


Announcements and Such

- Today's Music: *Neil Young*
 - HW #4 is due on Thursday @ 4pm, usual drill (chapter 4 — proofs).
 - I've posted my solutions for HW #2 and HW #3.
 - Grade Curve (so far). Take the average of:
 - (1) your average HW score (all on 100-point scale), and
 - (2) your mid-term score.
 - The approximate "curve" for the course is as follows:
A-ish (≥ 90), B-ish (80-90), C-ish (70-80), D-ish (60-70).
 - This should be a reasonably good (but not perfect) guide to where the (overall) grade curve will end-up for the entire course.
 - Today: Chapter 4, Continued
 - More on \sim and \vee rules. Sequent & Theorem Introduction (SI/TI).
-  **You should be doing as many proofs as you can.**

The Three Negation Rules

Negation Elimination (\sim E)	Negation Introduction (\sim I)
a_1, \dots, a_n (j) $\sim q$ \vdots b_1, \dots, b_u (k) q \vdots	j (j) p Assumption \vdots a_1, \dots, a_n (k) \wedge \vdots
$a_1, \dots, a_n, b_1, \dots, b_u$ (m) \wedge j, k \sim E	$\{a_1, \dots, a_n\}/j$ (m) $\sim p$ j, k \sim I

Double Negation (DN)

a_1, \dots, a_n	(j)	$\sim \sim p$
a_1, \dots, a_n	(k)	p j DN

The Two Disjunction Rules

a_1, \dots, a_n	(j)	p		a_1, \dots, a_n	(j)	q		
	⋮				⋮			
a_1, \dots, a_n	(k)	$p \vee q$	j \vee I	OR	a_1, \dots, a_n	(k)	$p \vee q$	j \vee I

a_1, \dots, a_n	(g)	$p \vee q$	
	⋮		
h	(h)	p	Assumption
	⋮		
b_1, \dots, b_u	(i)	r	
	⋮		
j	(j)	q	Assumption
	⋮		
c_1, \dots, c_w	(k)	r	
	⋮		
\mathcal{A}	(m)	r	g, h, i, j, k \vee E

$[\mathcal{A} = \{a_1, \dots, a_n\} \cup \{b_1, \dots, b_u\}/h \cup \{c_1, \dots, c_w\}/j]$

A Simple Example Involving \vee I and \vee E

- Here's a proof of the sequent: $A \vee B \vdash B \vee A$.

Problem is: $A \vee B \vdash B \vee A$

1	(1)	$A \vee B$	Premise
2	(2)	A	Assumption (\vee E)
2	(3)	$B \vee A$	2 \vee I
4	(4)	B	Assumption (\vee E)
4	(5)	$B \vee A$	4 \vee I
1	(6)	$B \vee A$	1,2,3,4,5 \vee E

An Example Involving \vee E and DN

- Here's a proof of the sequent: $A \vee B, \sim B \vdash A$.

Problem is: $A \vee B, \sim B \vdash A$

1	(1)	$A \vee B$	Premise
2	(2)	$\sim B$	Premise
3	(3)	$\sim A$	Assumption (for \sim I)
4	(4)	A	Assumption (for \vee E)
3,4	(5)	Δ	3,4 \sim E
6	(6)	B	Assumption (for \vee E)
2,6	(7)	Δ	2,6 \sim E
1,2,3	(8)	Δ	1,4,5,6,7 \vee E
1,2	(9)	$\sim \sim A$	3,8 \sim I
1,2	(10)	A	9 DN

A Tricky Example Involving \vee I and All 3 Negation Rules

- Here's a proof of the *theorem*: $\vdash A \vee \sim A$.

Problem is: $\vdash A \vee \sim A$

1	(1)	$\sim(A \vee \sim A)$	Assumption (\sim I)
2	(2)	A	Assumption (\sim I)
2	(3)	$A \vee \sim A$	2 \vee I
1,2	(4)	Δ	1,3 \sim E
1	(5)	$\sim A$	2,4 \sim I
1	(6)	$A \vee \sim A$	5 \vee I
1	(7)	Δ	1,6 \sim E
	(8)	$\sim\sim(A \vee \sim A)$	1,7 \sim I
	(9)	$A \vee \sim A$	8 DN

A Third Example Involving \vee I and \vee E

- Here's a proof of the sequent: $A \vee (B \& C) \vdash (A \vee B) \& (A \vee C)$.

