The Philosophical Significance of Gödel's Slingshot

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1. Introduction

A *collapsing argument* is an argument designed to show that there are fewer items of a given kind than might be supposed. Alonzo Church (1943a, 1956), W. V. Quine (1953c, 1953e, 1960), and Donald Davidson (1967a, 1967b, 1967c, 1969a, 1969b, 1990, forthcoming) have used collapsing arguments to undermine several philosophical theses, most notably (i) the thesis that there are *facts* to which true sentences correspond, (ii) the closely related thesis that sentences designate *propositions*, *states-of-affairs*, or *situations*, (iii) the seemingly unrelated thesis that expressions such as "necessarily", "possibly", "probably", "because", and "before" are (on some of their uses) non-truth-functional sentence connectives, and (iv) the thesis that quantifiers and modal operators may be fruitfully combined. In each case there is meant to be some sort of collapse: (i) the class of facts collapses into a singleton (the "Great Fact"); (ii) the class of items capable of serving as the designata of sentences collapses into a class of just two entities (which might as well be called "Truth" and "Falsity"); (iii) the class of sentence connectives satisfying a simple logical condition collapses into the boring class of truth-functional connectives; and (iv) modal distinctions collapse—i.e. "$p \leftrightarrow \Box p$" is valid—in systems that combine modality and quantification.

On the face of it, the threat of such collapses goes well beyond embarrassing certain approaches to natural language semantics. If there are no facts to which true sentences correspond, it is not obvious how facts can function (as some have suggested) as truth-makers, causal relata, and objects of knowledge. If sentences do not designate states-of-affairs or situations, then it is not obvious how such entities can be characterized in ways that make them (as some have suggested) the sorts of things that can be perceived, desired, and brought about by our actions (and inactions). And if words like "necessarily", "because", and "before" cannot

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be treated as non-truth-functional sentence connectives, and if there is no prospect of combining modality with quantification, how can philosophy's favourite non-truth-functional logics be used to elucidate appeals to events, times, causes, facts, states-of-affairs, situations, and propositions, or to tackle problems in metaphysics, ethics, and the philosophy of mind?[1]

According to Davidson (1984b, 1989, 1990, forthcoming) and Rorty (1992a, 1992b), the demise of facts produces further problems. Our thoughts, utterances, and inscriptions often are taken to have content in virtue of being *representations* of reality. These representations can be accurate or inaccurate: those that are accurate are said to be true, to correspond to the facts, to mirror reality (nature, the world). Davidson and Rorty find such locutions unfortunate: not only are they thoroughly intertwined with talk of facts, correspondence theories of truth, states-of-affairs, counterfactual circumstances, and possible worlds, they also underpin talk of scepticism, realism and anti-realism, the subjective-objective distinction, representational and computational theories of mind, and talk of alternative conceptual schemes that represent reality in different ways. Davidson and Rorty reject the representationalist presuppositions of modern philosophy. The time has come, they suggest, to see only folly in the idea of mental and linguistic representations of reality; and with this realization philosophy will be transformed as many of its staple problems and posits evaporate.

I do not want to dwell on these claims here, but I should stress one point: an examination of Davidson's case against representations must include an examination of his case against *facts*, for Davidson's position is basically this: in order to give any substance to the idea of representations of reality, reciprocal substance must be given to the idea that there are facts (that true utterances and beliefs represent). Once the case against facts is made, Davidson believes the case against representations (and the case against correspondence theories of truth) comes more or less free:

> The correct objection to correspondence theories [of truth] is ... that such theories fail to provide entities to which truth vehicles (whether we take these to be statements, sentences, or utterances) can be said to correspond. If this is right, and I am convinced it is, we ought also to question the popular assump-

[1] Russell, Wittgenstein, and Moore all made use of facts. More recently, ontologies of facts have been proposed by, e.g., Armstrong (1993), Barwise and Etchemendy (1989), Barwise and Perry (1981, 1983), Bennett (1988), Clark (1975), Fine (1982), Mellor (1991, 1995), Prior (1967), Searle (1995), Skyrms (1981), van Fraassen (1969), and Vendler (1967, 1967a). The ideas that situations or states of affairs serve as the designata of sentences has been explored by, e.g., Barwise and Perry (1981, 1983), Lewis (1943), Pollock (1984), and Taylor (1976, 1985).

