

(1)

$(F \rightarrow D) \,\&\, (D \rightarrow {\sim}R)$

$(R \rightarrow A) \,\&\, ({\sim}F \rightarrow A)$

$({\sim}F \rightarrow {\sim}R) \,\&\, (A \rightarrow R)$

$\therefore\ (R \rightarrow F) \,\&\, ({\sim}R \rightarrow F)$

is valid, since its corresponding conditional is a tautology.

(((F→D) & (D→ ~R)) & (((R→A) & (~F→A)) & ((~F→ ~R) & (A→R)))) → ((R→F) & (~R→F)))

A	D	F	R	F→D	&	D→	~R	&	R→A	&	~F	→A	&	~F	→	~R	&	A→R	→	R→F	&	~R	→F
⊤	⊤	⊤	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊤	⊤	**⊤**	⊤	⊤	⊥	⊤
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⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊤	**⊤**	⊥	⊥	⊥	⊤
⊥	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	**⊤**	⊤	⊥	⊤	⊥

- If we replace '$\sim F$' with 'G' throughout the argument, and then add the additional premise '$G \to \sim F$', then the resulting argument is *not* valid.

$(F \to D)\ \&\ (D \to \sim R)$

$(R \to A)\ \&\ (G \to A)$

(2) $(G \to \sim R)\ \&\ (A \to R)$ is *not* valid — see the truth-table on the following slide.

$G \to \sim F$ [implicit]

$\therefore\ (R \to F)\ \&\ (\sim R \to F)$

- What is needed is the other direction of '$G \to \sim F$', as in the following:

$(F \to D)\ \&\ (D \to \sim R)$

$(R \to A)\ \&\ (G \to A)$

(3) $(G \to \sim R)\ \&\ (A \to R)$

$\sim F \to G$ [implicit]

$\therefore\ (R \to F)\ \&\ (\sim R \to F)$

- As an exercise, use truth-table methods to show that (3) is valid. [Of course, the argument is also valid if we use the biconditional '$G \leftrightarrow \sim F$'.]

A	D	F	G	R	((((F→D)	&	(D→	~R))	&	(((R→A)	&	(G→A))	&	((G→	~R)	&	(A→R))))	&	(G→	~F))	→	((R→F)	&	(~R	→F))
⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊥	⊤
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⊤	⊤	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊤
⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊤	⊤	⊥	⊥	⊥	⊤
⊤	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥
⊤	⊤	⊥	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊥	⊥	⊤
⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥
⊤	⊥	⊤	⊤	⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊥	⊤
⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊤
⊤	⊥	⊤	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊥	⊤
⊤	⊥	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊤
⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊥	⊥	⊤	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊤	⊤	⊥	⊥	⊥	⊤
⊤	⊥	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥
⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊥	⊥	⊥	⊥	⊤
⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥
⊥	⊤	⊤	⊤	⊤	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊥	⊤
⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊤
⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊥	⊤
⊥	⊤	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊤
⊥	⊤	⊥	⊤	⊤	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊤	⊤	⊥	⊥	⊥	⊤
⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥
⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊥	⊥	⊤
⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊤	⊥
⊥	⊥	⊤	⊤	⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊥	⊤
⊥	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊤
⊥	⊥	⊤	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊥	⊤
⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊤
⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊤	⊤	⊥	⊥	⊥	⊤
⊥	⊥	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥
⊥	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊥	⊥	⊤
⊥	⊥	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊤	⊥

Chapter 2 in Light of Chapter 3: Example #2

Suppose no two contestants enter; then there will be no contest. No contest means no winner. Suppose all contestants perform equally well. Still no winner. There won't be a winner unless there's a loser. And conversely. Therefore, there will be a loser only if at least two contestants enter and not all contestants perform equally well.

- Here are the atomic sentences:

  T: At least two contestants enter. C: There is a contest.

  P: All contestants perform equally well.

  W: There is a winner. L: There is a loser.

- The resulting sentential form of the argument is as follows:

  $\sim T \to \sim C$. $\sim C \to \sim W$. $P \to \sim W$. $\sim L \leftrightarrow \sim W$. Therefore, $L \to (T \,\&\, \sim P)$.

