Homework #5 Solutions
Philosophy 12A
May 10, 2010

Page 158 #6. 'If Fermat was a French mathematician, then he was famous.' Our domain of discourse ($\mathcal{D}$), predicates (R, M, F), and individual constant (*i.e.*, the proper name of the individual person Fermat) (f) are:

$M\_$: _ is a mathematician	$R\_$: _ is French
$F\_$: _ is famous	f : Fermat
$\mathcal{D}$: people	

In "Loglish," we have 'If Rf and Mf, then Ff'. In LMPL, this becomes: '$(Rf \,\&\, Mf) \to Ff$'.

Page 158 #16. 'If no wealthy economist exists then no famous mathematician exists.' Our domain of discourse ($\mathcal{D}$) and predicates (W, E, F, M) are:

$W\_$: _ is wealthy	$E\_$: _ is an economist
$F\_$: _ is famous	$M\_$: _ is a mathematician
$\mathcal{D}$: people	

In "Loglish," we have 'If there does not exist an x such that both Wx and Ex, then there does not exist an x such that both Fx and Mx'. In LMPL, this becomes the following: '$\sim(\exists x)(Wx \,\&\, Ex) \to \sim(\exists x)(Fx \,\&\, Mx)$'.

Page 165 #5. 'If it rains, only the killjoys will be happy.' Our domain of discourse ($\mathcal{D}$), predicates (K, H), and atomic sentence letter (R) are as follows:

$K\_$: _ is a killjoy	R : 'It rains.'
$H\_$: _ is happy	$\mathcal{D}$: people

In "Loglish," we have 'If R, then only the K's will be H'. Or, in other words, 'If R, then all H's will be (the) K's'. In LMPL, this is: '$R \to (\forall x)(Hx \to Kx)$'. Here, '$R \to (\forall x)(Hx \leftrightarrow Kx)$' is also defensible, since the English sentence says '***the*** killjoys'.

Page 165 #15. 'No voter will be satisfied unless some politician who is elected is incorrupt.' Lexicon:

$E\_$: _ is elected	$P\_$: _ is a politician
$C\_$: _ is corrupt	$V\_$: _ is a voter
$S\_$: _ is satisfied	$\mathcal{D}$: people

This sentence says: $\ulcorner p$ unless $q\urcorner$, where p says 'There does not exist an x such that Vx and Sx', and q says 'There exists an x such that Ex and Px and not Cx'. In LMPL, p is '$\sim(\exists x)(Vx \,\&\, Sx)$', and q is '$(\exists x)[(Px \,\&\, Ex) \,\&\, \sim Cx]$'. Recall, $\ulcorner p$ unless $q\urcorner$ is symbolized *either* as $\ulcorner \sim q \to p\urcorner$ (p. 23) *or* as $\ulcorner p \vee q\urcorner$ (p. 57). So, both:

$$\text{`}\sim(\exists x)[(Px \,\&\, Ex) \,\&\, \sim Cx] \to \sim(\exists x)(Vx \,\&\, Sx)\text{'}$$

and

$$\text{`}\sim(\exists x)(Vx \,\&\, Sx) \vee (\exists x)[(Px \,\&\, Ex) \,\&\, \sim Cx]\text{'}$$

are acceptable.

Page 179 #5.
The existential claim '$(\exists x)(Ix \to Hx)$' is true on $\mathcal{I}$, because its instance '$Ia \to Ha$' is true on $\mathcal{I}$, since $\alpha \notin \text{Ext}(I)$.

Page 179 #9. The universal claim '$(\forall x)(\exists y)[Fx \to (Hx \vee Jy)]$' is true on $\mathcal{I}$, since all three of its instances are true on $\mathcal{I}$: (*i*) the existential claim '$(\exists y)[Fa \to (Ha \vee Jy)]$' is true on $\mathcal{I}$ because its instance '$Fa \to (Ha \vee Ja)$' is true on $\mathcal{I}$, since $\alpha \in \text{Ext}(H)$. (*ii*) '$(\exists y)[Fb \to (Hb \vee Jy)]$' is true on $\mathcal{I}$ because its instance '$Fb \to (Hb \vee Ja)$' is true on $\mathcal{I}$, since $\beta \in \text{Ext}(H)$. Finally, (*iii*) '$(\exists y)[Fc \to (Hc \vee Jy)]$' is true on $\mathcal{I}$ because its instance '$Fc \to (Hc \vee Ja)$' is true on $\mathcal{I}$, since $\alpha \in \text{Ext}(J)$.

