Announcements and Such

- Today's Music: *Pink Floyd*
- HW #4 is due on Thursday @ 4pm, usual drill (chapter 4 — proofs).
- I've posted my solutions for HW #2 and HW #3.
- Grade Curve (so far). Take the average of:
  1. your average HW score (all on 100-point scale), and
  2. your mid-term score.
     - The approximate "curve" for the course is as follows: A-ish (≥ 90), B-ish (80–90), C-ish (70–80), D-ish (60-70).
- This should be a reasonably good (but not perfect) guide to where the (overall) grade curve will end-up for the entire course.
- Today: Chapter 4, Finalé + Chapter 5, Intro.

☞ **You should be doing as many proofs as you can.**

The Rule of Definition for the Biconditional

Rule of Definition for ↔ (Df): If $\ulcorner(p \rightarrow q) \,\&\, (q \rightarrow p)\urcorner$ occurs as the entire formula at line j, then at line k we may write $\ulcorner p \leftrightarrow q\urcorner$, labeling the line 'j Df' and writing on its left the same numbers as are on the left of j. Conversely, if $\ulcorner p \leftrightarrow q\urcorner$ occurs as the entire formula at a line j, then at line k we may write $\ulcorner(p \rightarrow q) \,\&\, (q \rightarrow p)\urcorner$, labeling the line 'j Df' and writing on its left the same numbers as are on the left of j.

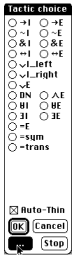
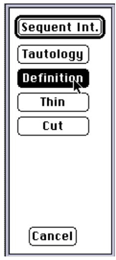
$$\begin{array}{llll} \mathrm{a}_1,\ldots,\mathrm{a}_n & (\mathrm{j}) & (p \rightarrow q) \,\&\, (q \rightarrow p) & \\ & \vdots & & \\ \mathrm{a}_1,\ldots,\mathrm{a}_n & (\mathrm{k}) & p \leftrightarrow q & \mathrm{j\ Df} \end{array}$$

OR

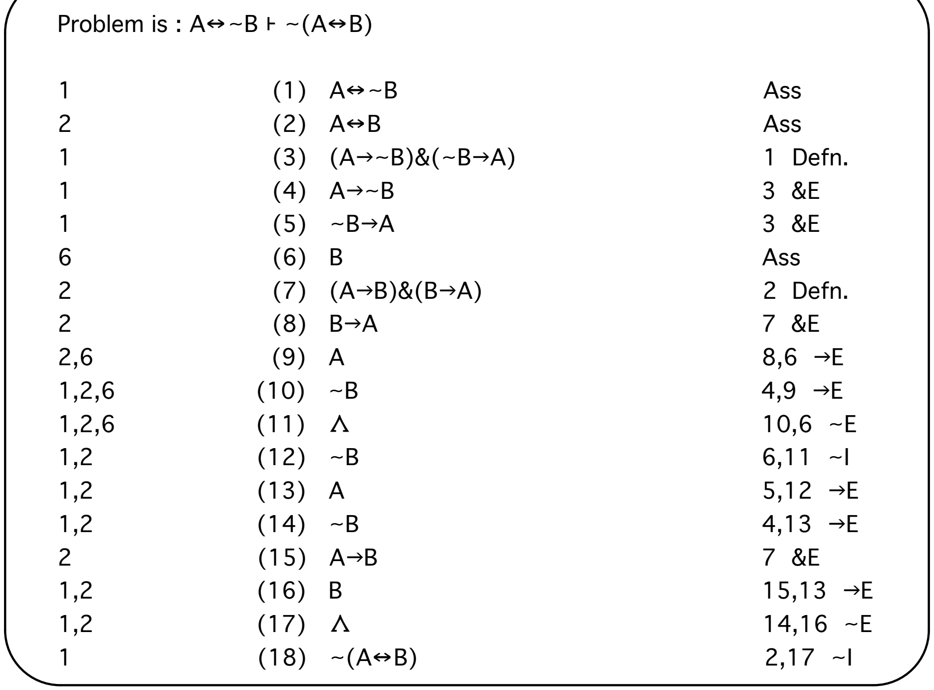
$$\begin{array}{llll} \mathrm{a}_1,\ldots,\mathrm{a}_n & (\mathrm{j}) & p \leftrightarrow q & \\ & \vdots & & \\ \mathrm{a}_1,\ldots,\mathrm{a}_n & (\mathrm{k}) & (p \rightarrow q) \,\&\, (q \rightarrow p) & \mathrm{j\ Df} \end{array}$$

Using ↔ in MacLogic

- Using the Definition strategy of MacLogic (accessed *via* the `…` button), we can implement our Df. rule for ↔. *Do not use* ↔I *or* ↔E!
- Using MacLogic's Definition strategy is much simpler than using its Tautology strategy (I did that last time, which was cumbersome).