- This is a valid form, as can be seen via the following truth-table, which shows that its corresponding conditional is tautologous:

C	L	P	T	W	((~T	→	~C)	&	(~C	→	~W)	&	(P →	~W)	&	(~L	↔	~W))	→	(L →	(T &	~P))
⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊥	⊥
⊤	⊤	⊤	⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊥	⊥	⊥
⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊥	⊥
⊤	⊤	⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊥	⊥	⊥
⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊤
⊤	⊤	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊤
⊤	⊤	⊥	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊥	⊤
⊤	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊥	⊥	⊤
⊤	⊥	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊥
⊤	⊥	⊤	⊤	⊥	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊥
⊤	⊥	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊥
⊤	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊥	⊥
⊤	⊥	⊥	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊤
⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤
⊤	⊥	⊥	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊤
⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊥	⊤
⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊥	⊥
⊥	⊤	⊤	⊤	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊥	⊥	⊥
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⊥	⊤	⊤	⊥	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊥	⊥	⊥
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⊥	⊤	⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊤
⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊥	⊤
⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊥	⊥	⊤
⊥	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊥
⊥	⊥	⊤	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊥
⊥	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊥
⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊥
⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊤
⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤
⊥	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊤
⊥	⊥	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤

- We saw the following premise our last argument: 'There won't be a winner unless there's a loser. And conversely.' I symbolized it as:
  - "Logish": If not L, then not W, *and conversely*. [*i.e.*, not L *iff* not W.]
  - LSL: '$\sim L \leftrightarrow \sim W$', *equivalently*: '$(\sim L \rightarrow \sim W) \,\&\, (\sim W \rightarrow \sim L)$'.
- Why not interpret the "and conversely" to be operating on the *unless* operator itself? This yields the following *different* symbolization:
  - "Logish": not W unless L, and L unless not W.
  - LSL: '$(\sim L \rightarrow \sim W) \,\&\, (\sim\sim W \rightarrow L)$', *equivalently*: '$(\sim L \rightarrow \sim W) \,\&\, (W \rightarrow L)$'.
- Answer: This is a *redundant* symbolization in LSL, since '$\sim L \rightarrow \sim W$' *is equivalent to* '$W \rightarrow L$'. Moreover, the resulting argument *isn't* valid.
- If we replace '$\sim L \leftrightarrow \sim W$' with '$\sim L \rightarrow \sim W$', then the resulting sentential form is not valid — see the truth-table on the following slide.
- **Principle of Charity**. If an argument $\mathscr{A}$ has two *plausible but semantically distinct* LSL symbolizations (where neither is *obviously* preferable) — and *only one of them is valid — choose the valid one*.

C L P T W | ((~ T → ~ C) & (~ C → ~ W) & (P → ~ W) & (~ L → ~ W)) → (L → (T & ~ P))

C	L	P	T	W	~T	→	~C	&	~C	→	~W	&	→	~W	&	~L	→	~W	**→**	→	&	~P
⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	**⊤**	⊥	⊥	⊥
⊤	⊤	⊤	⊤	⊥	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊤	**⊥**	⊥	⊥	⊥
⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	**⊤**	⊥	⊥	⊥
⊤	⊤	⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊤	⊤	**⊤**	⊥	⊥	⊥
⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊤	⊥	**⊤**	⊤	⊤	⊤
⊤	⊤	⊥	⊤	⊥	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊤	**⊤**	⊤	⊤	⊤
⊤	⊤	⊥	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊤	⊥	**⊤**	⊥	⊥	⊤
⊤	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊤	⊤	**⊤**	⊥	⊥	⊤
⊤	⊥	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊥	**⊤**	⊤	⊥	⊥
⊤	⊥	⊤	⊤	⊥	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	**⊤**	⊤	⊥	⊥
⊤	⊥	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊥	**⊤**	⊤	⊥	⊥
⊤	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊤	⊤	**⊤**	⊤	⊥	⊥
⊤	⊥	⊥	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	**⊤**	⊤	⊤	⊤
⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	**⊤**	⊤	⊤	⊤
⊤	⊥	⊥	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	**⊤**	⊤	⊥	⊤
⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊤	⊤	**⊤**	⊤	⊥	⊤
⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	**⊤**	⊥	⊥	⊥
⊥	⊤	⊤	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊤	**⊥**	⊥	⊥	⊥
⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	**⊤**	⊥	⊥	⊥
⊥	⊤	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊤	**⊥**	⊥	⊥	⊥
⊥	⊤	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊤	⊥	**⊤**	⊤	⊤	⊤
⊥	⊤	⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊤	**⊤**	⊤	⊤	⊤
⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊤	⊥	**⊤**	⊥	⊥	⊤
⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊤	**⊥**	⊥	⊥	⊤
⊥	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊥	**⊤**	⊤	⊥	⊥
⊥	⊥	⊤	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	**⊤**	⊤	⊥	⊥
⊥	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊥	**⊤**	⊤	⊥	⊥
⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	**⊤**	⊤	⊥	⊥
⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	**⊤**	⊤	⊤	⊤
⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	**⊤**	⊤	⊤	⊤
⊥	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	**⊤**	⊤	⊥	⊤
⊥	⊥	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	**⊤**	⊤	⊥	⊤