> tion that sentences, or their spoken tokens, or sentence-like entities or configurations in our brains, can properly be called "representations," since there is nothing for them to represent. If we give up facts as entities that make sentences true, we ought to give up representations at the same time, for the legitimacy of each depends on the legitimacy of the other. (1990, p. 304)

Davidson's critique of the fact-representation distinction is a challenge to the presuppositions of much work in modern philosophy, a challenge that can be met by the construction of a viable theory of facts.[2] But there is a collapsing argument, says Davidson, that precludes the articulation of such a theory. The style of argument—used earlier by Church and Quine—is sometimes called the "Frege Argument", a label that has something to do with the fact that Church and Davidson see it in Frege's work, and something to do with the fact that Frege's way of maintaining intuitively plausible compositionality assumptions involves postulating just two entities ("Truth" and "Falsity") to serve as the designata of sentences. In deference to the minimal machinery and presuppositions of the argument, Barwise and Perry (1981, 1983) have dubbed it the "slingshot." In view of the difficulty involved in attributing the argument to Frege, I shall use this label.

There is now an extensive literature on the slingshot, but to my mind most of it is confused: friends and foes alike seem to commit themselves to needlessly strong, often unmotivated, and occasionally ridiculous theses concerning well-formedness, variable-binding, transparency, extensionality, substitutivity, identity, existence, rigidity, direct reference, aboutness, analyticity, causation, intensional entities, domain purification, class membership, the semantics-pragmatics distinction, the semantics of class abstracts and definite descriptions, logical equivalence, logical consequence, logical truth, and logical

[2] The demise of representations is meant also to herald the collapse of conceptual relativism:

> Beliefs are true or false, but they represent nothing. It is good to be rid of representations, and with them the correspondence theory of truth, for it is thinking that there are representations that engenders thoughts of relativism (Davidson, 1989, pp. 162–3).

The idea here is that talk of relativism is encouraged by the idea that a viable distinction can be made between representations and things represented, a distinction that is untenable. In the framework of Davidson's (1984b) earlier work, the intelligibility of relativism presupposes a dualism of *conceptual scheme* and *empirical content*. His central argument against this dualism takes the form of four parallel subarguments, which are meant jointly to undermine the four and only ways of making it viable. As Davidson recognizes, a key premise in one of the subarguments—the argument against schemes *fitting reality*—is that there are no *facts* to which true utterances (or beliefs) correspond.

constants.[3] The way to clear the air and answer the questions raised by slingshot arguments is to reflect carefully upon (i) rules of inference, (ii) the possible semantic treatments of definite descriptions, and (iii) an elegant slingshot proof suggested by Gödel (1944), which has received relatively little attention.[4] According to Gödel, any theory that posits facts to which true sentence correspond must either give up an intuitive principle of compositionality or else presuppose Russell's Theory of Descriptions—or a similar non-referential theory—in order to avoid the "Eleatic" conclusion that all true sentences stand for the same fact. In a footnote, Gödel provides assumptions from which he claims this might be "proved rigorously." It will be extremely rewarding to reconstruct Gödel's proof in detail: (1) the assumptions it employs are less contentious than those employed by Church, Quine, and Davidson; (2) Gödel sees very clearly that the philosophical utility of the slingshot turns crucially on the semantics any would-be giant-slayer is going to ascribe to devices of description (or abstraction);[5] (3) the proof can be converted (uncontroversially) into a proof that anyone who wishes to posit facts, situations, states-of-affairs, or propositions—*whether or not such items are to serve as the designata of sentences*—must take a firm position on the semantics of descriptions; (4) a careful examination of the proof yields virtually everything that is needed to settle a number of vexed questions in philosophical logic and to expose much nonsense in discussions of particular nonextensional logics; (5) the proof can be converted into an elegant test for examining certain philosophical claims and the logical properties of philosophically important linguistic contexts.