☞ To get to Definition, first:  then .

- Here is a non-trivial example: $A \leftrightarrow {\sim}B \vdash {\sim}(A \leftrightarrow B)$. Let's try to tackle this one, using MacLogic's Definition strategy for our Df.
- The shortest proof I've been able to find is 18 steps (next slide). Forbes gives a 20-stepper in his discussion of this example (p. 118).



Problem is : A↔~B ⊢ ~(A↔B)

1	(1)	A↔~B	Ass
2	(2)	A↔B	Ass
1	(3)	(A→~B)&(~B→A)	1 Defn.
1	(4)	A→~B	3 &E
1	(5)	~B→A	3 &E
6	(6)	B	Ass
2	(7)	(A→B)&(B→A)	2 Defn.
2	(8)	B→A	7 &E
2,6	(9)	A	8,6 →E
1,2,6	(10)	~B	4,9 →E
1,2,6	(11)	Λ	10,6 ~E
1,2	(12)	~B	6,11 ~I
1,2	(13)	A	5,12 →E
1,2	(14)	~B	4,13 →E
2	(15)	A→B	7 &E
1,2	(16)	B	15,13 →E
1,2	(17)	Λ	14,16 ~E
1	(18)	~(A↔B)	2,17 ~I

Sequent and Theorem Introduction: I

- You may have noticed that certain important sequents or theorems tend to get proven over and over again in different problems.
- For instance, the sequent $X \vee Y, {\sim}X \vdash Y$ is a very useful thing to know, as are the sequents $X \rightarrow Y, {\sim}Y \vdash {\sim}X$, $\curlywedge \vdash X$, and many others.
- It would be nice if we had a rule that allowed us to say "OK, I've proven this sequent already, so I don't have to prove it again here".
- We have two such rules. They are called *Sequent Introduction* (SI) for sequents, and *Theorem Introduction* (TI) for theorems.
- SI and TI allow us to avoid having to re-solve certain sub-problems that we already know how to solve. This makes proofs shorter.
- We will have a fixed list of sequents and theorems that we'll be allowed to use in conjunction with SI and TI.

Sequent and Theorem Introduction: II

- Forbes lists a bunch of sequents and Theorems on page 123 that we may use with SI or TI. There's a MacLogic file containing all of them.
- Here are a few of the sequents and theorems that tend to be useful:

$$p \vee q, {\sim}p \vdash q\text{; or; } p \vee q, {\sim}q \vdash p \tag{DS}$$

$$p \rightarrow q, {\sim}q \vdash {\sim}p \tag{MT}$$

$$p \vdash q \rightarrow p\text{; or; } {\sim}p \vdash p \rightarrow q \tag{PMI}$$

$$\vdash p \vee {\sim}p \tag{LEM}$$

$${\sim}(p \,\&\, q) \dashv\vdash {\sim}p \vee {\sim}q \tag{DEM}$$

$${\sim}(p \vee q) \dashv\vdash {\sim}p \,\&\, {\sim}q \tag{DEM}$$

$${\sim}({\sim}p \vee {\sim}q) \dashv\vdash p \,\&\, q \tag{DEM}$$

$${\sim}({\sim}p \,\&\, {\sim}q) \dashv\vdash p \vee q \tag{DEM}$$

$$\curlywedge \vdash p \tag{EFQ}$$

$$p \,\&\, (q \vee r) \dashv\vdash (p \,\&\, q) \vee (p \,\&\, r) \tag{DIST}$$

Sequent and Theorem Introduction: III

- Remember the proof for #9 above: $\vdash (A \rightarrow B) \vee (B \rightarrow A)$.

1	(1)	~((A→B)∨(B→A))	Assumption (~I)
2	(2)	B	Assumption (→I)
3	(3)	~A	Assumption (~I)
4	(4)	A	Assumption (→I)
2	(5)	A→B	4,2 →I
2	(6)	(A→B)∨(B→A)	5 ∨I
1,2	(7)	⋏	1,6 ~E
1,2	(8)	~~A	3,7 ~I
1,2	(9)	A	8 DN
1	(10)	B→A	2,9 →I
1	(11)	(A→B)∨(B→A)	10 ∨I
1	(12)	⋏	1,11 ~E
	(13)	~~((A→B)∨(B→A))	1,12 ~I
	(14)	(A→B)∨(B→A)	13 DN

Sequent and Theorem Introduction: IV

- Using TI and SI, we can obtain the following much simpler proof:

	(1)	A∨~A	TI (LEM)
2	(2)	A	Assumption (∨E)
2	(3)	B→A	2 SI (PMI)
2	(4)	(A→B)∨(B→A)	3 ∨I
5	(5)	~A	Assumption (∨E)
5	(6)	A→B	5 SI (PMI)
5	(7)	(A→B)∨(B→A)	6 ∨I
	(8)	(A→B)∨(B→A)	1,2,4,5,7 ∨E

- Here, LEM is the theorem $\vdash A \vee {\sim}A$ (which we have already proven), and PMI stands for either of the sequents ${\sim}A \vdash A \rightarrow B$ (used at line 6), or $A \vdash B \rightarrow A$ (used at line 3), both of which we've proven.
- SI allows you to use (*any* substitution instance of) *any* sequent that you've already proven to make an inference at any stage of a proof.
- TI allows you to write down (*any* substitution instance of) *any* theorem that you have already proven at *any* stage of a proof.

The Formal Definitions of SI and TI

- **Sequent Introduction** (SI). Suppose $r_1, \ldots, r_n \vdash s$ is a *substitution-instance* of the sequent $p_1, \ldots, p_n \vdash q$ which we have already proved, and that the formulae $r_1, \ldots, r_n$ occur at lines $\mathrm{j}_1, \ldots, \mathrm{j}_n$ in a proof. Then we may infer s at line k, labeling the line '$\mathrm{j}_1, \ldots, \mathrm{j}_n$ SI (Identifier)' and writing on the left all numbers which appear on the left of lines $\mathrm{j}_1, \ldots, \mathrm{j}_n$.
- **Theorem Introduction** (TI). If $\vdash s$ is a *substitution-instance* of some theorem $\vdash q$ which we have already proved, we may introduce a new line k into a proof with the formula s at it and no numbers on its left, labeling the line 'TI (Identifier)'.
- 'Identifier' stands for the name of a sequent or theorem that has already been proven (*e.g.*, MT, DS, PMI, LEM, *etc*). See Forbes's list.
- Note: TI is just a *special case* of SI (with $n = 0$).

SI and TI: A Relatively Easy Example

- Use SI/TI to find a "short" proof of: $\sim(A \to (B \vee C)) \vdash (B \vee C) \to A$.

Problem is : ~(A→(B∨C)) ⊢ (B∨C)→A

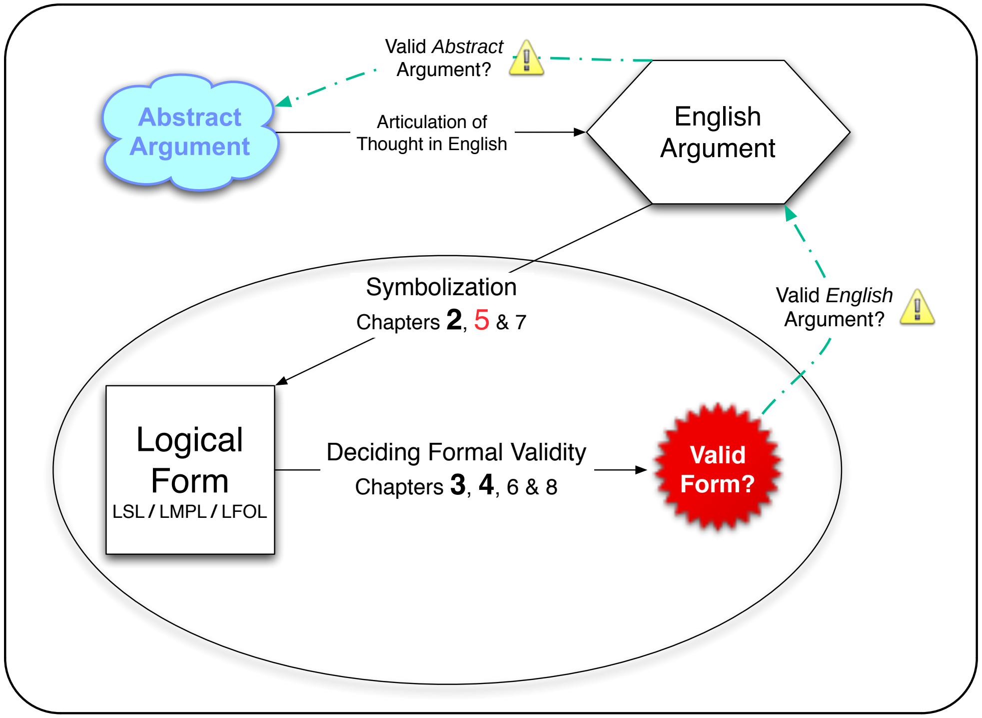
1	(1)	~(A→(B∨C))	Premise
1	(2)	A&~(B∨C)	1 SI Neg-Imp1
1	(3)	A	2 &E
1	(4)	(B∨C)→A	3 SI PMI1

SI and TI: A More Challenging Example

- Use SI/TI to find a "short" proof of: $A \to (B \vee C) \vdash (A \to B) \vee (A \to C)$.

