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THE ONLY NECESSITY IS VERBAL NECESSITY *

ARE there necessities in nature? The nominalists, and subsequently the empiricists, answered that all necessities are reducible to logical necessity (taken broadly, to include necessity *ex vi terminorum*). What is physically necessary is the same, on this view, as what is logically implied by some tacit antecedent—say, the laws of physics.

There are two problems here. The *second* concerns the status of that tacit antecedent. Is there an objective distinction between physical laws and accidental regularities? Or is any such distinction theory-relative? But the *first* problem is whether, logically, any such reduction is possible. Regardless of what status these laws have, can we really define what is physically necessary as what is logically implied by the laws?

A similar problem about "ought" was broached by A. R. Anderson.[1] He compared deontic logic and alethic logic, and concluded that the former was reducible to the latter by defining 'It ought to be the case that A' as 'It is necessary that, if not A then S'. Here S is a propositional constant, of which we require nothing more than that it not be (or stand for) a contradiction. The letter 'S' was chosen to suggest the word 'sanction': that A ought to be, is true exactly if the falsity of A implies a certain, fixed, bad consequence.

There were here of course two problems, the second following on the first. The *second* problem concerns the status of S. Can we take S to state the simple fact that the generally accepted moral code is violated? Or is the bad thing an objective fact, the violation of natural rights and duties? Or, more tangibly still, hellfire and brim-

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[1] "A Reduction of Deontic Logic to Alethic Modal Logic," *Mind*, LXVII, 265 (January 1958): 100–103.

stone raining down as on Sodom and Gomorrah? Anderson did not solve this second problem, nor even address himself to it. He solved only the first problem: can 'ought' be tenably explicated in terms of implication and special facts?

Of course, he did not solve the problem with which I began either. He reduced deontic modality to alethic modality. Physical and logical necessity are both types of alethic necessity. The problem I wish to consider is whether the alethic necessities generally can be tenably explicated in terms of implication and special facts. The answer will involve the claim that assertions of physical necessity are not only tacitly conditional, but indexical.

I

The dispute has two protagonists: Ray, a realist, and Gnome. Ray's explanation of physical necessity is very orthodox. In each possible world, he says, there are certain real necessities. These may be constituted by individual natures, dispositions, propensities, or simply lawlike facts about the world as a whole. When the real necessities in α are also real necessities in β, then β is physically possible relative to α. What is physically necessary in α is exactly what is the case in every world physically possible relative to α.

Gnome's reaction is that this is correct by and large, but misleading. First, Ray has divided facts into two classes (in each world): those which just are the case, and those which must be the case. This division is acceptable (as long as we postpone discussion of the grounds of division, the way it is made). But Ray's terminology of "real necessities" and "laws" is misleading because it suggests that the two sorts of facts are somehow different in themselves. Surely a real necessity in α is just an ordinary fact that is the case in α by necessity? What *those* facts imply with logical necessity is what is called physically necessary.

Ray counters that his terminology is meant to indicate that the division of facts into two sorts is objective, a matter of fact alone. (He is suspicious that Gnome will say that the division is theory-relative or language-dependent, for example.) But, in any case, what those real necessities imply is *not* the same as what is physically necessary. There are two relations which Gnome is confusing:

$\alpha R\beta$: the real necessities in α are real necessities in β
$\alpha R'\beta$: the real necessities in α are the case in β

This makes quite a difference to logic, for R is transitive but R' is not.

Gnome admits his mistake. What is physically necessary in α is what is implied by the fact that the real necessities in α *are* real necessities. He is not happy with this at all. For it seems that he is admitting something more than "ordinary" facts into world α. Still, he says, as far as necessity is concerned, physical necessity consists in being implied by some suitably chosen facts—and this implication is a matter of logical necessity (in a broad sense).

Ray presses the attack home. No, he says, you must still be confusing things. For if physical necessity were conditional logical necessity, then there would in principle be a sentence QR such that we could give the definition

$$(1.1) \qquad \boxed{R}A =_{\mathrm{df}} \Box(QR \supset A)$$

where $\boxed{R}$ and $\Box$ are the physical necessity and strict necessity operators, respectively. Since R is not symmetric, $\boxed{R}$ should not be of S5 type, whereas $\Box$ is. Yet, as defined, R must also be of S5 type, as an easy calculation will show. There can be no such sentence as QR.

