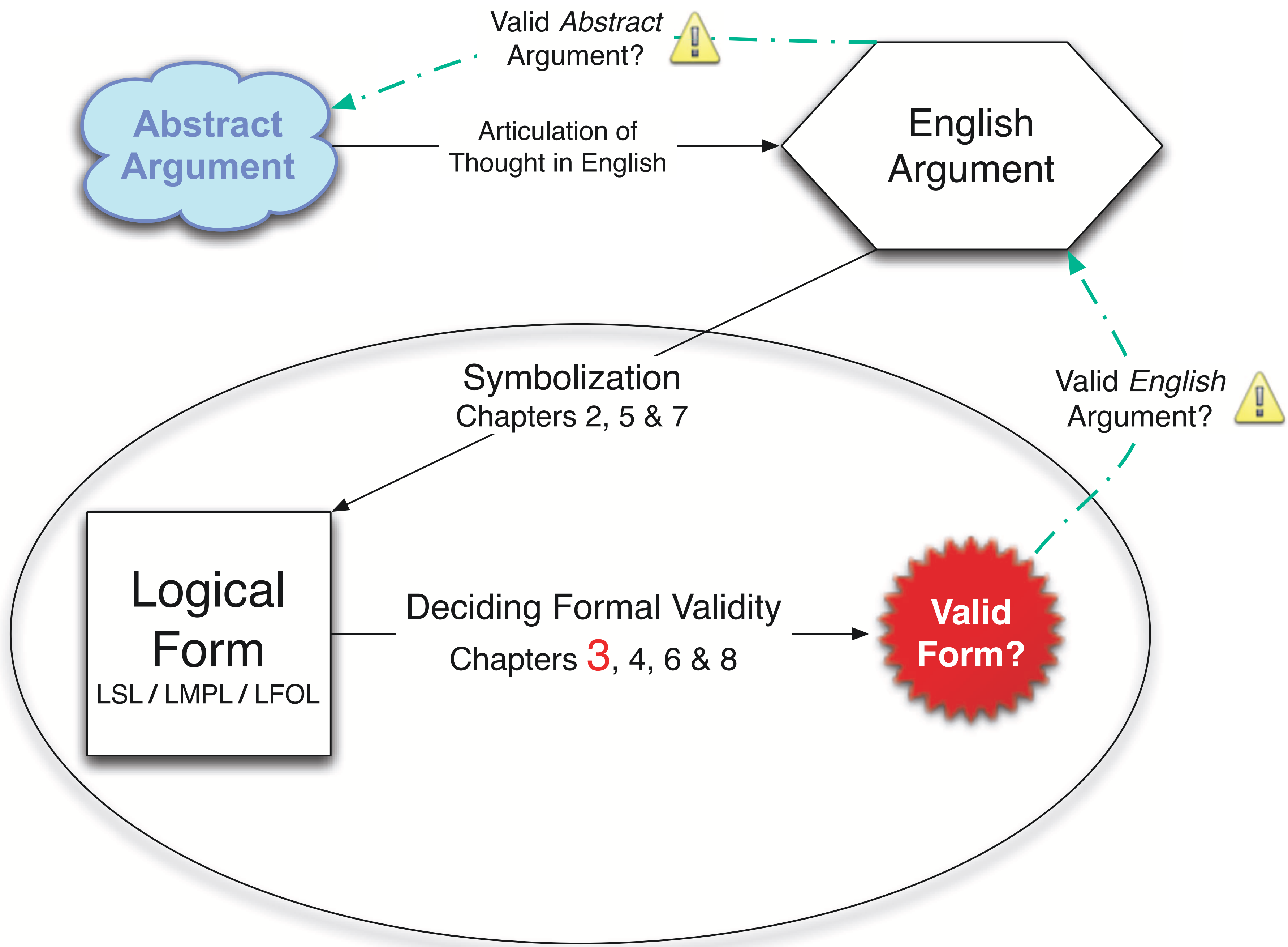
Overview of Today's Lecture

- Today's Music: *Black Roots*
- Richard will have extra office hours from 3:30–5 today.
- My office hours are 4–5:30 on Wednesdays.
- **HW #2 due Friday @ 5pm in the 12A Drop Box** (outside 301 Moses).
  ☞ Make sure you follow the guidelines/hints on my "HW Tips" Handout
- The mid-term is next Thursday (in class). I'll post a sample tomorrow. And, I will discuss the sample exam in lecture next Wednesday.
- I have posted a handout on the "short" method for testing LSL validity. I will be going over this handout in lecture very soon.
- More 12A Practice Problems can be found in: *Schaum's Outline of Logic*
- Today: Chapter 3, Continued