Problem is : A→(B∨C) ⊢ (A→B)∨(A→C)

1	(1)	A→(B∨C)	Premise
1	(2)	~A∨(B∨C)	1 SI IMP1
3	(3)	~A	Assumption (∨E)
3	(4)	A→B	3 SI PMI2
3	(5)	(A→B)∨(A→C)	4 ∨I_left
6	(6)	B∨C	Assumption (∨E)
7	(7)	B	Assumption (∨E)
7	(8)	A→B	7 SI PMI1
7	(9)	(A→B)∨(A→C)	8 ∨I_left
10	(10)	C	Assumption (∨E)
10	(11)	A→C	10 SI PMI1
10	(12)	(A→B)∨(A→C)	11 ∨I_right
6	(13)	(A→B)∨(A→C)	6,7,9,10,12 ∨E
1	(14)	(A→B)∨(A→C)	2,3,5,6,13 ∨E



Valid *Abstract* Argument?

Abstract Argument

Articulation of Thought in English

English Argument

Symbolization

Chapters **2**, **5** & 7

Valid *English* Argument?

Logical Form

LSL / LMPL / LFOL

Deciding Formal Validity

Chapters **3**, **4**, 6 & 8

Valid Form?

Chapter 5: Predication and Quantification

- Consider the following two arguments:

① Socrates is wise.
∴ Someone is wise.

② Everyone is happy.
∴ Plato is happy.

- Intuitively, both ① and ② are *valid* (*why*?). But, if we try to translate these into LSL, we get the *in*valid LSL forms:

①$_{\text{LSL}}$ S
$\therefore W$

②$_{\text{LSL}}$ H
$\therefore P$

- In LSL, we are not able to capture the *logical structure* shared between premises and conclusions of these kinds of arguments.
- If it's not *atomic sentences* that the premises and conclusions of such arguments have in common (structurally), then what *is* it?
- This is what Chapter 5 is about. . .

Predication and Quantification: II

- We need a *richer language* than LSL — one which accurately captures the deeper *logical structure* of arguments like ① and ②. New Jargon:
- A **predicate** is something which *applies to* an object or *is true of* an object or which an object *satisfies. E.g.*, Socrates satisfies the predicate **(is) Wise**.
- A **proper name** is a word or a phrase which *stands for*, or *refers to*, or *denotes* a specific person, place, or thing. *E.g.*, 'Socrates' is a proper name.
- **Quantifier phrases** specify *quantities. E.g.*, 'someone' means *at least one* person and 'everyone' means *all* people. 'Some' and 'all' are **quantifiers**.
- The collection of objects to which the quantifiers in a statement are *relativized* is called the **domain of discourse** of the statement (*e.g.*, 'some*one*' quantifies only over *people*, 'some*time*' quantifies over *times*).
- Chapter 5 introduces the logical language LMPL (the Language of Monadic Predicate Logic) that contains these elements (and a few more tricks).

Symbolization in LMPL I: New Atomic Sentences

- Among the atomic sentences of LMPL (*in addition to LSL sentence letters*) are (new) strings of the form $\ulcorner Xn \urcorner$, where 'X' is a (monadic) predicate, and 'n' is an individual constant (*i.e.*, a proper name).
- We will use the lower-case letters 'a'–'s' as *individual constants* ('t'–'z' are used as *variables* — much more on variables later).
- Some examples of these new kinds of atomic sentences:
  - 'Branden is tall.' ↦ 'Tb'.
  - 'Honda is an automobile manufacturer.' ↦ 'Ah'.
  - 'New York is a city.' ↦ 'Cn'.
- As in LSL, we can *combine* different LMPL atomic sentences using the sentential connectives to yield complex sentences. For instance:
  - 'Branden is tall, but Ruth is not tall.' ↦ '$Tb \,\&\, {\sim}Tr$'.

Symbolization in LMPL II: The Role of Variables

- So far, we can only symbolize sentences about *particular things*. We also want to be able to symbolize sentences like '*Someone* is wise.'
- 'Someone is wise' is called an *existentially quantified* sentence. This is because it asserts that *there exists at least one wise person*.
- Such statements are *not* about *particular* individuals. So, it would *not* be right to symbolize 'Someone is wise' as 'Ws', since 's' is an *individual constant*. This is where *variables* ('t'–'z') enter LMPL symbolizations.
- Intuitively, what want to be able to say is something like:

  There exists at least one person x such that x is wise.