Now Gnome is very unhappy. He decides to try to make his position clearer. The people in α have a sentence, he says, which may be something brief like 'What happen to be our laws of nature, taken *in extenso*, *are* laws of nature'. Whatever that implies is physically necessary for them, and nothing else. Moreover, if what that sentence says were spelled out in full, it would simply divide ordinary facts into two classes, and then say of one class (pointing): "*these* are the necessary facts." So let us call the full spelling out of that crucial sentence, the *law sentence* of α. Surely the people in α hold to be physically necessary exactly what their law sentence implies? What else could they mean?

Ray smiles. There simply cannot be such a thing as that law sentence, he says. Suppose first that, fully spelled out, it actually contains every fact that is really necessary as a conjunct. Then the fully spelled-out structure of $\boxed{R}$ A would be

$$\Box[(\_\_\_\_\_\& A \&\_\_\_\_\_) \supset A]$$

which is a tautology. But surely no assertion of physical necessity is a tautology? On the other hand, if the law sentence simply says that the laws of α, *whatever they may be*, are true, then it gives no useful information at all. For, as far as logical necessity is concerned, the laws of α could be just anything! And then no assertion of physical necessity, except the most trivial, would be true.

Gnome is taken aback; but he does not give up. The law sentence does assert that certain ordinary facts are the case, he says, and these imply tautologically whatever ordinary fact is a real necessity in α. But that those "certain" facts do tautologically imply a specific fact, is not itself a tautology. For indeed, those "certain" facts could have been anything. He looks around for an analogy: perhaps, he says, it is like saying that of course 'snow' has got to denote snow, although the English word 'snow' could have meant rain or sleet or hail.

Ray does not know what to make of this example, and does not really care. There just cannot be any such thing as your law sentence, he says. The people in α and in β would say it the same way: "Our physical laws are physical laws," but, in α, this would have to mean that one class of facts is the class of real necessities, and in β it would have to say about another class of facts that they play the role of real necessities.

This diagnosis is exactly what Gnome needs to make his point. Yes, he says, the law sentence is a sentence that expresses one thing in α and another in β. It is indexical. All assertions of physical necessity are tacitly conditional *and* tacitly indexical. *Being uttered in* α is a contextual factor that goes into the determination of what the law sentence says when it is uttered in α. In just the same way 'I am here' expresses a different proposition depending on who says it when and where. The little proof Ray gave had a hidden assumption: that the sentence QR is not indexical, and that the logic to be used was normal modal logic as opposed to *two-dimensional* modal logic.[2] This logic is the formal part of pragmatics, as opposed to mere semantics.

II

I have talked quite freely about possible worlds and relations among them. Before I go on with this, a disclaimer. In philosophy of logic and of language, we construct models of language. The criteria we wish to satisfy concern mainly the account given of patterns of inference. In principle, these criteria could no doubt be satisfied in different ways; we try to construct models that are "nice" in secondary ways—ease of use, picturability. The items in the models, such as possible worlds, I regard with a suspension of disbelief, as similar to the ropes and pulleys, threads, and little billiard balls that were introduced in nineteenth-century physics. Not everyone views them this way, but I shall not argue the point further.

In the interpretation of a language, we must first model the nonlinguistic part: the candidates for what expressions "stand for." This is where we introduce model structures: mathematical objects consisting of a set K of worlds, various relations R on K, and perhaps some further components.

[2] All bibliographical information about this subject will be given in the last section.

A proposition may be true in some worlds, false in others. It is now customary to identify a proposition with the set of worlds in which it is true. Propositions are what sentences "stand for."

In normal modal logic, modal qualifiers of sentences (such as 'it is necessary that') were correlated with operators on propositions. Three examples are:

$$(2.1) \qquad \Box X = \begin{cases} K & \text{if } X = K \\ \Lambda & \text{otherwise} \end{cases}$$

$$(2.2) \qquad \boxed{R} X = \{\alpha \in K : R(\alpha) \subseteq X\}$$

where $R(\alpha) = \{\beta \in K : \alpha R \beta\}$.

$$(2.3) \qquad X \dashv Y = \Box(\bar{X} \cup Y) = \begin{cases} K & \text{if } X \subseteq Y \\ \Lambda & \text{otherwise} \end{cases}$$

The operators listed here are operators on propositions, not on sentences. They are perfectly good operators; and they may even have some connection with modal qualifiers in English. But just how direct that connection is, is debatable.

