Philosophy 12A Homework Assignment #6

April 21, 2010

LPML *Proofs*

Answer the following twelve (12) questions from pages 194 and 203 of the text. You may use sequent and theorem introduction (SI/TI) on these problems.

1. p. 194 #2
2. p. 194 #4
3. p. 194 #5
4. p. 194 #6
5. p. 194 #8
6. p. 194 #11
7. p. 203 #1
8. p. 203 #3
9. p. 203 #4
10. p. 203 #7
11. p. 203 #9
12. p. 203 #11

Below, I have attached pages 194 and 203 from Frobes's text.

1	(1)	$(\forall x)(Fx \rightarrow Gx)$	Premise
2	(2)	Fa	Premise
1	(3)	$Fa \rightarrow Ga$	1 $\forall$E
1,2	(4)	Ga	3,4 $\rightarrow$E
1,2	(5)	$\sim\sim Ga$	4 SI (DN^{+})
1,2	(6)	$\sim Ga \rightarrow Ha$	5 SI (PMI)
1,2	(7)	$(\exists x)(\sim Gx \rightarrow Hx)$	6 $\exists$I ◆

This proof illustrates the standard way in which SI is used in quantificational NK: the instructions for applying the rule do not change at all, but more sequents are available as substitution instances, since we can put sentences of LMPL for the sentence-letters of the LSL sequents we use for SI. But it must be *closed* sentences of LMPL which are substituted for sentence-letters. For example, we do not permit the move from '$(\forall x)Fx$' to '$(\forall x)\sim\sim Fx$' in one line by SI, citing DN or SDN, since this involves substituting the open sentence 'Fx' for 'A' in $A \vdash_{NK} \sim\sim A$. To move from '$(\forall x)Fx$' to '$(\forall x)\sim\sim Fx$' we must instead apply $\forall$E to '$(\forall x)Fx$' to obtain, say, 'Fa', *then* use SI (DN^{+}), which yields '$\sim\sim Fa$', and then obtain '$(\forall x)\sim\sim Fx$' by $\forall$I. On the other hand, we *are* permitted to move from '$(\forall x)Fx$' to '$\sim\sim(\forall x)Fx$' in one line by SI, citing DN, since this just involves a straightforward substitution of the closed sentence '$(\forall x)Fx$' for 'A' in $A \vdash_{NK} \sim\sim A$. And of course we can also use SDN to move in one line from, say, '$(\forall x)Fx \rightarrow (\forall x)Gx$' to '$(\forall x)Fx \rightarrow \sim\sim(\forall x)Gx$', since again this just involves replacing a sentence-letter with the closed sentence '$(\forall x)Gx$'.

The rationale for restricting substitutions to closed sentences is twofold. First, while it is in fact possible to formulate a version of SI which allows replacement by open sentences, stating this version is rather involved. Secondly, there are not many occasions when the lack of this more complicated version of SI is sorely missed. But there are some, and so in the interests of keeping the frustration level down, we will later make two entirely *ad hoc* extensions to Sequent Introduction that will prove very convenient.

❑ Exercises

I Show the following.

(1) $(\forall x)Fx \;\&\; (\forall x)Gx \dashv\vdash_{NK} (\forall x)(Fx \;\&\; Gx)$
(2) $(\forall x)\sim Fx \vdash_{NK} (\exists x)(Fx \rightarrow Gx)$
*(3) $(\forall x)(Fx \rightarrow Gx) \vdash_{NK} (\forall x)Fx \rightarrow (\forall x)Gx$
(4) $(\forall x)Fx \vee (\forall x)Gx \vdash_{NK} (\forall x)(Fx \vee Gx)$
(5) $\sim(\exists x)Fx \vdash_{NK} (\forall x)\sim Fx$
(6) $(\exists x)Fx \rightarrow (\forall x)Gx \vdash_{NK} (\forall x)(Fx \rightarrow Gx)$
*(7) $(\exists x)Fx \rightarrow Ga \vdash_{NK} (\exists x)(Fx \rightarrow Gx)$
(8) $(\forall x)(\sim Fx \rightarrow \sim Kx) \vdash_{NK} (\exists x)((Fx \;\&\; Kx) \vee \sim Kx)$
(9) $(\forall x)(A \;\&\; Fx) \dashv\vdash_{NK} A \;\&\; (\forall x)Fx$
(10) $(\forall x)(A \rightarrow Fx) \dashv\vdash_{NK} A \rightarrow (\forall x)Fx$
(11) $(\forall x)(\forall y)(Fx \rightarrow Gy) \vdash_{NK} (\forall x)(Fx \rightarrow (\forall y)Gy)$
(12) $(\forall x)(\forall y)(Gy \rightarrow Fx) \vdash_{NK} (\forall x)((\forall y)Gy \rightarrow Fx)$

Why does this proof work where the previous one did not? The crucial difference is that at line 10, ∀I is being applied to the name 'c' in (9), and 'c' does not occur in either (1) or (3), on which (9) depends, so *this* application of ∀I is acceptable. Note also that the application of ∃E at line 7 is not in violation of any restriction on ∃E, for the term *t* to which restrictions apply is the one introduced in the instance at line 5, which is 'd', not 'c'; and (6) does not depend on any premise or assumption other than (5) containing 'd'. Finally, the key to this proof is the double application of ∀E to line 1. It is intrinsic to the meaning of the universal quantifier that we can infer as many instances of a universal sentence as we wish, replacing the bound variable with any name we please. This possibility is easy to overlook, but is often the way to solve harder problems.