- 'x' is a *variable* which ranges over *all* of the objects (*viz.*, *people*) in our domain of discourse. It does *not* denote any *particular* person.
- So far, so good. But, how do we deal with the *quantifier* 'some' *itself*?

Symbolization in LMPL III: The Existential Quantifier

- The quantifier ‘some’ is captured using the new symbol ‘$\exists$’ of LMPL.
- For instance, ‘Someone is wise’ gets symbolized as ‘$(\exists x)Wx$’ in LMPL.
- One must be careful about the *scope* of the existential quantifier. For instance, consider the following two (*non*-equivalent!) sentences:

(1) Someone is happy and someone is wise.
(2) Someone is happy and wise.

- Sentence (1) is symbolized in LMPL as ‘$(\exists x)Hx \,\&\, (\exists x)Wx$’.
- Sentence (2) is symbolized in LMPL as ‘$(\exists x)(Hx \,\&\, Wx)$’.
- How would you symbolize the following sentence in LMPL

‘Some economists are wealthy and some are not.’

using the following dictionary?

E__ : __ is an economist. L__ : __ is wealthy.

Symbolization in LMPL IV: More on $\exists$

- What, exactly, does ‘$\exists$’ *mean*?
- When I say ‘Someone in this room is wise’, I’m asserting the *disjunction* ‘Either Branden is wise, or Mike is wise, or …’ (*i.e.*, ‘$Wb \vee Wm \vee \ldots$’).
- In *finite* domains of discourse, we can always express existentially quantified sentences as disjunctions. But, in *infinite* domains, we cannot, since our language does not permit infinitely long formulas.
- We can use negation, together with the existential quantifier, to express other kinds of quantifiers. For instance, we may symbolize

‘No unwise person is happy.’

in LMPL as: ‘$\sim(\exists x)(\sim Wx \,\&\, Hx)$’.

- How would you symbolize the following sentence?

‘If a wealthy economist exists, then so does a famous mathematician.’

Symbolization in LMPL V: Free *vs* Bound and Open *vs* Closed

- In ‘Wx’, the variable ‘x’ is said to be *free*. But, in ‘$(\exists x)Wx$’, the variable ‘x’ is said to be *bound* by the existential quantifier.
- Formulas like ‘Wx’ which contain free variables are called *open sentences*. Formulas with *no* free variables are called *closed sentences*.
- Only closed sentences assert things that can be either true or false of some particular individuals in the domain of discourse (or the domain).
- For instance, ‘Wx’ says ‘x is wise’. But, since the ‘x’ in ‘Wx’ does not refer to any particular thing, ‘Wx’ can be neither true nor false.
- But, when we *existentially quantify* ‘Wx’, we end-up with ‘$(\exists x)Wx$’, which clearly *does* make an assertion that is either true or false (depending on whether any *particular* person in the domain happens to be wise).
- Which of the following are open/closed? [NOTE: ‘$\exists$’ binds like ‘$\sim$’!]

(1) ‘Ha’	(2) ‘Wx’	(3) ‘$(\exists x)Hx$’	(4) ‘$(\exists x)Hx \,\&\, Wx$’	(5) ‘$(\exists x)Hx \,\&\, Wb$’

Symbolization in LMPL VI: More Examples with $\exists$

- Let’s symbolize the following sentences. Whenever we symbolize in LMPL, we must state our dictionary of monadic predicates, and we must also say what the domain of discourse is over which we are quantifying.

1. No smoggy city is unpolluted.
2. Vampires do not exist.
3. If ghosts and vampires do not exist, then nothing can be a ghost without being a vampire.

- If the dictionary is (where the domain is people in this classroom now):

S__ : __ is standing up at the podium.
W__ : __ is wealthy.
b : Branden

then what do the following two LMPL sentences assert (in English)?

$\sim(\exists x)(Sx \,\&\, Wx)$ $\sim Wb$

Symbolization in LMPL VII: Back to ① and ②

- Now, we are in a position to symbolize in LMPL the argument ① that we saw at the beginning of this lecture:

$$①_{\text{LMPL}} \quad \begin{array}{l} Ws \\ \therefore (\exists x)Wx \end{array}$$

- Since there are only finitely many people, we can see why this argument is valid, by representing its conclusion as a long (but finite!) disjunction, in which its only premise is a disjunct:

$$① \quad \begin{array}{l} Ws \\ \therefore Wa \vee \cdots \vee Ws \vee \cdots \end{array}$$

- We can use a similar trick for argument ②. In that case, it's premise ['$(\forall x)Hx$'] entails a conjunction ['$Ha \,\&\, \cdots \,\&\, Hp \,\&\, \cdots$'], and its conclusion ['Hp'] is one of the conjuncts of that conjunction.