1	(1)	$A \vee (B \& C)$	Premise
2	(2)	A	Assumption (\vee E)
2	(3)	$A \vee B$	2 \vee I
2	(4)	$A \vee C$	2 \vee I
2	(5)	$(A \vee B) \& (A \vee C)$	3,4 $\&$ I
6	(6)	$B \& C$	Assumption (\vee E)
6	(7)	B	6 $\&$ E
6	(8)	$A \vee B$	7 \vee I
6	(9)	C	6 $\&$ E
6	(10)	$A \vee C$	9 \vee I
6	(11)	$(A \vee B) \& (A \vee C)$	8,10 $\&$ I
1	(12)	$(A \vee B) \& (A \vee C)$	1,2,5,6,11 \vee E

Another Example Involving \vee

- Let's do a proof of: $(A \& B) \vee (A \& C) \vdash A \& (B \vee C)$

1	(1) $(A \& B) \vee (A \& C)$	Premise
2	(2) $A \& B$	Ass ($\vee E$)
2	(3) A	2 &E
4	(4) $A \& C$	Ass ($\vee E$)
4	(5) A	4 &E
1	(6) A	1,2,3,4,5 $\vee E$
2	(7) B	2 &E
2	(8) $B \vee C$	7 $\vee I$
4	(9) C	4 &E
4	(10) $B \vee C$	9 $\vee I$
1	(11) $B \vee C$	1,2,8,4,10 $\vee E$
1	(12) $A \& (B \vee C)$	6,11 &I

A Final Example Involving \vee and \sim

- Let's do a proof of: $\sim A \vee B \vdash A \rightarrow B$

Problem is : $\sim A \vee B \vdash A \rightarrow B$

1	(1) $\sim A \vee B$	Premise
2	(2) A	Assumption (\rightarrow I)
3	(3) $\sim A$	Assumption (\vee E)
4	(4) $\sim B$	Assumption (\sim I)
2,3	(5) Δ	3,2 \sim E
2,3	(6) $\sim \sim B$	4,5 \sim I
2,3	(7) B	6 DN
8	(8) B	Assumption (\vee E)
1,2	(9) B	1,3,7,8,8 \vee E
1	(10) $A \rightarrow B$	2,9 \rightarrow I

General Tips on Proof Strategy and Planning

- As a first line of attack, always try to prove your conclusion by using the introduction rule for its main connective as the main strategy.
- This will indicate what assumptions, if any, need to be made and what other formulae will need to be derived. This is “working backward”.
- If these other formulae also contain connectives, then try to prove them by introducing their main connectives. Work backward, as far as possible.
- When this technique can no longer be applied, inspect your current stock of premises and assumptions to see if they have any *obvious* consequences.
- If your current premises and assumption contain a disjunction $\lceil r \vee s \rceil$, see if you can prove your current goal formula p from *each* of its disjuncts r and s (using your current premises and assumptions). If you think you can, then try using $\vee E$ to prove p . If no disjunction appears anywhere in your current of premises/assumptions, then $\vee E$ is probably not a good strategy.
- If you have tried everything you can think of to prove your current goal p , try assuming $\lceil \sim p \rceil$ and aim for $\lceil \sim \sim p \rceil$ by $\sim E$, $\sim I$; then use DN.