❑ Exercises

I Show the following:

(1) $(\exists x)Fx, (\forall x)(Fx \rightarrow Gx) \vdash_{NK} (\exists x)Gx$
*(2) $(\exists x)Fx \vee (\exists x)Gx \dashv\vdash_{NK} (\exists x)(Fx \vee Gx)$ (only left-to-right solution given)
(3) $(\exists x)(Fx \;\&\; {\sim}Gx), (\forall x)(Hx \rightarrow Gx) \vdash_{NK} (\exists x)(Fx \;\&\; {\sim}Hx)$
(4) $(\forall x){\sim}Fx \vdash_{NK} {\sim}(\exists x)Fx$
(5) $(\forall x)(Fx \rightarrow (\forall y){\sim}Fy) \vdash_{NK} {\sim}(\exists x)Fx$
(6) $(\exists x)(Fx \;\&\; {\sim}Gx) \vdash_{NK} {\sim}(\forall x)(Fx \rightarrow Gx)$
(7) $(\exists x)(Fx \;\&\; Gx), (\forall x)[(\exists y)Fy \rightarrow Rx], (\forall x)[(\exists y)Gy \rightarrow Sx] \vdash_{NK} (\forall x)(Rx \;\&\; Sx)$
*(8) $(\exists x)(Fx \vee (Gx \;\&\; Hx)), (\forall x)({\sim}Gx \vee {\sim}Hx) \vdash_{NK} (\exists x)Fx$
(9) $(\exists x)(Fx \;\&\; (Gx \vee Hx)) \vdash_{NK} (\exists x)(Fx \;\&\; Gx) \vee (\exists x)(Fx \;\&\; Hx)$
(10) $(\exists x)(Fx \leftrightarrow Gx), (\forall x)(Gx \rightarrow (Hx \rightarrow Jx)) \vdash_{NK}$
$(\exists x)Jx \vee ((\forall x)Fx \rightarrow (\exists x)(Gx \;\&\; {\sim}Hx))$
(11) $(\forall x)(Fx \;\&\; (\exists y)Gy) \vdash_{NK} (\exists x)(Fx \;\&\; Gx)$
(12) $(\exists x)(Fx \;\&\; {\sim}Fx) \dashv\vdash_{NK} (\forall x)(Gx \;\&\; {\sim}Gx)$
*(13) $(\exists x)Gx \vdash_{NK} (\forall x)(\exists y)(Fx \rightarrow Gy)$
(14) $(\forall x)(Fx \rightarrow (\exists y)Gy), (\forall x)({\sim}Fx \rightarrow (\exists y)Gy) \vdash_{NK} (\exists z)Gz$
(15) $\vdash_{NK} (\forall x)((Fx \rightarrow Gx) \vee (Gx \rightarrow Fx))$
(16) $(\exists x)Fx \rightarrow (\exists x)Gx \vdash_{NK} (\exists x)(Fx \rightarrow Gx)$
*(17) $\vdash_{NK} (\exists x)(\forall y)(Fx \rightarrow Fy)$
(18) $(\forall x)(\exists y)(Fx \rightarrow Gy), (\forall x)(\exists y)({\sim}Fx \rightarrow Gy) \vdash_{NK} (\exists z)Gz$
(19) $(\forall x)(\forall y)(Gy \rightarrow Fx) \dashv\vdash_{NK} (\forall x)((\exists y)Gy \rightarrow Fx)$
(20) $(\exists x)(\forall y)(Fx \rightarrow Gy) \dashv\vdash_{NK} (\exists x)(Fx \rightarrow (\forall y)Gy)$
(21) $(\exists x)(\forall y)(Fy \rightarrow Gx) \vdash_{NK} (\forall x)(\exists y)(Fx \rightarrow Gy)$
(22) $(\exists x)(\forall y)(Fx \rightarrow Gy) \dashv\vdash_{NK} (\forall x)Fx \rightarrow (\forall x)Gx$

II Show the following:

(1) $(\exists x)(A \;\&\; Fx) \dashv\vdash_{NK} A \;\&\; (\exists x)Fx$
*(2) $(\exists x)(A \vee Fx) \dashv\vdash_{NK} A \vee (\exists x)Fx$ (only left-to-right solution given)
(3) $(\forall x)(A \vee Fx) \dashv\vdash_{NK} A \vee (\forall x)Fx$
(4) $(\forall x)Fx \rightarrow A \vdash_{NK} (\exists x)(Fx \rightarrow A)$