Some Symbolizations Involving ∃

E__ : __ is an even number

P__ : __ is a prime number

G__ : __ is greater than the number 2

a : the number 2

Domain : natural numbers (ℕ)

(1) There exists a prime number and there exists an even number.

$(\exists x)Px \,\&\, (\exists x)Ex$

(2) There exists an even prime number. [$(\exists x)(Px \,\&\, Ex)$]

(3) 2 is an even prime number. [$Ea \,\&\, Pa$]

(4) If 2 is prime, then there are some even primes. [$Pa \to (\exists x)(Px \,\&\, Ex)$]

(5) No number is even if it is prime. [$\sim(\exists x)(Px \,\&\, Ex)$]

- Careful with this one! Why *isn't* this '$\sim(\exists x)(Px \to Ex)$'?
- Compare: No number is even if it is prime and greater than 2.
  - ∗ In LMPL, this is: '$\sim(\exists x)[(Px \,\&\, Gx) \,\&\, Ex]$', which is *true*. *Why?*
  - ∗ Note: '$\sim(\exists x)[(Px \,\&\, Gx) \to Ex]$' is *false*! *Why?*

The *Universal* Quantifier ∀

- To symbolize English sentences like 'Everyone is happy', we will need the *universal* quantifier '∀' (which means 'every' or 'all').

(*i*) We begin with the raw English sentence: 'Everyone is happy'.

(*ii*) Then, we move to the *Logish* form: 'For every x, x is happy'.

(*iii*) Finally, we have the full LMPL symbolization: '$(\forall x)Hx$'.

- As with the existential quantifier, we must be careful with the *scope* of '∀'. How would we symbolize the following two sentences?

(1) 'Everyone is happy and everyone is wise.'

(2) 'Everyone is happy and wise.'

- These sentences get symbolized differently, because they have different (syntactic) *structures*. But, do they have different *meanings*? In Chapter 6, we'll *prove* the answer to this question.

The Universal Quantifier II

- How should one symbolize the following English sentence?

  (3) 'Everyone who is happy is wise.'

- Note: Unlike (1) and (2) above, (3) does *not* have the consequence that *everyone* is happy. So, what, exactly, *does* (3) say?
- (3) says that *if* a person is happy, *then* that person is wise. This suggests the following *Logish* form (*wrt* the domain of people):

  'For every x, if x is happy then x is wise.'

- Now, we are ready for the full LMPL symbolization:

$$(\forall x)(Hx \to Wx)$$

- We will use this same trick to symbolize sentences like 'Every happy person is a wise person' or 'If someone is happy then he/she is wise', which both assert the same thing as (3).

The Universal Quantifier III

- How should one symbolize the following English sentence?

  (4) 'Only happy people are wise.'

- Note: (4) does *not* say that *all* happy people are wise. That is, unlike (3), (4) does *not* say that a person is wise *if* he/she is happy. Rather, (4) says that a person is wise *only if* he/she is happy.
- This suggests the following *Logish* form (domain of people):

  'For every x, x is wise *only if* x is happy.'

- Now, we are ready for the full LMPL symbolization:

$$(\forall x)(Wx \to Hx)$$

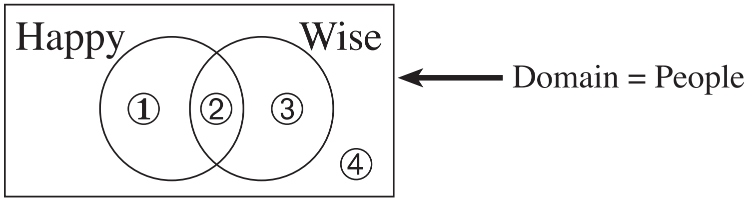
- Here, we have the usual distinction between necessary and sufficient conditions. (3) says that happiness is *sufficient* for wisdom. But, (4) says that happiness is *necessary* for wisdom.

The Universal Quantifier IV, and Venn Diagrams

- Consider the following English sentence:

  (5) 'No one who is unhappy is wise.'

- When trying to paraphrase or symbolize sentences like this in LMPL, it is useful to *picture* what they say using a *Venn Diagram*:



- (5) says that region ③ in the Venn Diagram is empty. So, (5) asserts the same thing as the following LMPL sentence:

  (5.1) $(\forall x)(Wx \to Hx)$

The Intimate Relationship Between ∃ and ∀

- What we have just shown (informally) is:

  '$\sim(\exists x)(Wx \,\&\, \sim Hx)$' *is equivalent to* '$(\forall x)(Wx \to Hx)$'

- This is just a *special case* of the following *general equivalences*:

  $\ulcorner \sim(\exists \nu)\sim\phi\nu \urcorner$ *is equivalent to* $\ulcorner (\forall \nu)\phi\nu \urcorner$
  and
  $\ulcorner \sim(\forall \nu)\sim\phi\nu \urcorner$ *is equivalent to* $\ulcorner (\exists \nu)\phi\nu \urcorner$