Valid *Abstract* Argument?
Abstract Argument
Articulation of Thought in English
English Argument
Symbolization
Chapters 2, 5 & 7
Valid *English* Argument?
Logical Form
LSL / LMPL / LFOL
Deciding Formal Validity
Chapters 3, 4, 6 & 8
Valid Form?

p	$\sim p$
⊤	⊥
⊥	⊤

p	q	$p \,\&\, q$
⊤	⊤	⊤
⊤	⊥	⊥
⊥	⊤	⊥
⊥	⊥	⊥

p	q	$p \vee q$
⊤	⊤	⊤
⊤	⊥	⊤
⊥	⊤	⊤
⊥	⊥	⊥

p	q	$p \rightarrow q$
⊤	⊤	⊤
⊤	⊥	⊥
⊥	⊤	⊤
⊥	⊥	⊤

p	q	$p \leftrightarrow q$
⊤	⊤	⊤
⊤	⊥	⊥
⊥	⊤	⊥
⊥	⊥	⊤

Chapter 3 — Semantics of LSL: Truth Functions VII

- If our truth-functional semantics for '→' doesn't perfectly capture the English meaning of 'if ... then ...', then why do we define it this way?
- The answer has two parts. First, our semantics is *truth-functional*. This is an *idealization* — it yields the *simplest* ("Newtonian") semantics.
- And, there are only $2^4 = 16$ possible binary truth-functions. Why?
- So, unless one of the *other* 15 binary truth-functions is *closer* to the English conditional than '→' is, it's *the best we can do*, *truth-functionally*.
- More importantly, there are certain *logical properties* that the conditional *must* have. It can be shown that our definition of '→' is the *only* binary truth-function which satisfies all three of the following:

(1) *Modus Ponens* [p and $\ulcorner p \to q \urcorner \therefore q$] is a valid sentential form.

(2) Affirming the consequent [q and $\ulcorner p \to q \urcorner \therefore p$] is *not* a valid form.

(3) All sentences of the form $\ulcorner p \to p \urcorner$ are logical truths.

Chapter 3 — Semantics of LSL: Truth Functions VIII

- Here are all of the 16 possible binary truth-functions. I've given them all names or descriptions. [Only a few of these names were made up by me.]

p	q	$\top$	NAND	$\rightarrow$	$\sim p$	FI ($\leftarrow$)	$\sim q$	$\leftrightarrow$	NOR	$\vee$	NIFF	q	NFI	p	NIF	&	$\bot$
$\top$	$\top$	$\top$	$\bot$	$\top$	$\bot$	$\top$	$\bot$	$\top$	$\bot$	$\top$	$\bot$	$\top$	$\bot$	$\top$	$\bot$	$\top$	$\bot$
$\top$	$\bot$	$\top$	$\top$	$\bot$	$\bot$	$\top$	$\top$	$\bot$	$\bot$	$\top$	$\top$	$\bot$	$\bot$	$\top$	$\top$	$\bot$	$\bot$
$\bot$	$\top$	$\top$	$\top$	$\top$	$\top$	$\bot$	$\bot$	$\bot$	$\bot$	$\top$	$\top$	$\top$	$\top$	$\bot$	$\bot$	$\bot$	$\bot$
$\bot$	$\bot$	$\top$	$\top$	$\top$	$\top$	$\top$	$\top$	$\top$	$\top$	$\bot$	$\bot$	$\bot$	$\bot$	$\bot$	$\bot$	$\bot$	$\bot$
(1)?				Yes													
(2)?				Yes													
(3)?				Yes													

- Exercise: fill-in the three rows at the bottom (except for $\rightarrow$, which I have done for you already) concerning (1), (2), and (3) from the previous slide.

