Homework #3 Solutions

Philosophy 12A
March 8, 2010

Part One

1. $A \to C$
   $B \to C$
   $A \vee B$
   $\therefore C$

   is a *valid* argument. There are no rows in which all of its premises are true and its conclusion is false:

A	B	C	$A \to C$	$B \to C$	$A \vee B$	C
⊤	⊤	⊤	⊤	⊤	⊤	⊤
⊤	⊤	⊥	⊥	⊥	⊤	⊥
⊤	⊥	⊤	⊤	⊤	⊤	⊤
⊤	⊥	⊥	⊥	⊤	⊤	⊥
⊥	⊤	⊤	⊤	⊤	⊤	⊤
⊥	⊤	⊥	⊤	⊥	⊤	⊥
⊥	⊥	⊤	⊤	⊤	⊥	⊤
⊥	⊥	⊥	⊤	⊤	⊥	⊥

2. $I \to N$
   $(\sim K \vee D) \leftrightarrow N$
   $D \to \sim I$
   $\therefore \sim I \to (N \to K)$

   is an *in*valid argument. Here's a row that makes all its premises true and its conclusion false:

D	I	K	N	$I \to N$	$(\sim K \vee D) \leftrightarrow N$	$D \to \sim I$	$\sim I \to (N \to K)$
⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊥

3. $(\sim O \to \sim S) \,\&\, (O \to (M \,\&\, \sim I))$
   $\sim I \to \sim M$
   $\therefore \sim S$

   is a *valid* argument. For the conclusion '$\sim S$' to be false, we require 'S' to be true. Now, for the first premise of this argument to be true, its first conjunct '$\sim O \to \sim S$' must be true. Hence, the antecedent '$\sim O$' of this first conjunct must be false (since its consequent '$\sim S$' has already been assumed to be false). So 'O' must be true. But, if 'O' is true, then in order for the second conjunct of the first premise to be true, its consequent '$M \,\&\, \sim I$' must be true. This forces 'M' to be true and 'I' to be false. But, that forces the second premise '$\sim I \to \sim M$' to be false. Therefore, there is no way to make both premises true while the conclusion is false, and the argument is valid.

Part Two: Page 66 I

1. **Answer**. $A \to B, B \to (C \vee D), \sim D \vDash A \to C$.
   "Short Method" Explanation. For the conclusion '$A \to C$' to be false, we require 'A' to be true, and 'C' to be false. For premise '$\sim D$' to be true, we require 'D' to be false. Therefore, for premise '$B \to (C \vee D)$' to be true we require its antecedent 'B' to be false (since our assumptions so far already force its consequent '$C \vee D$' to be false). But, all of this forces premise '$A \to B$' to be false, which means it's impossible for all the premises of this sequent to be true while its conclusion is false. Therefore, the conclusion *is* a semantic consequence of the premises.

5. Answer. $A \vee (B \,\&\, C), C \vee (D \,\&\, E), (A \vee C) \to (\sim B \vee \sim D) \nvDash B \,\&\, D$.
   "Short Method" Explanation. For the conclusion to be false at least one of 'B' and 'D' must be false, so at least one of 'A' and 'C' must be true if the first two premises are to be true. This means premise three '$(A \vee C) \to (\sim B \vee \sim D)$' has a true antecedent, but its consequent is true anyway, so the conclusion is *not* a semantic consequence of the premises. Specifically, here is an interpretation that makes all the premises true and the conclusion false.

A	B	C	D	E	$A \vee (B \,\&\, C)$	$C \vee (D \,\&\, E)$	$(A \vee C) \to (\sim B \vee \sim D)$	$B \,\&\, D$
⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊥

8. **Answer**. $(A \leftrightarrow B) \vee (B \leftrightarrow C) \nvDash A \leftrightarrow (B \vee C)$.
   "Short Method" Explanation. For the conclusion to be false, either (a) 'A' is true and '$B \vee C$' is false, or (b) 'A' is false and '$B \vee C$' is true. In case (a) the premise is true since its second disjunct '$B \leftrightarrow C$' is true (because 'B' and 'C' are both false), so the conclusion is *not* a semantic consequence of the premises. Specifically, here is an interpretation that makes all the premises true and the conclusion false.

A	B	C	$(A \leftrightarrow B) \vee (B \leftrightarrow C)$	$A \leftrightarrow (B \vee C)$
⊤	⊥	⊥	⊤	⊥