- Here, 'ϕ' is a *metavariable* ranging over formulas of LMPL (thought of as functions of ν), and 'ν' ranges over variable symbols of LMPL.
- It follows from the second general equivalence above that $\sim(\exists x)(Wx \,\&\, \sim Hx)$ is equivalent to $\sim\sim(\forall x)\sim(Wx \,\&\, \sim Hx)$. But, this is equivalent to $(\forall x)\sim(Wx \,\&\, \sim Hx)$, hence $(\forall x)(Wx \to Hx)$.
- Our formal semantics will make these relationships more precise.

- Here's *why* (informally) $\ulcorner \sim(\exists \nu)\sim\phi\nu \urcorner$ and $\ulcorner (\forall \nu)\phi\nu \urcorner$ are equivalent.
- Start with the existential claim inside the negation $\ulcorner \sim(\exists \nu)\sim\phi\nu \urcorner$:

$$(\exists \nu)\sim\phi\nu$$

- Next, note that, informally, $(\exists \nu)\sim\phi\nu$ asserts a *disjunction*:

$$\sim\phi a \vee \sim\phi b \vee \ldots$$

- So, by DeMorgan, its negation '$\sim(\exists \nu)\sim\phi\nu$' asserts a *conjunction*:

$$\sim\sim\phi a \,\&\, \sim\sim\phi b \,\&\, \ldots$$

- Then, by Double Negation (DN), we can see this is equivalent to:

$$\phi a \,\&\, \phi b \,\&\, \ldots$$

- But, this just asserts that *every* individual has ϕ. In other words, this says the same thing that the universal claim $(\forall \nu)\phi\nu$ says!
- Therefore, $\ulcorner \sim(\exists \nu)\sim\phi\nu \urcorner$ *is equivalent to* $\ulcorner (\forall \nu)\phi\nu \urcorner$. *QED*.
- We can run a parallel argument for $\ulcorner \sim(\forall \nu)\sim\phi\nu \urcorner$ and $\ulcorner (\exists \nu)\phi\nu \urcorner$.

Further Symbolization Problems

- If someone says “all athletes are not superstars” (another example: “all that glitters is not gold”), they are not to be symbolized exactly as read.
    - Sounds like $(\forall x)(Ax \to \sim Sx)$, but it’s really $\sim(\forall x)(Ax \to Sx)$.
    - Note: this is equivalent to $(\exists x)(Ax \,\&\, \sim Sx)$.
- “The only” gets symbolized like “All”. Example:
    - “The only animals in this canyon are skunks” is $(\forall x)((Ax \,\&\, Cx) \to Sx)$. Where Ax: x is an animal, Cx: x is in this canyon, and Sx: x is a skunk.
    - Clearly, $(\forall x)(Sx \to (Ax \,\&\, Cx))$ is *not* what’s intended. Why?
- “None but”, “none except” and “no … except” are like “Only”. Examples:
    - “None but the brave deserve a Purple Heart” is $(\forall x)(Px \to Bx)$. Where Bx: x is brave, Px: x deserves a Purple Heart.
    - “No birds except peacocks are proud of their tails” is equivalent to “Only peacocks are birds that are proud of their tails”.

LMPL Symbolizations: Summary and Tips

- Some general symbolization forms we’ve seen so far:
    - All Fs are Gs. LMPL: $(\forall x)(Fx \to Gx)$.
    - An F is a G. LMPL: $(\forall x)(Fx \to Gx)$.
    - Fs are Gs. LMPL: $(\forall x)(Fx \to Gx)$.
    - Only Fs are Gs. LMPL: $(\forall x)(Gx \to Fx)$.
    - The only Fs are Gs. LMPL: $(\forall x)(Fx \to Gx)$.
    - Some Fs are Gs. LMPL: $(\exists x)(Fx \,\&\, Gx)$.
    - No Fs are Gs. LMPL: $\sim(\exists x)(Fx \,\&\, Gx)$.
    - Nothing is an F if it’s G. $\sim(\exists x)(Gx \,\&\, Fx)$. [**NOT** $\sim(\exists x)(Gx \to Fx)$!]
    - If anything is an F, then Gs are. LMPL: $(\exists x)Fx \to (\forall x)(Gx \to Fx)$.
    - ‘All Fs are not Gs’ can sometimes *really* be $\sim(\forall x)(Fx \to Gx)$.
    - None but Fs are Gs (or None except Fs are Gs). $(\forall x)(Gx \to Fx)$.
- Remember: $\ulcorner \sim(\exists v)\sim\phi v \urcorner$ *is equivalent to* $\ulcorner(\forall v)\phi v\urcorner$ and $\ulcorner \sim(\forall v)\sim\phi v \urcorner$ *is equivalent to* $\ulcorner(\exists v)\phi v\urcorner$. You should be able to use these proficiently.