III

In normal modal logic, each sentence A stands for a proposition $|A|$, which is a set of worlds. The sentence 'It is necessary that A' stands, then, for the proposition $\Box|A|$. And this is all there is to it.

A number of writers have explored the idea that which proposition a sentence stands for, will vary from context to context. The first was perhaps Strawson, in whose terminology the same sentence could be used on different occasions to make different statements. Strawson's observation is obviously true, and if we ask how logicians have abstracted from this complication, various answers are possible. If on a particular occasion we display a given sentence, and ask whether it is true, what we wish to know is not obscure so long as we take the question also to bear on this same occasion. The answer is *yes* if the statement made (the proposition expressed) on this occasion is true; and *no* otherwise. Thus we have two determinants: the occasion or *context*, and the *facts*. Stalnaker has argued that we must keep these determinants clearly separate: the context determines which proposition is expressed, and the facts determine whether that proposition is true. I am not so convinced of their separability, though I believe this is correct in the main.

In ordinary logic, and in semantics, we can deal with sentences containing indexical expressions just to the extent that the indexicality makes no difference. We consider these *interpretations:* these assign to each sentence a truth value in each world. Thus the contextual factors (who the speaker is, when he speaks, and so on) are fixed in the interpretation. At that level of analysis, 'I am here' is treated exactly like sentences of sufficiently similar structure like 'Peter is in Rome', 'Paul is in Corinth'. In pragmatics, not quite so much gets built into the interpretation. I do not wish to consider cases in which some sentences get to express nothing at all. Therefore, the sort of model I will consider here is to be conceived as follows. We have a single speaker (let it be me). In each world, this speaker is equipped with a certain context. The context determines what proposition is expressed by a sentence A if spoken by the speaker in that context. Next, the world contains facts; these determine whether or not that proposition is true.

Therefore, I am regarding both context and facts as part of a possible world (our actual world, for example), and say that the world determines first what proposition is expressed, and then whether that proposition is true. So I put the intuitive assertion

> A is used on occasion s to make the statement that $\sim p$, which is made true by the facts in the world of occasion s

into the regimented form

> A expresses the proposition that $\sim p$ in world α, and world α makes that proposition true

as the complex, analyzed counterpart of

> Sentence A is true

Moreover, I shall continue to reify propositions as sets of worlds; that is, identify each proposition with the set of worlds in which it is true.

At this point our semantics has been broadened into a pragmatics, and we have a new freedom. The notion of truth as applied to sentences is no longer taken as very basic, being the resultant of two other notions. It is in fact a special case of a more general relation of sentences to *pairs* of worlds:

(a) world β makes true the proposition expressed by A in α
(b) what A expresses in α, is true in β
(c) what A expresses in α, is true in α

Here (a) and (b) are equivalent, and (c) is a special case: A is *true* in α, *simpliciter*.

Frege arrived at the conclusion that truth values and propositions may be identified: there are only the True and the False. But now we must distinguish the proposition expressed by A at α (the *referent*) from the *truth value* of A at α. Let us call this map the *sense* of A. Alternatively and equivalently, we may take the sense of A to be the relation among worlds which sentence A establishes in fashion (a) or (b) above. I shall use the same notation $[A]$ for the map or the relation, either of which may be called the *sense of A*, and write (b) alternatively as

(3.1) $$\beta \in [A](\alpha)$$

(3.2) $$\alpha[A]\beta$$

whichever happens to be convenient at the time.