When to Make Assumptions, and When *Not* to

- In constructing a proof, any assumptions you make must eventually be discharged, so you should only make assumptions in connection with the three rules which discharge assumptions.
- In other words, if you make an assumption p in a proof, you *must* be able to give one of the following three reasons:
 1. p is the antecedent of a conditional ' $p \rightarrow q$ ' you are trying to derive using the \rightarrow I rule (then, try to prove q).
 2. You are trying to derive ' $\sim p$ ', so you assume p with an eye toward using the \sim I rule (then, try to prove \wedge).
 3. p is one of the disjuncts of a disjunction ' $p \vee q$ ' (*somewhere in your current stock of premises and assumptions!*) to which you will be applying \vee E (then, try to prove some r from each).
- Remember, only the three rules \rightarrow I, \sim I, and \vee E involve making assumptions. *No other rules can discharge assumptions.*

10 More Examples Involving \vee I and \vee E

1. $(A \& B) \vee (A \& C) \vdash A$ [p. 111, ex. 2]
2. $(A \rightarrow \perp) \vee (B \rightarrow \perp), B \vdash \sim A$ [p. 116, §4.5, ex. 11]
3. $(A \vee B) \vee C \vdash A \vee (B \vee C)$ [p. 116, ex. 19]
4. $A \vee B \vdash (A \rightarrow B) \rightarrow B$ [p. 116, ex. 10]
5. $A \& B \vdash \sim(\sim A \vee \sim B)$ [p. 116, ex. 14 (\vdash)]
6. $A \vee B \vdash \sim(\sim A \& \sim B)$ [p. 116, ex. 13]
7. $\sim(A \& B) \vdash \sim A \vee \sim B$ [p. 116, ex. 16 (\dashv)]
8. $\sim C \vee (A \rightarrow B) \vdash (C \& A) \rightarrow B$ [not in text]
9. $\vdash (A \rightarrow B) \vee (B \rightarrow A)$ [not in text]
10. $\sim(A \vee B) \vdash \sim A \& \sim B$ [not in text]

Proof of Example #1

Problem is: $(A \& B) \vee (A \& C) \vdash A$

1	(1) $(A \& B) \vee (A \& C)$	Premise
2	(2) $A \& B$	Assumption ($\vee E$)
2	(3) A	2 &E
4	(4) $A \& C$	Assumption ($\vee E$)
4	(5) A	4 &E
1	(6) A	1,2,3,4,5 $\vee E$

Proof of Example #2

Problem is: $(A \rightarrow \perp) \vee (B \rightarrow \perp), B \vdash \sim A$

1	(1) $(A \rightarrow \perp) \vee (B \rightarrow \perp)$	Premise
2	(2) B	Premise
3	(3) A	Assumption ($\sim I$)
4	(4) $A \rightarrow \perp$	Assumption ($\vee E$)
3,4	(5) \perp	4,3 $\rightarrow E$
6	(6) $B \rightarrow \perp$	Assumption ($\vee E$)
2,6	(7) \perp	6,2 $\rightarrow E$
1,2,3	(8) \perp	1,4,5,6,7 $\vee E$
1,2	(9) $\sim A$	3,8 $\sim I$

Proof of Example #3

Problem is: $(A \vee B) \vee C \vdash A \vee (B \vee C)$

1	(1)	(A \vee B) \vee C	Premise
2	(2)	A \vee B	Assumption (\vee E)
3	(3)	A	Assumption (\vee E)
3	(4)	A \vee (B \vee C)	3 \vee I
5	(5)	B	Assumption (\vee E)
5	(6)	B \vee C	5 \vee I
5	(7)	A \vee (B \vee C)	6 \vee I
2	(8)	A \vee (B \vee C)	2,3,4,5,7 \vee E
9	(9)	C	Assumption (\vee E)
9	(10)	B \vee C	9 \vee I
9	(11)	A \vee (B \vee C)	10 \vee I
1	(12)	A \vee (B \vee C)	1,2,8,9,11 \vee E

Proof of Example #4

Problem is : $A \vee B \vdash (A \rightarrow B) \rightarrow B$

1	(1) $A \vee B$	Premise
2	(2) $A \rightarrow B$	Ass (\rightarrow I)
3	(3) A	Ass (\vee E)
2,3	(4) B	2,3 \rightarrow E
5	(5) B	Ass (\vee E)
1,2	(6) B	1,3,4,5,5 \vee E
1	(7) $(A \rightarrow B) \rightarrow B$	2,6 \rightarrow I