- You should be able to do this pretty soon (within the next week) . . .

Chapter 3 — Semantics of LSL: Additional Remarks on →

- Above, I explained *why* our conditional → behaves "like a disjunction":
  1. We want a *truth-functional* semantics for →. This is a simplifying *idealization*. Truth-functional semantics are the simplest compositional semantics for sentential logic. [A "Newtonian" semantic model.]
  2. Given (1), the *only* way to define → is *our* way, since it's the *only* binary truth-function that has the following three essential *logical* properties:
     - (i) *Modus Ponens* [p and $\ulcorner p \to q \urcorner \therefore q$] is a valid sentential form.
     - (ii) Affirming the consequent [q and $\ulcorner p \to q \urcorner \therefore p$] is *not* a valid form.
     - (iii) All sentences of the form $\ulcorner p \to p \urcorner$ are logical truths.
- There are *non*-truth-functional semantics for the English conditional.
- These may be "closer" to the English *meaning* of "if". But, they agree with our semantics for →, when it comes to the crucial *logical* properties (i)–(iii). Indeed, our → captures *most* of the (intuitive) *logical* properties of "if".

Constructing Truth-Tables for LSL Sentences

- With the truth-table definitions of the five connectives in hand, we can now construct truth tables for arbitrary compound LSL statements.
- The procedure for constructing the truth-table of p is as follows:

1. Determine the number of rows in the truth-table. This is 2^n, where n is the number of atomic sentences in the compound statement p.
2. The table will have $n + 1$ main columns: n columns for the atomic sentences in p, and one for the truth-values of p itself.
3. The table will also have some "quasi-columns" — one for each LSL statement occurring in the compound p — which needn't be drawn explicitly, but which go into the determination of p's truth values.
4. Place the atomic letters in the left most columns, in alphabetical order from left to right. And, place p in the right most column.
5. Write in all possible combinations of truth-values for the atomic statements. There are 2^n of these — one for each row of the table.

6. Convention: start on the nth column (farthest down the alphabet) with the pattern $\top\bot\top\bot\ldots$ repeated until the column is filled. Then, go $\top\top\bot\bot\ldots$ in the $n-1$st column, $\top\top\top\top\bot\bot\bot\bot\ldots$ in the $n-2$nd column, etc..., until the very first column has been completed.

7. Finally, we compute the truth-values of p in each row of the table. Here, we start from the inside-out. We first copy the truth-values of the atoms, then we compute the negations, conjunctions, etc. which compose p. Finally, we will be in a position to compute the value of the main connective of p, at which point we'll be done with the table.

- Example: Step-By-Step Truth-Table Construction of '$A \leftrightarrow (B \& A)$.'

A	B	A	$\leftrightarrow$	$(B$	$\&$	$A)$
$\top$	$\top$	$\top$	$\top$	$\top$	$\top$	$\top$
$\top$	$\bot$	$\top$	$\bot$	$\bot$	$\bot$	$\top$
$\bot$	$\top$	$\bot$	$\top$	$\top$	$\bot$	$\bot$
$\bot$	$\bot$	$\bot$	$\top$	$\bot$	$\bot$	$\bot$

Interpretations and the Relation of Logical Consequence

- An *interpretation* of an LSL formula p is an assignment of truth-values to all of the sentence letters in p — *i.e.*, a row in p's truth-table.
- A formula p is a *logical consequence* of a set of formulae S [written $S \vDash p$] just in case there is no interpretation (*i.e.*, no row in the joint truth-table of S and p) on which all the members of S are $\top$ but p is $\bot$.
- $S \vDash p$ is another way of saying that the argument from S to p is *valid*.
- Two LSL sentences p and q are said to be *logically equivalent* [written $p \dashv\vDash q$] iff they have the same truth-value on all (joint) interpretations.
- That is, p and q are logically equivalent iff *both* $p \vDash q$ *and* $q \vDash p$.
- I will often express $\ulcorner p \vDash q \urcorner$ by saying that $\ulcorner p$ *entails* $q\urcorner$. This is easier than saying that $\ulcorner q$ is a logical consequence of $p\urcorner$.
- The logical consequence relation $\vDash$ is our central theoretical relation.