Logic too gains a new freedom. Consider, for example, the idea that & ("and") is truth-functional. Who would deny it? But that only means that

(3.3) $(A \mathbin{\&} B)$ is true in α iff A is true in α and B is true in α

(3.4) $\alpha[A \mathbin{\&} B]\alpha$ iff $\alpha[A]\alpha$ and $\alpha[B]\alpha$

which is a far cry from the idea that springs to every logician's mind, namely

(3.5) $$[A \mathbin{\&} B] = [A] \cap [B]$$

This equation (3.5) is not forced by truth-functionality, which leaves us free to consider alternatives. This freedom, which may yet be helpful in such extremely nonclassical areas as the logic of relevant implication, I shall not utilize here at all. I shall assume that (3.5) is chosen, as well as the corresponding equation $[\sim A] = K^2 - [A]$, where K is the set of all worlds, for negation. But the very idea that truth-functionality might become such a weak constraint, will indicate what sorts of liberties we are able to take.

IV

There are many modes of modality in pragmatics, even if we remain at the level of logical or verbal notions (utilizing only the sentences, contexts, and facts). Consider, for example,

(4.1) A is true in every world

(4.2) $[A]$ is a reflexive relation: $\alpha[A]\alpha$ for all α

This is necessity, of a sort. The sentence 'I am here' enjoys that status, if every context specifies a speaker, a time of utterance, and a place of utterance. The sentence cannot be false, for if it expresses a proposition at all (and I make the simplifying assumption that all sentences do express something in each world) then it expresses a true proposition. But the proposition expressed is that van Fraassen is in Toronto on April 5, 1976, say; and that is not itself a necessary proposition.

So there is another sort of necessity; namely,

(4.3) What A expresses (in α) is a necessary proposition

$$[A](\alpha) = K \tag{4.4}$$

Note that this is a status that A may have in one world and not in another. A would have both the first status and the second status if its sense was universal:

(4.5) A expresses a necessary proposition in each world

(4.6) For all α, $[A](\alpha) = K$

(4.7) For all α and β, $\alpha[A]\beta : [A] = K^2$

The same multiplication of meanings will occur equally at the object-language level, when we introduce connectives that reflect the status of being true or necessary in one sense or other.

Let us begin with truth. Let WA express the proposition that A is true. The latter proposition is identified here as the set of worlds in which A is true; and WA expresses *that* proposition in every world. For brevity, I shall omit square brackets where I can

$$[\mathrm{W}A](\alpha) = \{\beta : \beta A \beta\} \tag{4.8}$$

$$\alpha[\mathrm{W}A]\beta \text{ iff } \beta A \beta \tag{4.9}$$

Now let $\boxtimes A$ express in α the proposition that A expresses a necessary proposition in α (see 4.3 above). *That* proposition is the set of worlds β such that $A(\alpha) = K$:

$$[\boxtimes A](\alpha) = \{\beta : A(\alpha) = K\} = \begin{cases} K & \text{if } A(\alpha) = K \\ \Lambda & \text{otherwise} \end{cases} \tag{4.10}$$

On the other hand, the proposition that sentence A expresses a necessary proposition, is the set of worlds in which A expresses a necessary proposition. That is the set of worlds β such that $A(\beta) = K$. This gives rise to a second necessity connective, which is, however, definable:

$$[\boxdot A](\alpha) = \{\beta : A(\beta) = K\} = [\mathrm{W}\boxtimes A](\alpha) \tag{4.11}$$

For $\alpha[\mathrm{W}\boxtimes A]\beta$ iff $\beta[\boxtimes A]\beta$ iff $A(\beta) = K$. Finally, we can have a connective that reflects the universal necessity defined by (4.5), and this is also definable:

$$[\Box A](\alpha) = \begin{cases} K & \text{if } [A] = K^2 \text{ and} \\ \Lambda & \text{otherwise} \end{cases} = [\boxtimes\Box A](\alpha) \tag{4.12}$$

For $\alpha[\boxtimes\Box A]\beta$ iff $\Box A(\alpha) = K$ iff, for all β, $A(\beta) = K$. Finally, there are sentences like 'I am here' which enjoy the curious status that they *must be true, but what they say is not necessary:*

(4.13) $$\boxtimes \mathrm{W}A \;\&\; {\sim}\boxtimes A$$

which status could be reflected in yet another connective; but I think we have enough.

We may note in passing that a sentence of form ($\mathrm{W}B \equiv B$) has exactly the status of A in (4.13): it must be true, because of its form, but it does not express a necessary proposition.