Proof of Example #5

Problem is: $A \& B \vdash \sim(\sim A \vee \sim B)$

1	(1)	A&B	Premise
2	(2)	$\sim A \vee \sim B$	Assumption ($\sim I$)
3	(3)	$\sim A$	Assumption ($\vee E$)
1	(4)	A	1 &E
1,3	(5)	Δ	3,4 $\sim E$
6	(6)	$\sim B$	Assumption ($\vee E$)
1	(7)	B	1 &E
1,6	(8)	Δ	6,7 $\sim E$
1,2	(9)	Δ	2,3,5,6,8 $\vee E$
1	(10)	$\sim(\sim A \vee \sim B)$	2,9 $\sim I$

Proof of Example #6

Problem is : $A \vee B \vdash \sim(\sim A \& \sim B)$

1	(1) $A \vee B$	Premise
2	(2) $\sim A \& \sim B$	Ass ($\sim I$)
3	(3) A	Ass ($\vee E$)
2	(4) $\sim A$	2 &E
2,3	(5) Δ	4,3 $\sim E$
6	(6) B	Ass ($\vee E$)
2	(7) $\sim B$	2 &E
2,6	(8) Δ	7,6 $\sim E$
1,2	(9) Δ	1,3,5,6,8 $\vee E$
1	(10) $\sim(\sim A \& \sim B)$	2,9 $\sim I$

Proof of Example #7

Problem is: $\sim(A \& B) \vdash \sim A \vee \sim B$

1	(1)	$\sim(A \& B)$	Premise
2	(2)	$\sim(\sim A \vee \sim B)$	Assumption ($\sim I$)
3	(3)	$\sim A$	Assumption ($\sim I$)
3	(4)	$\sim A \vee \sim B$	3 $\vee I$
2,3	(5)	Δ	2,4 $\sim E$
2	(6)	$\sim \sim A$	3,5 $\sim I$
2	(7)	A	6 DN
8	(8)	$\sim B$	Assumption ($\sim I$)
8	(9)	$\sim A \vee \sim B$	8 $\vee I$
2,8	(10)	Δ	2,9 $\sim E$
2	(11)	$\sim \sim B$	8,10 $\sim I$
2	(12)	B	11 DN
2	(13)	$A \& B$	7,12 $\& I$
1,2	(14)	Δ	1,13 $\sim E$
1	(15)	$\sim \sim(\sim A \vee \sim B)$	2,14 $\sim I$
1	(16)	$\sim A \vee \sim B$	15 DN

Proof of Example #8

Problem is: $\sim C \vee (A \rightarrow B) \vdash (C \& A) \rightarrow B$

1	(1)	$\sim C \vee (A \rightarrow B)$	Premise
2	(2)	$C \& A$	Assumption ($\rightarrow I$)
3	(3)	$\sim B$	Assumption ($\sim I$)
4	(4)	$\sim C$	Assumption ($\vee E$)
2	(5)	C	2 &E
2,4	(6)	Δ	4,5 $\sim E$
7	(7)	$A \rightarrow B$	Assumption ($\vee E$)
2	(8)	A	2 &E
2,7	(9)	B	7,8 $\rightarrow E$
2,3,7	(10)	Δ	3,9 $\sim E$
1,2,3	(11)	Δ	1,4,6,7,10 $\vee E$
1,2	(12)	$\sim \sim B$	3,11 $\sim I$
1,2	(13)	B	12 DN
1	(14)	$(C \& A) \rightarrow B$	2,13 $\rightarrow I$

Proof of Example #9

Problem is: $\vdash (A \rightarrow B) \vee (B \rightarrow A)$

1		(1) $\sim((A \rightarrow B) \vee (B \rightarrow A))$	Assumption (\sim I)
2		(2) B	Assumption (\rightarrow I)
3		(3) $\sim A$	Assumption (\sim I)
4		(4) A	Assumption (\rightarrow I)
2		(5) $A \rightarrow B$	4,2 \rightarrow I
2		(6) $(A \rightarrow B) \vee (B \rightarrow A)$	5 \vee I
1,2		(7) Δ	1,6 \sim E
1,2		(8) $\sim \sim A$	3,7 \sim I
1,2		(9) A	8 DN
1		(10) $B \rightarrow A$	2,9 \rightarrow I
1		(11) $(A \rightarrow B) \vee (B \rightarrow A)$	10 \vee I
1		(12) Δ	1,11 \sim E
		(13) $\sim \sim((A \rightarrow B) \vee (B \rightarrow A))$	1,12 \sim I
		(14) $(A \rightarrow B) \vee (B \rightarrow A)$	13 DN

