Homework Assignment #4

March 18, 2010

Answer the following six (6) exercises from pp. 127–128 of the text. You *may* (but, you do not *have to*) use Theorem and/or Sequent Introduction on any of these problems. The only Theorems/Sequents you can use are those listed on page 123 of the text.

[NOTE: Two (2) extra credit points (for each proof) will be awarded to those of you who find the *shortest* proof in the class (all those *tied* for shortest will get extra credit).]

1. III.3
2. III.5
3. III.7
4. III.9
5. III.14
6. IV.3

Just to make sure we're on the same page (some people have an earlier printing of the text), I have included the entire problem set from the textook, below:

III Show the following. Wherever you apply Sequent Introduction, be sure to indicate which previously proved sequent you are using.

(1) $\sim A \vdash_{NK} \sim B \rightarrow \sim(A \vee B)$
(2) $A \rightarrow (B \vee \sim C), \sim A \rightarrow (B \vee \sim C), \sim B \vdash_{NK} \sim C$
(3) $\vdash_{NK} A \vee (A \rightarrow B)$
*(4) $\sim B \rightarrow A \vdash_{NK} (B \rightarrow A) \rightarrow A$
(5) $\vdash_{NK} (A \rightarrow B) \rightarrow [(A \rightarrow \sim B) \rightarrow \sim A]$
(6) $\sim[A \rightarrow (B \vee C)] \vdash_{NK} (B \vee C) \rightarrow A$
(7) $A \rightarrow B, (\sim B \rightarrow \sim A) \rightarrow (C \rightarrow D), \sim D \vdash_{NK} \sim C$
*(8) $(A \vee B) \rightarrow (A \vee C) \vdash_{NK} A \vee (B \rightarrow C)$
(9) $(A \,\&\, B) \leftrightarrow C, \sim(C \vee \sim A) \vdash_{NK} \sim B$
(10) $A \rightarrow (B \vee C) \dashv\vdash_{NK} (A \rightarrow B) \vee (A \rightarrow C)$

(11) $\sim(A \,\&\, \sim B) \vee \sim(\sim D \,\&\, \sim E), \sim(E \vee B), C \rightarrow (\sim E \rightarrow (\sim D \,\&\, A)) \vdash_{NK} \sim C$
(12) $(A \vee B) \rightarrow (C \,\&\, D), (\sim E \vee C) \rightarrow [(F \vee G) \rightarrow H],$
$(\sim I \rightarrow J) \rightarrow [G \,\&\, (H \rightarrow \sim K)] \vdash_{NK} K \rightarrow (\sim A \vee \sim I)$
*(13) $(A \leftrightarrow B) \leftrightarrow (C \leftrightarrow D) \vdash_{NK} (A \leftrightarrow C) \leftrightarrow (B \leftrightarrow D)$
(14) $(A \vee B) \,\&\, (C \vee D) \vdash_{NK} (B \vee C) \vee (A \,\&\, D)$
(15) $\sim(A \leftrightarrow B), \sim(B \leftrightarrow C), \sim(C \leftrightarrow A) \vdash_{NK} \curlywedge$

IV Symbolize the following arguments (state your dictionary explicitly). Then give proofs of the resulting argument-forms.

(3) If God is willing to prevent evil but is unable to do so, He is impotent. If God is able to prevent evil but unwilling to do so, He is malevolent. If He is neither able nor willing, then he is both impotent and malevolent. Evil exists if and only if God is unwilling or unable to prevent it. God exists only if He is neither impotent nor malevolent. Therefore, if God exists evil does not.