The two important unary connectives, as we have now seen, are W and $\boxtimes$. The others introduce subtle distinctions that will play a role only in fairly complex cases. Arthur Prior and Hans Kamp studied similar distinctions in tense logic. The word 'now', if modifying a simple sentence, seems to make no difference at all.

(4.14) It is raining.

(4.15) It is raining now.

These two sentences are true at exactly the same time. Each can be inferred from the other; they are, in that sense, logically equivalent. But they are not substitutable *salva veritate.*

(4.16) (Yesterday) it was the case that it would rain.

(4.17) (Yesterday) it was the case that it would rain now.

If P and F are the past-tense and future-tense connective, respectively, and A the sentence (4.14), then tense logicians symbolized (4.16) as PFA. But substituting (4.15) for (4.14) therein would then give the nonequivalent sentence (4.17). There are similar points to be made about other indexical words; consider, for example:

(4.18) I would like to have more money than I *actually* have.

We can now represent such connections quite generally.

Let us call A and B *materially identical* if $[\mathrm{W}A] = [\mathrm{W}B]$. This means that A and B are true in exactly the same worlds. It does not guarantee that A and B have the same sense—that would be the condition that $[A] = [B]$—and therefore does not warrant substitution *salva veritate* everywhere. What it does license is the inference from A to B and conversely, in any deductive argument. This means also that A and B will play the same role in inferences in which they occur as separate sentences. That is, if $X, A \vdash C$ then $X, B \vdash C$ and also if $X, C \vdash A$ then $X, C \vdash B$. The general metarule that does fail now concerns embedding of inferentially equivalent sentences in large contexts:

(4.19) $A \vdash B$, $B \vdash A$; therefore $\phi(A) \vdash \phi(B)$

For $A \vdash B$ holds if B is true whenever A is true. Therefore the premises establish only that $[\mathrm{W}A] = [\mathrm{W}B]$. But substitution *salva veritate* requires a stronger relationship, namely, $[A] = [B]$, sameness of sense. In our present connectives we already have cases like this: $\boxdot A$ and $\Box A$ are materially identical, but they rarely express the same proposition.

The importance of this point lies in the way it directs our attention to the underdetermination of many of our concepts, meant to apply to our language, but developed in a narrowly conceived logic. For example, I am not sure what the criteria are for being a necessity connective. But, if these criteria concern the role a connective plays in inferences in which no other modal connectives occur (just think of the axioms of various normal modal logics), then they are not likely to distinguish among sentences that are materially identical, but different nevertheless.

V

What is the correct treatment of implication in pragmatics? The first temptation is to analogize old familiar definitions. This suggests that we could symbolize English conditionals by such formulas as $\boxtimes(A \supset B)$ or $\boxtimes(\mathrm{W}A \supset B)$ and so on.

To check these suggestions, we must consider examples in which the antecedent or consequent is an indexical sentence. Making up those examples, we must be careful not to have cross references between antecedent and consequent, because we are talking only about propositional logic. The relation between 'I am here' and 'I am in Toronto', for example, like that between 'John is in Canada' and 'John is in Toronto', cannot be represented at this low level of analysis.

But consider the sentence 'If I am here, then van Fraassen is in Toronto'. Spoken here and now by me, it must have the same truth value as 'If van Fraassen is in Toronto, then van Fraassen is in Toronto'. The latter has the form 'if A, then A', and so is clearly necessary. Thus the first sentence expresses a necessary proposition. Yet it is not a priori true; looking at the sentence by itself will not tell us that it is true. Taking a cue from the last section, we may try to express all this by symbolizing the sentence C. I. Lewis fashion as $\boxtimes(A \supset B)$ and adding that $\sim\boxtimes\mathrm{W}(A \supset B)$. The exact opposite status, in other words, from 'I am here'!