Logical Truth, Logical Falsity, and Contingency: Definitions

- A statement is said to be logically true (or tautologous) if it is $\top$ on all interpretations. *E.g.*, any statement of the form $p \leftrightarrow p$ is tautological.

p	p	$\leftrightarrow$	p
$\top$	$\top$	$\top$	$\top$
$\bot$	$\bot$	$\top$	$\bot$

- A statement is logically false (or self-contradictory) if it is $\bot$ on all interpretations. *E.g.*, any statement of the form $p \mathbin{\&} {\sim}p$ is logically false:

p	p	$\&$	$\sim$	p
$\top$	$\top$	$\bot$	$\bot$	$\top$
$\bot$	$\bot$	$\bot$	$\top$	$\bot$

- A statement is contingent if it is *neither* tautological *nor* self-contradictory. Example: 'A' (or *any* basic sentence) is contingent.

A	A
$\top$	$\top$
$\bot$	$\bot$

Logical Truth, Logical Falsity, and Contingency: Problems

- Classify the following statements as logically true (tautologous), logically false (self-contradictory), or contingent:

1. $N \to (N \to N)$
2. $(G \to G) \to G$
3. $(S \to R) \mathbin{\&} (S \mathbin{\&} {\sim}R)$
4. $((E \to F) \to F) \to E$
6. $(M \to P) \vee (P \to M)$
11. $[(Q \to P) \mathbin{\&} ({\sim}Q \to R)] \mathbin{\&} {\sim}(P \vee R)$
12. $[(H \to N) \mathbin{\&} (T \to N)] \to [(H \vee T) \to N]$
15. $[(F \vee E) \mathbin{\&} (G \vee H)] \leftrightarrow [(G \mathbin{\&} E) \vee (F \mathbin{\&} H)]$

Equivalence, Contradictoriness, Consistency, and Inconsistency

- Statements p and q are equivalent [$p \dashv\vDash q$] if they have the same truth-value on all interpretations. For instance, '$A \to B$' and '$\sim A \vee B$'.

A	B	A	$\to$	B	$\sim$	A	$\vee$	B
⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤
⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥
⊥	⊤	⊥	⊤	⊤	⊤	⊥	⊤	⊤
⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊤	⊥

- Statements p and q are contradictory [$p \dashv\vDash \sim q$] if they have opposite truth-values on all interpretations. For instance, '$A \to B$' and '$A \,\&\, \sim B$'.

A	B	A	$\to$	B	A	$\&$	$\sim$	B
⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊥	⊤
⊤	⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊥
⊥	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤
⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊤	⊥

- Statements p and q are inconsistent [$p \vDash \sim q$] if there is no interpretation on which they are both true. For instance, '$A \leftrightarrow B$' and '$A \,\&\, \sim B$' are inconsistent [Note: they are *not* contradictory!].

A	B	A	$\leftrightarrow$	B	A	$\&$	$\sim$	B
$\top$	$\top$	$\top$	$\top$	$\top$	$\top$	$\bot$	$\bot$	$\top$
$\top$	$\bot$	$\top$	$\bot$	$\bot$	$\top$	$\top$	$\top$	$\bot$
$\bot$	$\top$	$\bot$	$\bot$	$\top$	$\bot$	$\bot$	$\bot$	$\top$
$\bot$	$\bot$	$\bot$	$\top$	$\bot$	$\bot$	$\bot$	$\top$	$\bot$

- Statements p and q are consistent [$p \nvDash \sim q$] if there's an interpretation on which they are both true. *E.g.*, '$A \,\&\, B$' and '$A \vee B$' are consistent:

A	B	A	$\&$	B	A	$\vee$	B
$\top$	$\top$	$\top$	$\top$	$\top$	$\top$	$\top$	$\top$
$\top$	$\bot$	$\top$	$\bot$	$\bot$	$\top$	$\top$	$\bot$
$\bot$	$\top$	$\bot$	$\bot$	$\top$	$\bot$	$\top$	$\top$
$\bot$	$\bot$	$\bot$	$\bot$	$\bot$	$\bot$	$\bot$	$\bot$

Semantic Equivalence, Contradictoriness, *etc.*: Relationships

- What are the logical relationships between ⌜p and q are equivalent⌝, ⌜p and q are consistent⌝, ⌜p and q are contradictory⌝, and ⌜p and q are inconsistent⌝? That is, which of these entails which (and which don't)?

Equivalent	Contradictory
$\Downarrow$? $\Uparrow$	$\Downarrow$? $\Uparrow$
Consistent	Inconsistent

- Answers:
  1. Equivalent $\not\Rightarrow$ Consistent (*example*?)
  2. Consistent $\not\Rightarrow$ Equivalent (*example*?)
  3. Contradictory $\Rightarrow$ Inconsistent (*why*?)
  4. Inconsistent $\not\Rightarrow$ Contradictory (*example*?)

Semantic Equivalence: Example #1

- Recall that ⌜p unless q⌝ translates in LSL as ⌜$\sim q \to p$⌝.
- We've said that we can also translate ⌜p unless q⌝ as ⌜$p \vee q$⌝.
- This is because ⌜$\sim q \to p$⌝ is *semantically equivalent to* ⌜$p \vee q$⌝. We may demonstrate this, using the following joint truth-table.

p	q	$\sim q$	$\to$	p	$p \vee q$
⊤	⊤	⊥	⊤	⊤	⊤
⊤	⊥	⊤	⊤	⊤	⊤
⊥	⊤	⊥	⊤	⊥	⊤
⊥	⊥	⊤	⊥	⊥	⊥

- The truth-tables of ⌜$p \vee q$⌝ and ⌜$\sim q \to p$⌝ are the same.
- Thus, $\sim q \to p \mathrel{=\mkern-3mu|\mkern-3mu\models} p \vee q$.

Semantic Equivalence: Example #2

- $\ulcorner p \leftrightarrow q \urcorner$ is an *abbreviation* for $\ulcorner (p \to q) \,\&\, (q \to p) \urcorner$.
- The following truth-table shows it is a *legitimate* abbreviation:

p	q	$(p \to q)$	$\&$	$(q \to p)$	$p \leftrightarrow q$
$\top$	$\top$	$\top$	$\top$	$\top$	$\top$
$\top$	$\bot$	$\bot$	$\bot$	$\top$	$\bot$
$\bot$	$\top$	$\top$	$\bot$	$\bot$	$\bot$
$\bot$	$\bot$	$\top$	$\top$	$\top$	$\top$

- $\ulcorner p \leftrightarrow q \urcorner$ and $\ulcorner (p \to q) \,\&\, (q \to p) \urcorner$ have the same truth-table.
- Thus, $p \leftrightarrow q \mathrel{=\!\!|\!\vDash} (p \to q) \,\&\, (q \to p)$.

Semantic Equivalence: Example #3

- Intuitively, the truth-conditions for *exclusive or* ($\oplus$) are such that $\ulcorner p \oplus q \urcorner$ is true if and only if *exactly* one of p or q is true.
- I said that we could say something equivalent to this using our $\vee$, &, and $\sim$. Specifically, I said $p \oplus q \dashv\vDash (p \vee q) \,\&\, {\sim}(p \,\&\, q)$.
- The following truth-table shows that this is correct:

p	q	$(p \vee q)$	&	${\sim}(p \,\&\, q)$	$p \oplus q$
$\top$	$\top$	$\top$	$\bot$	$\bot$	$\bot$
$\top$	$\bot$	$\top$	$\top$	$\top$	$\top$
$\bot$	$\top$	$\top$	$\top$	$\top$	$\top$
$\bot$	$\bot$	$\bot$	$\bot$	$\top$	$\bot$

- $\ulcorner p \oplus q \urcorner$ and $\ulcorner (p \vee q) \,\&\, {\sim}(p \,\&\, q) \urcorner$ have the same truth-table.

