Philosophy 12A Homework Assignment #5

April 8, 2010

1 LPML *Symbolizations*

Answer the following four (4) questions from pages 158 and 165 of the text.

1. p. 158 #6
2. p. 158 #16
3. p. 165 #5
4. p. 165 #15

2 Working with a *Given* LPML Interpretation

Answer the following three (3) questions from page 179 of the text.

5. #5
6. #9
7. #12

3 *Constructing* LPML Interpretations

Answer the following three (3) questions from page 184 of the text.

8. #6
9. #8
10. #21

I have attached pages 158, 165, 179, and 184 from the 4th printing of *Modern Logic* to the end of this homework assignment.

❑ Exercises

Symbolize each of the following sentences using names, predicates and the existential quantifier, as appropriate. State your dictionary and say what domain your quantifiers are relativized to. Show at least one intermediate step in Loglish for each example.

(1) Some mathematicians are famous.
(2) Some mathematicians are not famous.
*(3) There is no mathematician who is famous.
(4) Some Germans are famous mathematicians.
(5) Gödel was a famous German mathematician.
(6) If Fermat was a French mathematician, then he was famous.
(7) Ada Lovelace was a brilliant English mathematician but she was not famous.
(8) Some famous mathematicians are neither German nor French.
(9) New Orleans is polluted but not smoggy.
(10) Some cities are smoggy and polluted.
(11) Some polluted cities are smoggy.
*(12) Some polluted cities are smoggy and some aren't.
(13) No smoggy city is unpolluted.
(14) No city is smoggy if it is unpolluted.
*(15) If a wealthy economist exists so does a famous mathematician.
(16) If no wealthy economist exists then no famous mathematician exists.
(17) Vampires don't exist.
(18) Nothing is both a ghost and a vampire.
(19) There aren't any ghosts, nor vampires either.
(20) If ghosts and vampires don't exist then nothing can be a ghost without being a vampire.

3 More symbolizations: the universal quantifier

The other arguments of §1, B and D, contain the quantifier 'everyone'. In Loglish, 'every' becomes 'for every _' and so the premise of B, 'everyone is happy', is rendered

(1.a) For every x, x is happy

relativizing 'for every _' to the domain of people. In place of 'every' we may have 'each', or 'any', or 'all'. To turn (1.a) into a sentence of LMPL, we need a symbol for 'for every_'; the symbol we use is '$\forall$', and so we obtain

(1.s) $(\forall x)Hx$.

'$\forall$' is called the *universal* quantifier, and a sentence like (1.s) with '$\forall$' as its main connective is called a *universal sentence*.

The following two examples parallel (2.9) and (2.10) at the syntactic level:

(2) Everyone is happy and everyone is wise.

❑ Exercises

I Translate each of the following sentences in LMPL using names, predicates and quantifiers as appropriate. State your dictionary and the domain to which your quantifiers are relativized. Show at least one intermediate step in Loglish for each example.

(1) All donkeys are stubborn.
(2) Every inflationary economy is faltering. ('I_', 'E_', 'F_')
*(3) Only *private* universities are expensive. ('P_', 'U_', 'E_')
(4) Whales are mammals.
(5) If it rains, only the killjoys will be happy. ('A': it rains)
(6) It's always the men who are overpaid.
*(7) All that glitters is not gold. ('G_': _ glitters; 'O_': _ is gold)
(8) If a woman is elected, someone will be happy.
(9) If a woman is elected, she will be happy.
(10) If Mary is elected, the directors will all resign.
(11) No corrupt politicians were elected.
*(12) None but corrupt politicians were elected.
(13) If any elected politician is corrupt, no voter will be satisfied.
(14) If an elected politician is corrupt, he will not be re-elected.
(15) No voter will be satisfied unless some politician who is elected is incorrupt.
*(16) Invariably, a wealthy logician is a textbook author.
(17) Any logician who is a textbook author is wealthy.
(18) Occasionally, a logician who is a textbook author is wealthy.
(19) Among the wealthy, the only logicians are textbook authors.
(20) Except for textbook authors, no logicians are wealthy.

II With the same symbols as in (7), (2) and (5) respectively, translate the following formulae into idiomatic sentences of ordinary English.