Proof of Example #10

Problem is : $\sim(A \vee B) \vdash \sim A \& \sim B$

1	(1)	$\sim(A \vee B)$	Premise
2	(2)	A	Ass (\sim I)
2	(3)	$A \vee B$	2 \vee I
1,2	(4)	Δ	1,3 \sim E
1	(5)	$\sim A$	2,4 \sim I
6	(6)	B	Ass (\sim I)
6	(7)	$A \vee B$	6 \vee I
1,6	(8)	Δ	1,7 \sim E
1	(9)	$\sim B$	6,8 \sim I
1	(10)	$\sim A \& \sim B$	5,9 $\&$ I

The Rule of Definition for the Biconditional

Rule of Definition for \leftrightarrow (Df): If $\lceil (p \rightarrow q) \& (q \rightarrow p) \rceil$ occurs as the entire formula at line j , then at line k we may write $\lceil p \leftrightarrow q \rceil$, labeling the line ‘ j Df’ and writing on its left the same numbers as are on the left of j . Conversely, if $\lceil p \leftrightarrow q \rceil$ occurs as the entire formula at a line j , then at line k we may write $\lceil (p \rightarrow q) \& (q \rightarrow p) \rceil$, labeling the line ‘ j Df’ and writing on its left the same numbers as are on the left of j .

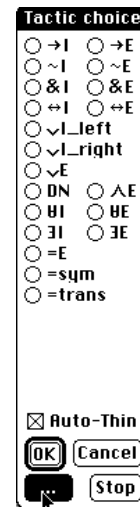
a_1, \dots, a_n	(j)	$(p \rightarrow q) \& (q \rightarrow p)$	
	⋮		
a_1, \dots, a_n	(k)	$p \leftrightarrow q$	j Df
OR			
a_1, \dots, a_n	(j)	$p \leftrightarrow q$	
	⋮		
a_1, \dots, a_n	(k)	$(p \rightarrow q) \& (q \rightarrow p)$	j Df

Using \leftrightarrow in MacLogic

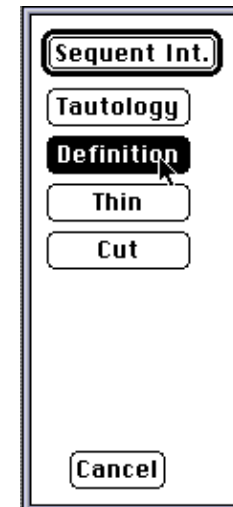
- Using the Definition strategy of MacLogic (accessed *via* the button), we can implement our Df. rule for \leftrightarrow . *Do not use \leftrightarrow I or \leftrightarrow E!*
- Using MacLogic's Definition strategy is much simpler than using its Tautology strategy (I did that last time, which was cumbersome).



To get to Definition, first:



then



- Here is a non-trivial example: $A \leftrightarrow \sim B \vdash \sim(A \leftrightarrow B)$. Let's try to tackle this one, using MacLogic's Definition strategy for our Df.
- The shortest proof I've been able to find is 18 steps (next slide). Forbes gives a 20-stepper in his discussion of this example (p. 118).