Equivalence, Contradictoriness, *etc.*: Some Problems

- Use truth-tables to determine whether the following pairs of statements are semantically equivalent, contradictory, consistent, or inconsistent.

  1. '$F \,\&\, M$' and '$\sim(F \vee M)$'
  2. '$R \vee \sim S$' and '$S \,\&\, \sim R$'
  3. '$H \leftrightarrow \sim G$' and '$(G \,\&\, H) \vee (\sim G \,\&\, \sim H)$'
  4. '$N \,\&\, (A \vee \sim E)$' and '$\sim A \,\&\, (E \vee \sim N)$'
  5. '$W \leftrightarrow (B \,\&\, T)$' and '$W \,\&\, (T \rightarrow \sim B)$'
  6. '$R \,\&\, (Q \vee S)$' and '$(S \vee R) \,\&\, (Q \vee R)$'
  7. '$Z \,\&\, (C \leftrightarrow P)$' and '$C \leftrightarrow (Z \,\&\, \sim P)$'
  8. '$Q \rightarrow \sim(K \vee F)$' and '$(K \,\&\, Q) \vee (F \,\&\, Q)$'

Some More Semantic Equivalences

- Here is a simultaneous truth-table which establishes that

$$A \leftrightarrow B \dashv\vDash (A \,\&\, B) \vee (\sim A \,\&\, \sim B)$$

A	B	A	$\leftrightarrow$	B	$(A$	$\&$	$B)$	$\vee$	$(\sim$	A	$\&$	$\sim$	$B)$
$\top$	$\top$	$\top$	$\top$	$\top$	$\top$	$\top$	$\top$	$\top$	$\bot$	$\top$	$\bot$	$\bot$	$\top$
$\top$	$\bot$	$\top$	$\bot$	$\bot$	$\top$	$\bot$	$\bot$	$\bot$	$\bot$	$\top$	$\bot$	$\top$	$\bot$
$\bot$	$\top$	$\bot$	$\bot$	$\top$	$\bot$	$\bot$	$\top$	$\bot$	$\top$	$\bot$	$\bot$	$\bot$	$\top$
$\bot$	$\bot$	$\bot$	$\top$	$\bot$	$\bot$	$\bot$	$\bot$	$\top$	$\top$	$\bot$	$\top$	$\top$	$\bot$

- Can you prove the following equivalences with truth-tables?
  - $\sim(A \,\&\, B) \dashv\vDash \sim A \vee \sim B$
  - $\sim(A \vee B) \dashv\vDash \sim A \,\&\, \sim B$
  - $A \dashv\vDash (A \,\&\, B) \vee (A \,\&\, \sim B)$
  - $A \dashv\vDash A \,\&\, (B \rightarrow B)$
  - $A \dashv\vDash A \vee (B \,\&\, \sim B)$

A More Complicated Equivalence (Distributivity)

- The following simultaneous truth-table establishes that

$$p \,\&\, (q \vee r) \dashv\vDash (p \,\&\, q) \vee (p \,\&\, r)$$

p	q	r	p	$\&$	$(q \vee r)$	$(p \,\&\, q)$	$\vee$	$(p \,\&\, r)$
⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤
⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊥
⊤	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊤
⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊥
⊥	⊤	⊤	⊥	⊥	⊤	⊥	⊥	⊥
⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊥
⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊥
⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥

- This is *distributivity* of & over ∨. It also works for ∨ over &.