It would be a mistake to conclude from this reasoning that $\boxtimes(A \supset B)$ is the correct symbolization of the conditional. For its only significant premise is that the two sentences mentioned must have the same truth value if spoken by me here and now. Therefore, the argument is no better than that which leads to the conclusion that 'It is raining' and 'It is raining now' have the same correct symbolization. Consider 'If it is the case that if I am here then van Fraassen is in Toronto, *then* van Fraassen is in Toronto'. This is surely true no matter who says it and where! The speaker would surely be relying only on something like modus ponens if he said this. But the above symbolization would yield $\boxtimes[\boxtimes A \supset B) \supset B]$. Granted that the antecedent is true, that expresses the same proposition as $\boxtimes B$, which is false. Indeed, this symbolized sentence expresses here the same proposition as any contradiction—the empty proposition.

The other suggestion I had at the beginning, the symbolization $\boxtimes(\mathrm{W}A \supset B)$, fares even worse. Since A is our friend 'I am here', WA expresses the necessary proposition, and so the symbolized sentence is true only if B expresses the necessary proposition, which it does not.

I have a proposal for the construal of conditionals such as these, which is not definable in terms W, $\boxtimes$, and truth functions:

$$(5.1)\quad [A \to B](\alpha) = \{\beta : A(\beta) \subseteq B(\alpha)\}$$

This is materially identical with $\boxtimes(A \supset B)$, and so the first sentence we considered will have the correct truth value (see below). The difference will come in embeddings. Thus 'If it is the case that, if I am here then van Fraassen is in Toronto, *then* van Fraassen is in Toronto' is symbolized as $(A \to B) \to B$. And here we have

$$\begin{aligned}(5.2)\quad \alpha[A \to B. \to B]\alpha \text{ iff } & [A \to B](\alpha) \subseteq B(\alpha)\\ \text{iff } & \{\beta : A(\beta) \subseteq B(\alpha)\} \subseteq B(\alpha)\end{aligned}$$

Now A is such that $\beta \in A(\beta)$ for all β (for A is 'I am here'). Therefore, if $A(\beta) \subseteq B(\alpha)$, then β is in $B(\alpha)$. This shows that the set on the left is included in the set on the right, hence that the sentence is indeed true in α. We see, therefore, that this new conditional, like 'I am here' has to be true, even though what it says is not necessary.

A different conclusion is to be reached about the simpler 'If I am here then van Fraassen is in Toronto' symbolized $A \to B$. In this case $A(\alpha) = B(\alpha)$, and so $\alpha[A \to B]\alpha$. Indeed, in general, $\beta[A \to B]\beta$ exactly if $\beta[\boxtimes(A \supset B)]\beta$. But there is absolutely nothing necessary about this sentence, in any sense; it is just materially identical with a necessary sentence.

Let us try the proposal on a third example: 'If van Fraassen is in Toronto then I am here'. This cannot be necessary, though it must be true in the present context, where I say it in Toronto (and only because of that). And this is so: $\alpha[B \rightarrow A]\alpha$ exactly if $B(\alpha) \subsetneq A(\alpha)$, and so also in this case where $B(\alpha) = A(\alpha)$. But this is not at all like the paradox of strict implication: although the consequent must be true, that is not what guarantees the truth of this conditional. Indeed, the corresponding conditional 'If van Fraassen is not in Toronto, then I am here' is false as spoken by van Fraassen in Toronto—as it should be—if we symbolize it as $\sim B \rightarrow A$. For in this context/world α, $B(\alpha) = A(\alpha)$ and so $K - B(\alpha)$, which is $\sim B(\alpha)$, is not included in $A(\alpha)$ at all.

More theoretically, is this arrow an implication connective? The criteria for that status cannot be too exact. No doubt the concept of an implication is a cluster concept; passing the test consists in having a good number of the earmarks. Those earmarks are inference rules: modus ponens, modus tollens, transitivity, weakening of the antecedent, and so on. It is noteworthy that none of these concern iterated or embedded conditionals. Although various theories of conditionals propose laws governing iterated conditionals, none of these laws have become generally accepted criteria for such theories. From this it follows that if two conditionals are materially identical, then they will pass the test equally well. Since $A \rightarrow B$ is materially identical with $\boxtimes(A \supset B)$, it will certainly have the earmarks I mentioned above, and various others (such as contraposition) as well. Thus the arrow is indeed an implication connective.