Page 179 #12.
'$(\forall x)(\forall y)[(Fx \leftrightarrow Gy) \leftrightarrow (\exists w)(\exists z)(Hw \,\&\, Jz)]$' is false on $\mathcal{I}$, since its instance (*i*) '$(\forall y)[(Fa \leftrightarrow Gy) \leftrightarrow (\exists w)(\exists z)(Hw \,\&\, Jz)]$' is false on $\mathcal{I}$. Instance (*i*) is false on $\mathcal{I}$, because *its* instance (*i*.1) '$(Fa \leftrightarrow Ga) \leftrightarrow (\exists w)(\exists z)(Hw \,\&\, Jz)$' is false on $\mathcal{I}$. The biconditional (*i*.1) is false, because its left-side '$Fa \leftrightarrow Ga$' is false [since $\alpha \in \text{Ext}(F)$ but $\alpha \notin \text{Ext}(G)$], but its right-side ($i.1r$) '$(\exists w)(\exists z)(Hw \,\&\, Jz)$' is true. ($i.1r$) is true on $\mathcal{I}$, because its instance ($i.1r.1$) '$(\exists z)(Ha \& Jz)$' is true on $\mathcal{I}$. Finally, ($i.1r.1$) is true on $\mathcal{I}$, because *its* instance ($i.1r.1.1$) '$Ha \,\&\, Ja$' is true on $\mathcal{I}$ [since both $\alpha \in \text{Ext}(H)$ and $\alpha \in \text{Ext}(J)$].

Page 184 #6.
Interpretation $\mathcal{I}_1$ establishes that:

$$(\exists x)(Fx \leftrightarrow Gx) \nvDash (\exists x)(Fx \vee Gx)$$

($\mathcal{I}_1$)

	F	G
α	$-$	$-$

$[\mathcal{D} = \{\alpha\}, \text{Ext}(F) = \varnothing = \text{Ext}(G)]$

On $\mathcal{I}_1$, the premise '$(\exists x)(Fx \leftrightarrow Gx)$' is true, because its instance '$Fa \leftrightarrow Ga$' is true, since $\alpha \notin \text{Ext}(F)$ and $\alpha \notin \text{Ext}(G)$. But, on $\mathcal{I}_1$, the conclusion '$(\exists x)(Fx \vee Gx)$' is *false*, because its instance '$Fa \vee Ga$' is false, since $\alpha \notin \text{Ext}(F)$ and $\alpha \notin \text{Ext}(G)$.

Page 184 #8.
Interpretation $\mathcal{I}_2$ establishes that:

$$(\forall x)Fx \to (\exists x)Gx \nvDash (\forall x)(Fx \to Gx)$$

($\mathcal{I}_2$)

	F	G
α	$+$	$-$
β	$-$	$-$

$[\mathcal{D} = \{\alpha, \beta\}, \text{Ext}(F) = \{\alpha\}, \text{Ext}(G) = \varnothing]$

On $\mathcal{I}_2$, the premise '$(\forall x)Fx \to (\exists x)Gx$' is true, because its antecedent '$(\forall x)Fx$' is false, since instance 'Fb' of the antecedent is false [$\beta \notin \text{Ext}(F)$]. But, on $\mathcal{I}_2$, the conclusion '$(\forall x)(Fx \to Gx)$' is *false*, because its instance '$Fa \to Ga$' is false, since $\alpha \in \text{Ext}(F)$ but $\alpha \notin \text{Ext}(G)$.

Page 184 #21.
Interpretation $\mathcal{I}_2$ *also* establishes that:

$$(\exists x)[Fx \to (\forall y)Gy] \nvDash (\exists x)Fx \to (\forall y)Gy$$

On $\mathcal{I}_2$, the premise '$(\exists x)[Fx \to (\forall y)Gy]$' is true, because its instance '$Fb \to (\forall y)Gy$' is true, since $\beta \notin \text{Ext}(F)$. But, on $\mathcal{I}_2$, the conclusion '$(\exists x)Fx \to (\forall y)Gy$' is *false*, because its antecedent '$(\exists x)Fx$' is true [$\alpha \in \text{Ext}(F)$]; but its consequent '$(\forall y)Gy$' is false [in fact, *neither* $\alpha \in \text{Ext}(G)$ *nor* $\beta \in \text{Ext}(G)$].