The Exhaustive Truth-Table Method for Testing Validity

- Remember, an argument is valid if it is *impossible* for its premises to be true while its conclusion is false. Let $p_1, \ldots, p_n$ be the premises of a LSL argument, and let q be the conclusion of the argument. Then, we have:

$$\begin{array}{l} p_1 \\ \vdots \\ p_n \\ \hline \therefore q \end{array}$$

is valid if and only if there is no row in the simultaneous truth-table of $p_1, \ldots, p_n$, and q which looks like the following:

atoms	premises			conclusion
$\cdots$	p_1	$\cdots$	p_n	q
$\cdots$	$\top$	$\top$	$\top$	$\bot$

- We will use simultaneous truth-tables to prove validities and invalidities. For example, consider the following valid argument:

A

$A \to B$

$\therefore B$

atoms		premises				conclusion
A	B	A	A	$\to$	B	B
$\top$	$\top$	$\top$	$\top$	$\top$	$\top$	$\top$
$\top$	$\bot$	$\top$	$\top$	$\bot$	$\bot$	$\bot$
$\bot$	$\top$	$\bot$	$\bot$	$\top$	$\top$	$\top$
$\bot$	$\bot$	$\bot$	$\bot$	$\top$	$\bot$	$\bot$

☞ VALID — there is no row in which A and $A \to B$ are both $\top$, but B is $\bot$.

- In general, we'll use the following procedure for evaluating arguments:
  1. Translate and symbolize the the argument (if given in English).
  2. Write out the symbolized argument (as above).
  3. Draw a simultaneous truth-table for the symbolized argument, outlining the columns representing the premises and conclusion.
  4. Is there a row of the table in which all premises are $\top$ but the conclusion is $\bot$? If so, the argument is invalid; if not, it's valid.
- We will practice this on examples. But, first, a "short-cut" method.

The "Short" Truth Table Method for Validity Testing I

- Consider the following LSL argument:

  $A \rightarrow (B \,\&\, E)$

  $D \rightarrow (A \vee F)$

  $\sim E$

  $\therefore D \rightarrow B$

- This argument has 3 premises and contains 5 atomic sentences. This would lead to a complete truth-table with 32 rows and 8 columns (this will be far more than 256 distinct computations).
- As such, the exhaustive truth-table method does not seem practical in this case. So, instead, let's try to construct or "reverse engineer" an invalidating interpretation.
- To do this, we "work backward" from the *assumption* that the conclusion is $\perp$ and all the premises are $\top$ on some row.

- Step 1: Assume there is an interpretation on which all three premises are ⊤ and the conclusion is ⊥. This leads to:

A	B	D	E	F	A	$\to$	$B \,\&\, E$	D	$\to$	$(A \vee F)$	$\sim E$	$D \to B$
						⊤			⊤		⊤	⊥

- Step 2: From the assumption that $\sim E$ is ⊤, we may infer that both E and $B \,\&\, E$ are ⊥. This fills-in two more cells:

A	B	D	E	F	A	$\to$	$B \,\&\, E$	D	$\to$	$(A \vee F)$	$\sim E$	$D \to B$
			⊥			⊤	⊥		⊤		⊤	⊥

- Step 3: Now, the only way that $A \to (B \,\&\, E)$ can be ⊤ (as we've assumed) is if its antecedent A is ⊥. This yields the following:

A	B	D	E	F	A	$\to$	$B \,\&\, E$	D	$\to$	$(A \vee F)$	$\sim E$	$D \to B$
⊥			⊥		⊥	⊤	⊥		⊤		⊤	⊥

- Step 4: Now, $D \to B$ can be $\bot$ (as we've been assuming) if and only if D is $\top$ and B is $\bot$ (just by the definition of $\to$). So:

A	B	D	E	F	A	$\to$	$B \& E$	D	$\to$	$(A \vee F)$	$\sim E$	$D \to B$
$\bot$	$\bot$	$\top$	$\bot$		$\bot$	$\top$	$\bot$	$\top$	$\top$		$\top$	$\bot$

- Step 5: Then, $D \to (A \vee F)$ can be $\top$ (as we've assumed) only if its consequent $A \vee F$ is $\top$, which gives the following:

A	B	D	E	F	A	$\to$	$B \& E$	D	$\to$	$(A \vee F)$	$\sim E$	$D \to B$
$\bot$	$\bot$	$\top$	$\bot$		$\bot$	$\top$	$\bot$	$\top$	$\top$	$\top$	$\top$	$\bot$

- Step 6: Finally, since A is $\bot$, the only way that $A \vee F$ can be $\top$ is if F is $\top$, which completes our construction!