Problem is : $A \leftrightarrow \sim B \vdash \sim(A \leftrightarrow B)$

1	(1)	$A \leftrightarrow \sim B$	Ass
2	(2)	$A \leftrightarrow B$	Ass
1	(3)	$(A \rightarrow \sim B) \& (\sim B \rightarrow A)$	1 Defn.
1	(4)	$A \rightarrow \sim B$	3 &E
1	(5)	$\sim B \rightarrow A$	3 &E
6	(6)	B	Ass
2	(7)	$(A \rightarrow B) \& (B \rightarrow A)$	2 Defn.
2	(8)	$B \rightarrow A$	7 &E
2,6	(9)	A	8,6 \rightarrow E
1,2,6	(10)	$\sim B$	4,9 \rightarrow E
1,2,6	(11)	Δ	10,6 \sim E
1,2	(12)	$\sim B$	6,11 \sim I
1,2	(13)	A	5,12 \rightarrow E
1,2	(14)	$\sim B$	4,13 \rightarrow E
2	(15)	$A \rightarrow B$	7 &E
1,2	(16)	B	15,13 \rightarrow E
1,2	(17)	Δ	14,16 \sim E
1	(18)	$\sim(A \leftrightarrow B)$	2,17 \sim I

Sequent and Theorem Introduction: I

- You may have noticed that certain important sequents or theorems tend to get proven over and over again in different problems.
- For instance, the sequent $X \vee Y, \sim X \vdash Y$ is a very useful thing to know, as are the sequents $X \rightarrow Y, \sim Y \vdash \sim X$, $\wedge \vdash X$, and many others.
- It would be nice if we had a rule that allowed us to say “OK, I’ve proven this sequent already, so I don’t have to prove it again here”.
- We have two such rules. They are called *Sequent Introduction* (SI) for sequents, and *Theorem Introduction* (TI) for theorems.
- SI and TI allow us to avoid having to re-solve certain sub-problems that we already know how to solve. This makes proofs shorter.
- We will have a fixed list of sequents and theorems that we’ll be allowed to use in conjunction with SI and TI.

Sequent and Theorem Introduction: II

- Forbes lists a bunch of sequents and Theorems on page 123 that we may use with SI or TI. There's a MacLogic file containing all of them.
- Here are a few of the sequents and theorems that tend to be useful:

$$p \vee q, \sim p \vdash q; \text{ or; } p \vee q, \sim q \vdash p \quad (\text{DS})$$

$$p \rightarrow q, \sim q \vdash \sim p \quad (\text{MT})$$

$$p \vdash q \rightarrow p; \text{ or; } \sim p \vdash p \rightarrow q \quad (\text{PMI})$$

$$\vdash p \vee \sim p \quad (\text{LEM})$$

$$\sim(p \& q) \sim p \vee \sim q \quad (\text{DEM})$$

$$\sim(p \vee q) \sim p \& \sim q \quad (\text{DEM})$$

$$\sim(\sim p \vee \sim q) p \& q \quad (\text{DEM})$$

$$\sim(\sim p \& \sim q) p \vee q \quad (\text{DEM})$$

$$\wedge \vdash p \quad (\text{EFQ})$$

$$p \& (q \vee r) (p \& q) \vee (p \& r) \quad (\text{DIST})$$

Sequent and Theorem Introduction: III

- Remember the proof for #9 above: $\vdash (A \rightarrow B) \vee (B \rightarrow A)$.

1		(1) $\sim((A \rightarrow B) \vee (B \rightarrow A))$	Assumption (\sim I)
2		(2) B	Assumption (\rightarrow I)
3		(3) $\sim A$	Assumption (\sim I)
4		(4) A	Assumption (\rightarrow I)
2		(5) $A \rightarrow B$	4,2 \rightarrow I
2		(6) $(A \rightarrow B) \vee (B \rightarrow A)$	5 \vee I
1,2		(7) Δ	1,6 \sim E
1,2		(8) $\sim \sim A$	3,7 \sim I
1,2		(9) A	8 DN
1		(10) $B \rightarrow A$	2,9 \rightarrow I
1		(11) $(A \rightarrow B) \vee (B \rightarrow A)$	10 \vee I
1		(12) Δ	1,11 \sim E
		(13) $\sim \sim((A \rightarrow B) \vee (B \rightarrow A))$	1,12 \sim I
		(14) $(A \rightarrow B) \vee (B \rightarrow A)$	13 DN

Sequent and Theorem Introduction: IV

- Using TI and SI, we can obtain the following much simpler proof:

	(1)	$A \vee \sim A$	TI (LEM)
2	(2)	A	Assumption ($\vee E$)
2	(3)	$B \rightarrow A$	2 SI (PMI)
2	(4)	$(A \rightarrow B) \vee (B \rightarrow A)$	3 $\vee I$
5	(5)	$\sim A$	Assumption ($\vee E$)
5	(6)	$A \rightarrow B$	5 SI (PMI)
5	(7)	$(A \rightarrow B) \vee (B \rightarrow A)$	6 $\vee I$
	(8)	$(A \rightarrow B) \vee (B \rightarrow A)$	1,2,4,5,7 $\vee E$

- Here, LEM is the theorem $\vdash A \vee \sim A$ (which we have already proven), and PMI stands for either of the sequents $\sim A \vdash A \rightarrow B$ (used at line 6), or $A \vdash B \rightarrow A$ (used at line 3), both of which we've proven.
- SI allows you to use (*any* substitution instance of) *any* sequent that you've already proven to make an inference at any stage of a proof.
- TI allows you to write down (*any* substitution instance of) *any* theorem that you have already proven at *any* stage of a proof.

The Formal Definitions of SI and TI

- **Sequent Introduction (SI).** Suppose $r_1, \dots, r_n \vdash s$ is a *substitution-instance* of the sequent $p_1, \dots, p_n \vdash q$ which we have already proved, and that the formulae r_1, \dots, r_n occur at lines j_1, \dots, j_n in a proof. Then we may infer s at line k , labeling the line ‘ j_1, \dots, j_n SI (Identifier)’ and writing on the left all numbers which appear on the left of lines j_1, \dots, j_n .
- **Theorem Introduction (TI).** If $\vdash s$ is a *substitution-instance* of some theorem $\vdash q$ which we have already proved, we may introduce a new line k into a proof with the formula s at it and no numbers on its left, labeling the line ‘TI (Identifier)’.
- ‘Identifier’ stands for the name of a sequent or theorem that has already been proven (*e.g.*, MT, DS, PMI, LEM, *etc*). See Forbes’s list.
- Note: TI is just a *special case* of SI (with $n = 0$).

SI and TI: A Relatively Easy Example

- Use SI/TI to find a “short” proof of: $\sim(A \rightarrow (B \vee C)) \vdash (B \vee C) \rightarrow A$.

Problem is : $\sim(A \rightarrow (B \vee C)) \vdash (B \vee C) \rightarrow A$

1	(1) $\sim(A \rightarrow (B \vee C))$	Premise
1	(2) $A \& \sim(B \vee C)$	1 SI Neg-Imp1
1	(3) A	2 &E
1	(4) $(B \vee C) \rightarrow A$	3 SI PMI1

SI and TI: A More Challenging Example

- Use SI/TI to find a “short” proof of: $A \rightarrow (B \vee C) \vdash (A \rightarrow B) \vee (A \rightarrow C)$.

Problem is : $A \rightarrow (B \vee C) \vdash (A \rightarrow B) \vee (A \rightarrow C)$

1	(1) $A \rightarrow (B \vee C)$	Premise
1	(2) $\sim A \vee (B \vee C)$	1 SI IMP1
3	(3) $\sim A$	Assumption ($\vee E$)
3	(4) $A \rightarrow B$	3 SI PMI2
3	(5) $(A \rightarrow B) \vee (A \rightarrow C)$	4 $\vee I_{\text{left}}$
6	(6) $B \vee C$	Assumption ($\vee E$)
7	(7) B	Assumption ($\vee E$)
7	(8) $A \rightarrow B$	7 SI PMI1
7	(9) $(A \rightarrow B) \vee (A \rightarrow C)$	8 $\vee I_{\text{left}}$
10	(10) C	Assumption ($\vee E$)
10	(11) $A \rightarrow C$	10 SI PMI1
10	(12) $(A \rightarrow B) \vee (A \rightarrow C)$	11 $\vee I_{\text{right}}$
6	(13) $(A \rightarrow B) \vee (A \rightarrow C)$	6,7,9,10,12 $\vee E$
1	(14) $(A \rightarrow B) \vee (A \rightarrow C)$	2,3,5,6,13 $\vee E$