(a) $(\forall x)(Gx \rightarrow \sim Ox)$ (b) $\sim(\exists x)((Fx \& Ex) \& \sim Ix)$ (c) $A \rightarrow (\forall x)(\sim Hx \rightarrow \sim Kx)$

4 The syntax of LMPL

We take the same perspective on the language of monadic predicate logic as we took on the language of sentential logic: LMPL is a language in its own right, with its own lexicon and its own formation rules for well-formed formulae. The lexicon includes the new categories of symbol which have been introduced in the previous examples:

The lexicon of LMPL:

> All items in the lexicon of LSL, now including '$\curlywedge$'; an unlimited supply of individual variables 'x', 'y', 'z', 'x′', 'y′', 'z′'...; an unlimited supply of individual constants 'a', 'b', 'c', 'a′', 'b′', 'c′'...; an unlimited supply of monadic predicate-letters 'A_1', 'B_1', 'C_1',...,'A_1'', 'B_1'', 'C_1'',...; quantifier symbols '$\forall$' and '$\exists$'.

body of the formula. This makes the search process longer. Since (6) is existential, we can find out whether it is true or false by finding out whether or not it has a true instance. The instances of (6) are

(6a) $(\forall x)(Ga \,\&\, (Jx \rightarrow (Ix \vee Fx)))$
(6b) $(\forall x)(Gb \,\&\, (Jx \rightarrow (Ix \vee Fx)))$
(6c) $(\forall x)(Gc \,\&\, (Jx \rightarrow (Ix \vee Fx)))$.

Each of these instances is a universal sentence, and each of them in turn has three instances. For example, the instances of (6b) are:

($6b_1$) $Gb \,\&\, (Ja \rightarrow (Ia \vee Fa))$
($6b_2$) $Gb \,\&\, (Jb \rightarrow (Ib \vee Fb))$
($6b_3$) $Gb \,\&\, (Jc \rightarrow (Ic \vee Fc))$

Had (6) been false, therefore, we would have had to consider a total of nine quantifier-free sentences to confirm this. Fortunately, (6) is true, and this is shown by its instance (6a).[2]

❑ Exercise

Evaluate the numbered formulae in the displayed interpretation. Explain your reasoning in the same way as in (1)–(6) above, accounting for the truth-values of quantified sentences in terms of the truth-values of their instances.

	F	G	H	I	J
α	+	–	+	–	+
β	+	–	+	–	–
γ	+	–	–	+	+

(1) $(Ha \vee Hc) \rightarrow Ib$
(2) $(Ha \,\&\, Hc) \vee (Ja \,\&\, Jc)$
*(3) $(\exists x)(Fx \,\&\, Gx)$
(4) $\sim(\exists x)Gx$
(5) $(\exists x)(Ix \rightarrow Hx)$
(6) $(\forall x)((Hx \vee Ix) \rightarrow Fx)$
(7) $(\forall x)((Fx \,\&\, Hx) \rightarrow Jx)$
*(8) $(\forall x)(Hx \rightarrow (\exists y)(Jx \,\&\, Iy))$
(9) $(\forall x)(\exists y)(Fx \rightarrow (Hx \vee Jy))$
(10) $(\exists x)Ix \rightarrow (\forall x)(Jx \rightarrow Ix)$
(11) $(\exists x)(Ix \rightarrow (\forall y)(Jy \rightarrow Iy))$
(12) $(\forall x)(\forall y)((Fx \leftrightarrow Gy) \leftrightarrow (\exists w)(\exists z)(Hw \,\&\, Jz))$

[2] The rules of this section explain the term 'logical constant' mentioned earlier, which is applied both to quantifiers and to sentential connectives. Domains and extensions of predicates *vary* from interpretation to interpretation, but the evaluation rules for connectives and quantifiers are *constant* across all interpretations. Any expression which has a constant evaluation rule is called a logical constant.

To summarize: '$(\exists x)(\forall y)(Fx \rightarrow Gy)$' has two instances, (i) '$(\forall y)(Fa \rightarrow Gy)$' and (ii) '$(\forall y)(Fb \rightarrow Gy)$', and it is true because (ii) is true. (ii) is true because it has two instances, '$Fb \rightarrow Ga$' and '$Fb \rightarrow Gb$' and both are true since both have false antecedents. On the other hand, '$(\forall x)(\exists y)(Fx \rightarrow Gy)$' is false. It has two instances, (iii) '$(\exists y)(Fa \rightarrow Gy)$' and (iv) '$(\exists y)(Fb \rightarrow Gy)$', and (iii) is false. (iii) is false because both its instances, '$(Fa \rightarrow Ga)$' and '$Fa \rightarrow Gb$', are false, since both have true antecedent and false consequent.