This essay should, I believe, answer all technical questions raised by slingshot arguments and encourage people to face the genuine philosoph-

[3] Discussions of the slingshot arguments by Church, Quine, and Davidson include those by Anscombe (1969), Altman *et al* (1979), Barwise and Perry (1981, 1983), Bennett (1988), Burge (1986), Cummins and Gottlieb (1972), Dale (1978), Davies (1978, 1981), Evnine (1991), Føllesdal (1964, 1965, 1966, 1983), Gottlieb and Davis (1974), Harré and Madden (1975), Hochberg (1978, 1984), Horgan (1978, 1982), Levin (1976), Lindström (1991), Lycan (1974), McGinn (1976), Mackie (1974), Mellor (1991), Morton (1969), Neale (1993), Olson (1987), Rosenberg and Martin (1979), Sainsbury (1990), Searle (1995), Sharvy (1970, 1972, 1983), Sleigh (1966), Taylor (1976, 1985), Trenholme (1975), Wagner (1986), Wallace (1969), Wedberg (1966, 1984), Wideker (1985), Williamson (1976), and Wilson (1974).

[4] The only published discussions of Gödel's slingshot I have come across are by Wedberg (1966, 1984), Wallace (1969), Burge (1986), Olson (1987), and Parsons (1990); the argument is mentioned in passing by Morton (1969), Widerker (1985), and Davidson (forthcoming).

[5] Church (1943b, 1956) also seems to be aware of this, but the picture is sharper in Gödel's discussion.

ical questions that Gödel's version poses: (i) Which rules of inference are valid in which linguistic contexts (for example, truth-functional, modal, and causal contexts)? (ii) What are the philosophical and logical consequences of rejecting Russell's Theory of Descriptions? (iii) Is it possible to have useful ontologies of propositions, states-of-affairs, situations or facts? (iv) What are the prospects for representationalist philosophy? It is to facts that we turn first.

2. *Russellian facts*

It is a familiar idea in philosophy that a sentence is a structured entity, and that for certain favoured sentence-forms the semantic powers of certain favoured constituents—so-called "singular terms"—derive from the fact that they stand for things in the world. Thus the occurrence of the name "Sophocles" in the sentence "Sophocles snored loudly" might be said to stand for Sophocles. However, it is much less usual, indeed it is very strained, to say that the occurrences in this sentence of "snored", "loudly", and "snored loudly" stand for things. One moral it might be tempting to draw from this observation is that it is futile to search for entities to correspond all of the parts of a sentence. However, it is common for philosophers and linguists to take such things as verbs, verb phrases, adverbs, connectives, and quantifiers to stand for things, for example properties, relations, sets, and functions. And, more importantly for present concerns, it is common to take whole sentences to stand for such things as truth-values, propositions, facts, states of affairs, or situations.

Frege had the idea that a sentence can stand for either Truth or Falsity, and a number of philosophers, including Church, Gödel, Quine, and Davidson, have claimed to see in Frege's work an argument to the effect that there is no viable alternative to the view that if sentences have references, then there is unique entity *A* for which every true sentence stands and a unique entity *B*, distinct from *A*, for which every false sentence stands. Whether or not such an argument can be found in Frege's work, it is clear that Russell wanted none of this.[6] On Russell's account, a true sentence stands for a *fact*. In "The Philosophy of Logical Atomism" he draws our

[6] Frege (1892) is commonly held to argue that the reference of a sentence is a truth-value, assuming a principle of compositionality to the effect that the reference of a complex expression is determined only by the references of its parts (and their syntactical organization). Since a sentence is a complex expression, the reference,

> ... must remain unchanged when a part of the sentence is replaced by an expression with the same reference. ... What feature except the truth-value can be found that belongs to ... sentences quite generally and remains unchanged by substitutions of the kind just mentioned? (1892, pp. 64–5)

attention to the first of series of "truisms ... so obvious that it is almost laughable to mention them".

> ... the world contains *facts*, which are what they are whatever we may choose to think about them, and ... there are also *beliefs*, which have reference to facts, and by reference to facts are either true or false. (1918, p. 182)

But what, exactly, are facts, and how are they to be individuated? The following propositions give the basic Russellian picture: (i) facts "just as much as particular chairs and tables, are part of the real world" (1919, p. 183); (ii) a fact is "the sort of thing expressed by a whole sentence, not by a single name. ... We express a fact, for example, when we say that a certain thing has a certain property, or that it has a certain relation to another thing" (1918, pp. 182–3); (iii) facts are "complexes" of *objects* (particulars) and *properties* (universals); (iv) the (major) "constituents" of a true sentence correspond to the "components" of the fact to which the sentence corresponds; (v) facts are individuated by their components and the way they are related to one another. We have here all the ingredients of what is often called a "correspondence theory" of truth. Chief amongst them (for our purposes) is the idea that true beliefs and sentences correspond to (stand for) *facts*, construed as non-linguistic entities.

