Homework #2 Solutions

February 22, 2010

Page 19: Exercises 1, 3, 5 *&* 12.

1. E: Van Gogh's pictures are the world's most valuable.
   P: Van Gogh's pictures are the world's most profound.
   Final symbolization: $E \,\&\, {\sim}P$.

3. Same lexicon as in (1).
   Final symbolization: ${\sim}E \,\&\, {\sim}P$.

5. D: Digital computers can simulate every aspect of human intelligence.
   N: Neural networks can simulate every aspect of human intelligence.
   E: Digital computers can simulate some aspects of human intelligence.
   O: Neural networks can simulate some aspects of human intelligence.
   Final symbolization: $({\sim}D \,\&\, {\sim}N) \,\&\, (E \,\&\, O)$.

12. R: It rains.
    P: It pours.
    Final symbolization: $R \rightarrow P$. [also acceptable: anything equivalent, since the form is not obvious from the English sentence — *e.g.*, ${\sim}(R \,\&\, {\sim}P)$]

Page 26: Exercises 2, 4, 6 *&* 16.

2. T: It is Tuesday.
   B: It is Belgium.
   L: I'm lost.
   Final symbolization: $(T \,\&\, {\sim}B) \rightarrow L$.

4. D: The economy declines.
   C: There is a change of leadership.
   R: There will be a recession.
   Final symbolization: $D \rightarrow ({\sim}C \rightarrow R)$. [also acceptable: $D \rightarrow (C \vee R)$]

6. A: Applicants may examine their dossiers.
   W: Applicants have already waived their right to examine their dossiers.
   R: Applicants' referees approve.
   Final symbolization: $A \rightarrow ({\sim}W \,\&\, R)$.

16. R: There is a right to smoke in public.
    H: Smoking in public significantly affects the health of others.
    Final symbolization: $(R \rightarrow {\sim}H) \,\&\, {\sim}{\sim}H$. [note: we want to stay as close to the English *form* as possible.]

Page 33: Exercises 1, 5, 10 *&* 12.

1. G: The government rigs the election.
   R: There will be riots.
   V: The government is guaranteed victory.
   Final symbolization (argument): $G \rightarrow R,\ {\sim}G \rightarrow V,\ {\sim}G \rightarrow {\sim}V \therefore R$

5. K: I know I exist.
   E: I exist.
   H: I think.
   N: I know I think.
   Final symbolization (argument): $K \rightarrow E,\ (N \rightarrow K) \,\&\, (H \rightarrow N),\ H \therefore E$

10. N: At least two contestants enter.
    C: There will be a contest.
    W: There will be a winner.
    E: All contestants perform equally well.
    L: There is a loser.
    Final symbolization (argument): ${\sim}N \rightarrow {\sim}C,\ {\sim}C \rightarrow {\sim}W,\ E \rightarrow {\sim}W,\ {\sim}L \leftrightarrow {\sim}W \therefore L \rightarrow (N \,\&\, {\sim}E)$

12. D: The Mayor is defeated.
S: Council members are involved in a financial scandal.
U: The urban middle class supports the Mayor.
Final symbolization (argument): $\sim D \rightarrow U,\ U \rightarrow \sim S \therefore D \leftrightarrow S$

Page 43, IV: 2 *&* 6

2. This is an erroneous usage of scare quotes. 'Rome' is not the largest city in Italy, since 'Rome' a *word*, *not a city*.
6. This is a correct usage of selective (corner) quotes. It's a true (metatheoretic) statement. [If we had used scare quotes here instead, then *that* would have been a mistake, since '$\sim p$' is *not* a wff of LSL, because there are only upper case letters in LSL.]

Page 57, I: 1 *&* 5

1. Truth-Table for '$A \rightarrow (B \rightarrow (A \& B))$' (main connective in red):

A	B	A	$\rightarrow$	$(B$	$\rightarrow$	$(A$	$\&$	$B))$
⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤
⊤	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊥
⊥	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤
⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊥	⊥

$\therefore$ '$A \rightarrow (B \rightarrow (A \& B))$' is *tautological* (it is true on all interpretations).

5. Truth-Table for '$((F \& G) \rightarrow H) \rightarrow ((F \vee G) \rightarrow H)$' (main connective in red):

F	G	H	$((F$	$\&$	$G)$	$\rightarrow$	$H)$	$\rightarrow$	$((F$	$\vee$	$G)$	$\rightarrow$	$H)$
⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤
⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊥
⊤	⊥	⊤	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊤
⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊥	⊥
⊥	⊤	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊤
⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊥	⊥
⊥	⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊥	⊥	⊥	⊤	⊤
⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥

$\therefore$ '$((F \& G) \rightarrow H) \rightarrow ((F \vee G) \rightarrow H)$' is *contingent* (it is true on some interpretations, false on others).

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II. Truth-Tables for the sentences in question (main connectives in red):

(1)

A	B	A	$\vee$	B
⊤	⊤	⊤	⊤	⊤
⊤	⊥	⊤	⊤	⊥
⊥	⊤	⊥	⊤	⊤
⊥	⊥	⊥	⊥	⊥

(2)

A	B	A	$\rightarrow$	B
⊤	⊤	⊤	⊤	⊤
⊤	⊥	⊤	⊥	⊥
⊥	⊤	⊥	⊤	⊤
⊥	⊥	⊥	⊤	⊥

(3)

A	B	$\sim$	$(A$	$\&$	$\sim$	$B)$
⊤	⊤	⊤	⊤	⊥	⊥	⊤
⊤	⊥	⊥	⊤	⊤	⊤	⊥
⊥	⊤	⊤	⊥	⊥	⊥	⊤
⊥	⊥	⊤	⊥	⊥	⊤	⊥

(4)

A	B	$\sim$	$(\sim$	A	$\&$	$\sim$	$B)$
⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊤
⊤	⊥	⊤	⊥	⊤	⊥	⊤	⊥
⊥	⊤	⊤	⊤	⊥	⊥	⊥	⊤
⊥	⊥	⊥	⊤	⊥	⊤	⊤	⊥

(5)

A	B	$\sim$	A	$\vee$	B
⊤	⊤	⊥	⊤	⊤	⊤
⊤	⊥	⊥	⊤	⊥	⊥
⊥	⊤	⊤	⊥	⊤	⊤
⊥	⊥	⊤	⊥	⊤	⊥

(6)

A	$\sim$	A	$\vee$	A
⊤	⊥	⊤	⊤	⊤
⊥	⊤	⊥	⊤	⊥

(7)

A	$(A$	$\rightarrow$	$(A$	$\&$	$\sim$	$A))$	$\rightarrow$	$\sim$	A
⊤	⊤	⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊤
⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊤	⊤	⊥

Therefore, we have the following equivalences:

- (6) and (7) are equivalent.
- (1) and (4) are equivalent.
- (2), (3), and (5) are equivalent.

Page 58, III

III. No, if p is not a tautology, it does *not* follow that $\ulcorner \sim p \urcorner$ is a tautology. This is equivalent to the metatheoretic question: "If $\nvDash p$, then does it follow that $\vDash \sim p$?". There are LSL sentences such that *both* $\nvDash p$ *and* $\nvDash \sim p$. Any atomic wff (*e.g.*, 'A') will do. More generally, any *contingent* sentence p will, by definition, be such that *both* $\nvDash p$ *and* $\nvDash \sim p$.