- Some equivalences:
    - ‘All Fs are Gs’ is equivalent to ‘No Fs are non-Gs’.
        - $(\forall x)(Fx \to Gx)$ is equivalent to $\sim(\exists x)(Fx \,\&\, \sim Gx)$.
    - ‘All Fs are Gs’ is equivalent to ‘All non-Gs are non-Fs’.
        - $(\forall x)(Fx \to Gx)$ is equivalent to $(\forall x)(\sim Gx \to \sim Fx)$.
    - ‘Some Fs are Gs’ is equivalent to ‘Some Gs are Fs’.
        - $(\exists x)(Fx \,\&\, Gx)$ is equivalent to $(\exists x)(Gx \,\&\, Fx)$.
    - ‘No Fs are Gs’ is equivalent to ‘No Gs are Fs’.
        - $\sim(\exists x)(Fx \,\&\, Gx)$ is equivalent to $\sim(\exists x)(Gx \,\&\, Fx)$.
- Some *non*-equivalences:
    - ‘All Fs are Gs’ is *not* equivalent to ‘All Gs are Fs’.
        - $(\forall x)(Fx \to Gx)$ is *not* equivalent to $(\forall x)(Gx \to Fx)$.
    - ‘Some Fs are non-Gs’ is *not* equivalent to ‘Some Gs are non-Fs’.
        - $(\exists x)(Fx \,\&\, \sim Gx)$ is *not* equivalent to $(\exists x)(Gx \,\&\, \sim Fx)$.
- The LSL equivalences + the general quantifier equivalences yield all.

Further Symbolizations Involving $\forall$ and $\exists$

- How should we paraphrase and/or symbolize the following sentence?

  (6) If anyone is wealthy, then economists are.

- At first blush, we might try to paraphrase (6) as follows:

  (6.1) If everyone is wealthy, then all economists are wealthy (which gives the LMPL symbolization: ‘$(\forall x)Wx \to (\forall x)(Ex \to Wx)$’).

- But, (6.1) *cannot* be right. If the antecedent of (6.1) is true, then *everybody* is wealthy (not just the economists!). In this sense, (6.1) is analogous to an LSL *tautology* — it’s true *in all possible worlds.* Is *that* all (6) asserts?
- In fact, (6) asserts something *much stronger* than (6.1). What (6) says is that all it takes for every economist to be wealthy is for there to exist *one* wealthy person. This leads to the following alternative paraphrase of (6):

  (6.2) If *someone* is wealthy, then all economists are wealthy (which gives the LMPL symbolization: ‘$(\exists x)Wx \to (\forall x)(Ex \to Wx)$’).

Still More Symbolizations Involving ∀ and ∃

- How should we paraphrase and/or symbolize the following sentence?

  (7) Every wealthy logician is happy.

- It helps to do a *Logish*, intermediate form first:

  (7.1) For every x, if x is wealthy and x is a logician, then x is happy.

- This leads to the following LMPL symbolization:

  (7.2) $(\forall x)((Wx \,\&\, Lx) \rightarrow Hx)$

- OK, but what about the following sentence?

  (8) No wealthy economists are happy.

- This time, the *Logish*, intermediate form is:

  (8.1) Not: there is at least one x such that x is wealthy, and x is an economist, and x is happy.

- Which leads to the following LMPL symbolization:

  (8.2) $\sim(\exists x)((Wx \,\&\, Ex) \,\&\, Hx)$

One Last Symbolization Involving ∀

(9) A fetus is a person, but an embryo is not.

- In this case, the domain of discourse must be *wider* than the domain of people (since we need to be able to say that some things are *not* persons). And, ‘is a person’ must then be included as a *predicate* in our dictionary.

  P__ : __ is a person

  F__ : __ is a fetus

  E__ : __ is an embryo

  Domain of Discourse : *all things*

- Now, it helps to do a *Logish*, intermediate form first:

  (9.1) For every x, if x is a fetus then x is a person, and for every x, if x is an embryo then x not a person.

- This leads to the following LMPL symbolization:

  (9.2) $(\forall x)(Fx \rightarrow Px) \,\&\, (\forall x)(Ex \rightarrow \sim Px)$

  which is *semantically equivalent* (as we will *prove* in Chapter 6) to:

  (9.3) $(\forall x)((Fx \rightarrow Px) \,\&\, (Ex \rightarrow \sim Px))$

  But, (9.2) is *preferred* over (9.3), since (9.2) is closer to the *structure* of (9).