A notation for representing facts, situations, states-of-affairs events, or propositions cannot solve philosophical problems; but sometimes it can serve a point. With a view to highlighting the fact that Russell's facts have objects and properties as components, I want to borrow a notation used by van Fraassen (1969) in a similar context.[7] Take a true atomic sentence

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This passage appears to admit of a weaker and a stronger interpretation. On the weaker reading, Frege is arguing that if the reference of a sentence can be altered only by replacing one of its parts *X* by an expression *Y* that does not have the same reference as *X*, then, all true sentences have the same reference, and similarly all false ones. On the stronger reading, Frege is arguing, given the same condition, that the reference of a sentence must be a truth-value because the truth-value of a sentence is the only *semantically relevant entity* associated with a sentence that survives all substitutions of coreferential expressions. The stronger reading appears to be supported by the fact that in the next paragraph Frege says,

> If now the truth-value of a sentence is its reference, then on the one hand all true sentences have the same reference and so, on the other hand, do all false sentences. (1892, p. 65)

There is more motivating Frege's idea that sentences refer to truth-values than is suggested by the argument just mentioned. For an illuminating discussion see Burge (1986).

[7] Van Fraassen aims to show that facts can provide a semantic explication of "tautological entailment" in the sense of Anderson and Belnap (1966). Nothing of consequence turns on using van Fraassen's notation (or his sketch of "conjunctive" facts). I do not mean to be committing myself to any of van Fraassen's theses

"Fa"; van Fraassen uses "$\langle F, a\rangle$" for the complex *that-Fa*, and says that the fact

$$\{\langle F, a\rangle\}$$

makes "*Fa*" true.[8] $\{\langle F, a\rangle\}$ has as its components F (the property for which the predicate *"F"* stands) and a (the object for which the term *"a"* stands).[9]

On Russellian account, then, the (true) sentence (1) stands for the fact given by (1′):

(1) Kurt is mortal

(1′) $\{\langle Kurt, mortal\rangle\}$.

This fact has as its components (i) Kurt (the person himself), corresponding to "Kurt", the singular term occupying the subject position of (1), and (ii) the property of being mortal (given here by "*mortal*"), corresponding to the predicate expression "is mortal". (1) might be said to "depict" (1′); and the structure of (1) might be said to "mirror" the structure of (1′). (While essential to the philosophical projects that Russell set himself, this is inessential to most the points I shall be making.)

An important question that must be faced by any theory that purports to get at truth by way of facts concerns quantified sentences. To what facts do the following (true) sentences correspond?

(2) Every human is mortal

(3) Some humans are mortal.

According to Russell, (2) corresponds to a *general* rather than a *particular* fact. In order to sidestep questions that are not relevant to present con-

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(or their denials), or even to a theory of facts at all. Similar quasi-set-theoretic notations are used by a number of philosophers to represent situations (e.g., Barwise and Perry, 1983), states-of-affairs (e.g., Taylor, 1976, 1985), events (e.g., Kim, 1993), and propositions (e.g., Kaplan, 1978). There is nothing wrong with such notation *per se*, but it is a mistake to read too much philosophy into it.

[8] If we are Russellians, we can think of "$\{\langle F, a\rangle\}$" as a definite description of a fact—"the fact that *Fa*"—though not a name of that fact.

[9] Why does van Fraassen put braces around "$\langle F, a\rangle$"? Consider a true non-atomic sentence, such as the conjunction *"Fa . Gb"* or the disjunction *"Fa* $\vee$ *Gb"*. Russell hoped to avoid postulating "conjunctive" or "disjunctive" facts (more generally, "molecular" facts) to which such sentences correspond. Nothing of vital importance to present concerns turns on any decision taken about such entities; but for thoroughness, continuity, and simplicity I propose to follow van Fraassen (and others who feel that fact theorists will probably need molecular facts) and say that $\{\langle F, a\rangle\}$ makes the disjunction *"Fa* $\vee$ *Gb"* true (and that $\{\langle G, b\rangle\}$ also makes it true), and that the "conjunctive" fact $\{\langle F, a\rangle, \langle G, b\rangle\}$ makes the conjunction *"Fa . Gb"* true.

cerns, we can adopt a neo-Russellian account of general facts rather than Russell's own suggestions. The general fact to which (2) corresponds might be represented as follows:

(2′) $\{\langle\langle every,\ human\rangle,\ mortal\rangle\}$.