VI

We are now finally in a position to return to physical necessity. What we need is a sentence which, if stated in α, expresses a proposition that is true in all and only those worlds which are physically possible relative to α. This is the *law sentence:* depending on what we take to be the appropriate analysis of physical necessity, this sentence will express in α one of three things. It may say that the laws of α hold (are not violated), or that they are laws, or that they are the only laws. In any case, letting R be the appropriate relation of relative physical necessity, the proposition to be expressed in α is

$$\{\beta : \alpha R \beta\}$$

We might as well symbolize the sentence as R; then we need

$$(6.1) \quad [R](\alpha) = \{\beta : \alpha R \beta\}$$

which also means that, if we view the sense $[R]$ of sentence R as a binary relation, then $[R] = R$.

Now we can evaluate the nominalist reduction that what is physically necessary is exactly what is implied, logically, by that law sentence. This is then the suggested definition:

$$(6.2) \qquad \boxed{R}A =_{df} R \rightarrow A$$

$$(6.3) \qquad [\boxed{R}A](\alpha) = [R \rightarrow A](\alpha) = \{\beta : R(\beta) \subseteq A(\alpha)\}$$

This means that what is expressed by 'It is physically necessary that A' in α is a proposition that is true in β exactly if what A says in α is true in every world that is physically possible relative to β.

We have two ways, basically, to check on this proposal. The first concerns truth *simpliciter:* will the defined sentence $\boxed{R}A$ be true, *simpliciter*, in all and only those cases in which we mean it to be true? The second concerns logic: will the logic of $\boxed{R}$ be exactly what we expected it to be on the basis of semantic (pre-pragmatic) considerations? Both these checks consist really in comparison with normal modal logic. We may expect to pass them, because a quick look at section II, equation (2.2) and equation (6.3) above shows that the two views of $\boxed{R}$ presented in these sections agree:

$$(6.4) \qquad [\boxed{R}A](\alpha) = \boxed{R}[A](\alpha)$$

so it is not inappropriate to have used the same symbol for that operator on propositions and the connective.

Let us look at truth first. $\boxed{R}A$ should be true in α if and only if A is true in every world physically possible relative to α:

$$(6.5) \qquad [\boxed{R}A]\alpha \text{ iff } \alpha A\beta \text{ for all } \beta \text{ such that } \alpha R\beta; \text{ iff for all } \beta, \text{ if } \alpha R\beta \text{ then } \alpha A\beta; \text{ iff } R(\alpha) \subseteq A(\alpha)$$

which is correct. What this means is that $\boxed{R}A$, as defined, is at least materially identical with the correct notion.

Secondly, the logic. Here we know from normal modal logic that logical system M must be correct if R is reflexive, B if in addition R is symmetric, S4 if R is reflexive and transitive, and S5 if R is an equivalence relation. Soundness and completeness proofs are called for. But here our job is very easy. For the soundness and completeness problems here are simply reducible to those in normal modal logic, which were solved by Kripke long ago. The reduction is this. First choose an actual world; call it π. Now define a *truth-value assignment* g as follows: $g(\alpha,A) = T$ if α is in $A(\pi)$, and F otherwise. In that case, we note that $g(\alpha,\boxed{R}A) = T$ exactly if, for all β, if $\alpha R\beta$ then $\pi A\beta$, which means exactly if $g(\beta,A) = T$ for all β such that $\alpha R\beta$. This reduces the model of two-dimensional modal logic to one of normal modal logic, and the soundness and completeness proofs transpose.

VII

We have now come to the end of the argument. I shall add only some remarks on the logical history of the idea. There is no way to assign historical priorities in such a recent confluence of so many rivulets, even if that were desirable. There is now in existence a formal pragmatics, and logics that need, and have been given, a pragmatic rather than (but analogous to) a semantic analysis. None of this was the case ten years ago, but the ideas that made this possible were there.