A	B	D	E	F	A	$\to$	$B \& E$	D	$\to$	$(A \vee F)$	$\sim E$	$D \to B$
$\bot$	$\bot$	$\top$	$\bot$	$\top$	$\bot$	$\top$	$\bot$	$\top$	$\top$	$\top$	$\top$	$\bot$

The "Short" Truth Table Method for Validity Testing II

- Consider the following LSL argument:

  $\sim A \vee (B \rightarrow C)$

  $E \rightarrow (B \,\&\, A)$

  $C \rightarrow E$

  $\therefore C \leftrightarrow A$

- Let's try our "short" truth table method on this one.
- Step 1: Assume there is an interpretation on which all three premises are ⊤ and the conclusion is ⊥. This leads to the following partial row:

A	B	C	E	$\sim A$	$\vee$	$(B \rightarrow C)$	E	$\rightarrow$	$(B \,\&\, A)$	$C \rightarrow E$	$C \leftrightarrow A$
					⊤			⊤		⊤	⊥

- Step 2: Now, there are *two* ways the conclusion $C \leftrightarrow A$ can be false:
  - Case 1: C is ⊤ and A is ⊥.
  - Case 2: C is ⊥ and A is ⊤.

- Step 2 (Case 1): C is $\top$ and A is $\bot$. This leads to the following:

A	B	C	E	$\sim A$	$\vee$	$(B \to C)$	E	$\to$	$(B \,\&\, A)$	$C \to E$	$C \leftrightarrow A$
$\bot$		$\top$		$\top$	$\top$			$\top$	$\bot$	$\top$	$\bot$

- Step 3 (Case 1): Now, the *only* way to make $E \to (B \,\&\, A)$ T is to make E $\bot$. But, this *contradicts* constraints already forced on our construction!

A	B	C	E	$\sim A$	$\vee$	$(B \to C)$	E	$\to$	$(B \,\&\, A)$	$C \to E$	$C \leftrightarrow A$
$\bot$		$\top$	$\bot$	$\top$	$\top$		$\bot$	$\top$	$\bot$	$\top$ / $\bot$!!	$\bot$

  - All this shows (so far) is that there is no row in which all the premises are $\top$ and the conclusion is $\bot$ *by way of C being $\top$ and A being $\bot$*. But, this does not yet show that the argument is valid! We must also check Case 2.

- Step 2 (Case 2): C is $\bot$ and A is $\top$. This leads to the following, initially:

A	B	C	E	$\sim A$	$\vee$	$(B \to C)$	E	$\to$	$(B \,\&\, A)$	$C \to E$	$C \leftrightarrow A$
$\top$		$\bot$			$\top$			$\top$		$\top$	$\bot$

- Step 3 (Case 2): If A is $\top$, then $\sim A$ is $\bot$. So, making $\sim A \vee (B \to C)$ $\top$ will require making $B \to C$ $\top$, which implies that B is $\bot$:

A	B	C	E	$\sim A$	$\vee$	$(B \to C)$	E	$\to$	$(B \,\&\, A)$	$C \to E$	$C \leftrightarrow A$
$\top$	$\bot$	$\bot$		$\bot$	$\top$	$\top$		$\top$		$\top$	$\bot$

- Step 4 (Case 2): Since B must be $\bot$, so must $B \,\&\, A$. As a result, the only way to make $E \to (B \,\&\, A)$ $\top$ is to make E $\bot$. Success!!

A	B	C	E	$\sim A$	$\vee$	$(B \to C)$	E	$\to$	$(B \,\&\, A)$	$C \to E$	$C \leftrightarrow A$
$\top$	$\bot$	$\bot$	$\bot$	$\bot$	$\top$	$\top$	$\bot$	$\top$	$\bot$	$\top$	$\bot$

  - So, we have found a row in which all premises are $\top$ and the conclusion is $\bot$. Thus, the argument is *invalid* after all!
  - The moral of this example is that we must exhaust all possible ways of constructing an invalidating row, until we either find one (invalid) or we have shown that all possible constructions lead to contradiction (valid).
  - Let's look at a case of a *valid* argument.