And we might think of this fact having as its components (i) the logical complex composed of (a) the property of being human, and (b) the *every*-relation (a relation that holds between pairs of properties $\langle P,Q\rangle$—here represented by "$\langle\langle$—, $P\rangle$, $Q\rangle$"— if and only if there is nothing that has P that does not also have Q), and (ii) the property of being mortal.[10]

Of course, talk of properties and relations as components of facts will not be to everyone's taste, especially when it is stressed that, on Russellian accounts, properties are not to be construed extensionally, i.e. coextensional predicates need not stand for the same property.

3. *Russellian descriptions*

For Russell, sentence (4) stands for a *general* fact because definite descriptions are treated quantificationally (rather than referentially):

(4) The king is mortal.

Considerable confusion will be avoided later if this point is spelt out immediately. According to Russell's Theory of Descriptions, (an utterance of) a sentence of the form ⌜the F is G⌝ is true if, and only if, every F is G and there is exactly one F. So whereas the "logical form" of a sentence of the form ⌜α is G⌝ can be given by a formula of the form ⌜$G\alpha$⌝, the logical form of a sentence of the form ⌜the F is G⌝ is given by a quantificational formula:

(5) $(\exists x)((\forall y)(Fy \leftrightarrow y = x) \bullet Gx)$.

In *Principia Mathematica*, a definite description ⌜the F⌝ is represented by a pseudo-term of the form "$(\iota x)Fx$", which can be read as "the unique x such that Fx." The *iota*-operator looks like a variable-binding operator for creating a term from a formula ϕ: a simple one-place predicate symbol G may be prefixed to a description ⌜$(\iota x)\phi$⌝ to form a formula ⌜$G(\iota x)\phi$⌝. But for Russell, a phrase of the form ⌜$(\iota x)\phi$⌝ is not a genuine term; it is an abbreviatory device that permits (provably legitimate) shortcuts in the course of proofs, and the use of pseudo-formulae that are sometimes easier to grasp

[10] For the Russellian, the fact corresponding to (3) differs from (2′) only in that its first component is a logical complex that has as a component not the *every*-relation but the *some*-relation—a relation that holds between pairs of properties $\langle P,Q\rangle$ if and only if there is something that has P that also has Q.

than the genuine formulae for which they go proxy. Thus "$G(\iota x)Fx$" is just shorthand for (5) above (in much the same way that "Ph.D." is shorthand (via Latin and abbreviation) for "Doctor of Philosophy").

Superficially, complications arise in the use of Russell's pseudo-terms because of matters of scope. The formula "$\neg G(\iota x)Fx$" is ambiguous as there is not, on Russell's account, a unique formula for which it is an abbreviation:

(6) $\neg(\exists x)((\forall y)(Fy \leftrightarrow y = x) \bullet Gx)$

(7) $(\exists x)((\forall y)(Fy \leftrightarrow y = x) \bullet \neg Gx)$.

When using their abbreviatory notation, Whitehead and Russell introduce a device for representing the scope of a description: they place a copy of it within square brackets appended to the front of the formula that constitutes its scope. Thus in the abbreviatory notation, (6) and (7) are represented as (6′) and (7′) respectively:

(6′) $\neg[(\iota x)Fx]\ G(\iota x)Fx$

(7′) $[(\iota x)Fx]\ \neg G(\iota x)Fx$.

Where a description has smallest possible scope, it is conventional to omit the scope marker; thus (6)/(6′) can be reduced to "$\neg G(\iota x)Fx$".

The main proposition of the Theory of Descriptions is *Principia Mathematica* *14.01:

*14.01 $[(\iota x)\phi]G(\iota x)\phi =_{\text{df}} (\exists x)((\forall y)(\phi \leftrightarrow y = x) \bullet Gx)$.