As I have mentioned, the basic idea that one sentence may be used to make different statements, on different occasions, was already propounded by Strawson, in his well-known critique of Russell's theory of descriptions. That idea was developed, in a more formal and precise way, in Nuel Belnap's theory of conditionals.[3] Belnap himself began with a quote from Quine, which suggested the possibility (but did not advocate) that a conditional says nothing at all if its antecedent is false. In a paper on this topic I showed that Belnap's treatment of conditionals along these lines can also be construed in terms of Strawson's presuppositions, and linked this to Kaplan's treatment of demonstratives (see below).[4]

In section III above I mentioned that within pragmatics we find a great deal of logical freedom. For example, we may insist that the ordinary connectives should obey the rules of truth-functional logic, and indeed, that the truth values of such complex sentences (conjunctions, disjunctions, and the like) should be the usual functions of those of their components, without being committed to the idea that the senses of the sentences form structurally a Boolean algebra. I used the sense/reference terminology here to indicate a link with Roman Suszko's non-Fregean Logic.[5] For that is the only logic in which I had seen such freedom explictly exercised. However, in retrospect, as I also indicated, we may see a similar feature in relevant logics. It is possible that both non-Fregean and relevant logics

[3] "Conditional Assertion and Restricted Quantification," *Noûs*, IV, 1 (February 1970): 1–12; see also J. M. Dunn, "Axiomatizing Belnap's Conditional Assertion," *Journal of Philosophical Logic*, IV, 4 (October 1975): 383–397; and R. Manor, "A Semantic Analysis of Conditional Assertion," *ibid.*, III, 1/2 (January/April 1974): 37–52.

[4] "Incomplete Assertion and Belnap Connectives," in D. Hockney *et al.*, eds., *Contemporary Research in Philosophical Logic and Linguistic Semantics* (Dordrecht: Reidel, 1975) pp. 43–70.

[5] "Abolition of the Fregean Axiom," in R. Parikh, ed., *Logic Colloquium: Symposium on Logic;* (New York: Springer-Verlag, 1975). See also S. L. Bloom and Suszko, "Investigations into the Sentential Calculus with Identity" *Notre Dame Journal of Formal Logic*, XIII, 3 (July 1972): 289–308.

may admit pragmatic analysis, or significant relations with two-dimensional modal logic; I do not know.

In my own acquaintance with the subject, Strawson and Belnap were followed by Stalnaker and Thomason. It is true that Montague called a certain theory he developed pragmatics; but I took that to be a generalized semantics. I became convinced that there was more to it by Stalnaker's papers on pragmatics.[6] This was followed by papers on presuppositions by Stalnaker and Thomason which showed the fruitfulness of the new framework.[7]

Meanwhile A. N. Prior and Hans Kamp had introduced pragmatic elements into the analysis of tense logic.[8] David Lewis gave an analysis of 'actual' as similar to 'now', and Frank Vlach was writing a dissertation on this general subject.[9] Åqvist proposed a new analysis of counterfactuals, which led Segerberg to a general formulation of "two-dimensional modal logic"; it was Segerberg who introduced this term.[10]

And again, meanwhile, David Kaplan was presenting successive versions of his analysis of demonstratives, which contains the most comprehensive scheme of formal pragmatics to date. At this point, the most important and comprehensive papers on the subject are Segerberg's, mentioned above, and Kaplan's on demonstratives (forthcoming). The present paper is meant as a simple application to a philosophical problem on which I have strong feelings. Another philosophical application, which I take to be related, on the a priori but nonnecessary status of the principles of semantics, is being prepared by Richmond Thomason.

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[6] R. C. Stalnaker, "Pragmatics," in G. Harman and D. Davidson, eds., *Semantics of Natural Language* (Dordrecht: Reidel, 1972), pp. 380–397.

[7] Stalnaker, "Pragmatic Presuppositions," in P. Unger and M. Munitz, eds., *Semantics and Philosophy* (New York: NYU Press, 1974); Thomason's paper is unpublished.

[8] Prior, "Now," *Noûs*, II, 2 (May 1968): 101–119; Kamp, "Formal Properties of 'Now'," *Theoria*, XXXVII, 3 (1971): 227–274.

[9] Lewis, "Anselm and Actuality," *Noûs*, IV, 2 (May 1970): 175–188; Vlach, doctoral dissertation, UCLA 1975.

[10] L. Åqvist, "Modal Logic with Subjunctive Conditionals and Dispositional Predicates" *Journal of Philosophical Logic*, II, 1 (February 1973): 1–76; K. Segerberg, "Two-dimensional Modal Logic," *ibid.*, 77–96.