It is noticeable that all our problems of showing failure of semantic consequence have been solved with small domains, whereas in Chapter 5, the domains with respect to which our symbolizations are relativized are large: people, places, things. But counterexamples with small domains to argument-forms derived from symbolizations of English relativized to large domains are not irrelevant to English arguments, for if the argument-form can be shown to be invalid by an interpretation with a small domain, then it *is* shown to be invalid, and if it is the form of an English argument, it follows that that English argument is monadically invalid. Moreover, a counterexample with a small domain can be 'blown up' into one with a large domain by a duplication process (see Exercise II.2), so our preference for simplicity does not entail irrelevance.

❑ Exercises

I Show the following, with explanations:

(1) $(\forall x)(Fx \rightarrow Gx) \nvDash (\forall x)(Gx \rightarrow Fx)$
(2) $(\forall x)(Fx \vee Gx), (\forall x)(Fx \vee Hx) \nvDash (\forall x)(Gx \vee Hx)$
(3) $(\forall x)(Fx \rightarrow \sim Gx), (\forall x)(Gx \rightarrow Hx) \nvDash (\forall x)(Fx \rightarrow \sim Hx)$
*(4) $(\forall x)((Fx \& Gx) \rightarrow Hx) \nvDash (\forall x)(Fx \vee Gx) \vee (\forall x)(Fx \vee Hx)$
(5) $(\exists x)(Fx \& \sim Hx), (\exists x)(Gx \& \sim Hx) \nvDash (\exists x)(Fx \& Gx)$
(6) $(\exists x)(Fx \leftrightarrow Gx) \nvDash (\exists x)(Fx \vee Gx)$
(7) $(\exists x)(Fx \& Gx), (\forall x)(Gx \rightarrow Hx) \nvDash (\forall x)(Fx \rightarrow Hx)$
(8) $(\forall x)Fx \rightarrow (\exists x)Gx \nvDash (\forall x)(Fx \rightarrow Gx)$
(9) $(\exists x)(Fx \vee Gx), (\forall x)(Fx \rightarrow \sim Hx), (\exists x)Hx \nvDash (\exists x)Gx$
(10) $(\forall x)(Fx \rightarrow Gx) \nvDash \sim(\forall x)(Fx \rightarrow \sim Gx)$
(11) $(\exists x)\sim Fx \nvDash \sim(\exists x)Fx$
(12) $\sim(\forall x)Fx \nvDash (\forall x)\sim Fx$
*(13) $(\forall x)(Fx \rightarrow Gx) \rightarrow (\forall x)(Hx \rightarrow Jx) \nvDash (\exists x)(Fx \& Gx) \rightarrow (\forall x)(Hx \rightarrow Jx)$
(14) $(\exists x)(Fx \rightarrow A), (\exists x)(A \rightarrow Fx) \nvDash (\forall x)(A \leftrightarrow Fx)$
(15) $\sim(A \rightarrow (\forall x)Fx) \nvDash (\forall x)(A \rightarrow \sim Fx)$
(16) $(\forall x)Fx \leftrightarrow A \nvDash (\forall x)(Fx \leftrightarrow A)$
(17) $(\forall x)Fx \rightarrow (\forall x)Gx \nvDash Fa \rightarrow (\forall x)Gx$
(18) $Fa \rightarrow (\exists x)Gx \nvDash (\exists x)Fx \rightarrow (\exists x)Gx$
(19) $(\forall x)Fx \leftrightarrow (\forall x)Gx \nvDash (\exists x)(Fx \leftrightarrow Gx)$
*(20) $(\forall x)Fx \rightarrow (\exists y)Gy \nvDash (\forall x)(Fx \rightarrow (\exists y)Gy)$
(21) $(\exists x)(Fx \rightarrow (\forall y)Gy) \nvDash (\exists x)Fx \rightarrow (\forall y)Gy$
(22) $\sim(\exists x)Fx \vee \sim(\exists x)Gx \nvDash \sim(\exists x)(Fx \vee Gx)$