On Russell's account, there is no possibility of a genuine referring expression failing to refer, so no predicate letter in the language of *Principia Mathematica* stands for "exists". Russell introduces a second abbreviatory symbol "E!" that may be combined with a description $\ulcorner(\iota x)\phi\urcorner$ to create a second type of pseudo-formula $\ulcorner \text{E!}(\iota x)\phi\urcorner$, which is also to be understood in terms of a contextual definition:

*14.02 $\text{E!}(\iota x)\phi =_{\text{df}} (\exists x)(\forall y)(\phi \leftrightarrow y = x)$.

By successive applications of *14.01 and *14.02, any well-formed formula containing a definite description—no matter how complex the matrix—can be replaced by a formula that is description-free. It is clear, then, that the addition of the definite description operator to an ordinary first-order language by way of Russell's contextual definition would not add to the expressive power of the language.

At the appropriate moment, I will examine alternative treatments of descriptions. Right now, I want to prepare for issues raised by Gödel and Quine. Russell's treatment of descriptions is sometimes attacked on the grounds that (i) it is too unfaithful to surface syntax to constitute a serious contribution to a semantic theory, (ii) it gives rise to "the well-known problem of scope" and (iii) it artificially deprives languages of definite descriptions."[11] Such objections are engendered by an insufficiently keen appreciation of the quantificational character of Russell's theory, the distinction between the theory itself and its formal implementation, and the concept of scope. Certainly Russell's implementation of the theory suggests a fairly significant mismatch between surface syntax and "logical form," but it has little to do with descriptions *per se*. In order to characterize the logical forms of quantified sentences such as "every human is mortal" or "some human is mortal" in standard first-order logic we have to use formulae containing sentence connectives, no counterparts of which occur in the surface forms of the sentences. And when we turn to a sentence like "just two men are wise", we have to use many more expressions that do not have counterparts in surface syntax, as well as repetitions of a number that do:

$$(\exists x)(\exists y)[(x \neq y) \bullet \text{man}(x) \bullet \text{man}(y) \bullet \text{wise}(x) \bullet \text{wise}(y) \bullet (\forall z)((\text{man}(z) \bullet \text{wise}(z)) \rightarrow (z = x) \vee (z = y))].$$

So there is no real problem of fidelity to surface syntax that is specific to *descriptions*. The case involving descriptions is a symptom of—and also helps us to see the severity of—a larger problem involving the use of standard first-order logic to characterize the logical forms of sentences of ordinary language. Similarly, where ambiguities of scope arise. If Russell's theory predicts ambiguity where there actually *is* ambiguity in natural language, this is a virtue rather than a vice, and if there is any "problem" it concerns only the fact that the use of Russell's abbreviatory conventions may, on occasion, require the insertion of scope indicators in order to make it clear which of two (or more) unambiguous formulae in primitive notation a particular pseudo-formula is abbreviating.

From the point of view of providing a systematic semantics for natural language there is no need to use Russell's notation (or even the notation of standard first-order logic) in order to capture his insights about the logic and semantics of descriptions. These can be captured perfectly well by treating "the" as a quantificational DETERMINER on a par with "every", "some", "no", "most", etc.[12] For example, we may assume that a determiner *DET* combines with a variable x_k (for any $k \geq 1$) and a formula ϕ to form a restricted quantifier $\ulcorner[\mathit{DET}\ x_k\!: \phi]\urcorner$ (e.g. $\ulcorner[\text{every } x_2\!: \text{man}(x_2)]\urcorner$) which combines with a formula ψ to form a formula $\ulcorner[\mathit{DET}\ x_k\!: \phi]\ \psi\urcorner$ (e.g. $\ulcorner[\text{every } x_2\!: \text{man}(x_2)]\ \text{snores}(x_2)\urcorner$). A truth-conditional semantic theory

[11] See (e.g.) Burge (1978, 1986), Carnap (1947), Føllesdal (1966), Kalish *et al.* (1973), Lambert (1991), Scott (1967), Thomason (1969), and Wedberg (1966, 1984).

[12] See (e.g.) Barwise and Cooper (1981), Evans (1977), Wiggins (1980), Neale (1990), Westerståhl (1989).

could contain axioms like the following (borrowing Tarski's procedure of approaching truth via satisfaction):

(i) $(\forall s, k, \phi, \psi)$ (s satisfies $\ulcorner$[every x_k: ϕ] $\psi\urcorner$ iff every sequence satisfying ϕ and differing from s at most in the k-th position also satisfies ψ)

(ii) $(\forall s, k, \phi, \psi)$ (s satisfies $\ulcorner$[the x_k: ϕ] $\psi\urcorner$ iff the sequence satisfying ϕ and differing from s at most in the k-th position also satisfies ψ).

(In a Russellian spirit, the right-hand side of (ii) is to be understood as shorthand for "there is exactly one sequence satisfying ϕ and differing from s at most in the k-th position and every such sequence satisfies ψ." For further discussion, see Neale (1993a).)[13]

Such an implementation of Russell's theory has a great deal to recommend it. For one thing, it draws out the syntactic and semantic similarities between "every", "some", "a", "the", and so on, and makes the scope of a description utterly transparent in the formal notation. For example (6)/(6′) and (7)/(7′) above will be rendered as (6″) and (7″) respectively:

(6″) $\neg$[the x: Fx] Gx

(7″) [the x: Fx] $\neg Gx$.

Similarly, if there are viable nonextensional sentence connectives in natural language—something we do not want to assume at this point—analogues of (6″) and (7″) can be used to represent the notorious ambiguities that are claimed to arise in natural language when such connectives co-occur with definite descriptions. For example, "the first person into space was necessarily Gagarin" has two readings:

(8) $\Box$[the x: Fx](x = Gagarin)

(9) [the x: Fx]$\Box$(x = Gagarin).

In view of the need to discuss certain "derived" rules of inference employed by Whitehead and Russell, I will continue to use standard logical notation supplemented with the *iota*-operator in much of the sequel. The introduction of restricted quantifier notation is meant to quell fears about the degree of mismatch between logical and grammatical form and to defuse a worry of Gödel's by indicating how Russell's theory of

[13] The viability of a formal language containing restricted quantifiers shows that the language of *Principia Mathematica* is not an essential ingredient of a theory of quantification and logical form; in particular, it is not an essential ingredient of the Theory of Descriptions, exposing once again the hollowness of the objections raised above. Russell's theory is often put forward as the paradigm case of a theory that invokes a distinction between *grammatical form* and *logical form*, but ironically there is a sense in which it preserves symmetry: the gap between grammatical form and logical form in the case of $\ulcorner$the F is $G\urcorner$ is no wider than it is in the case of $\ulcorner$every F is $G\urcorner$ or $\ulcorner$some F is $G\urcorner$ because "the" is of the same syntactical and semantical category as "every" and "some". The most promising non-Russellian treatments of descriptions will be examined in §12.

descriptions can function as a component of a general and systematic theory of quantified noun phrases in natural language.

Since it analyses descriptions in terms of the familiar devices of first-order quantification theory with identity, Russell's Theory of Descriptions *automatically* handles (i) descriptions whose matrices are satisfied by nothing, (ii) descriptions whose matrices are satisfied by more than one thing, and (iii) descriptions whose matrices contain a pronoun, or some other variable, bound by another quantified noun phrase (e.g. "the mother of every Englishman"). Additionally, the theory predicts and explains ambiguities of scope involving descriptions and sentence connectives like "necessarily" and "possibly". Any rival theory of descriptions must cover the same data, a fact that will be important later. Of course, there may turn out to be a better axiom for "the" than one that encodes Russell's theory—the more viable options will be canvassed in due course—but I do not know of one.

4. Quine on names and descriptions

Russell's Theory of Descriptions ought to be very attractive to those who laud the virtues of "extensionalism" and first-order logic. Besides its evident success, as Quine and others have stressed the theory requires the postulation of no new entities, avoids problematic existence assumptions and truth-value gaps, provides a treatment of descriptions within first-order quantification theory with identity, and captures scope ambiguities and a range of inferences involving descriptions as a matter of first-order logic (for example the fact that $\ulcorner$the F is $G\urcorner$ entails $\ulcorner$some F is $G\urcorner$, $\ulcorner$there is at least one $F\urcorner$, $\ulcorner$there is at most one $F\urcorner$, and $\ulcorner$there is at least one $G\urcorner$).[14]

These evident virtues of Russell's theory have led to numerous examinations of its potential application to expressions other than phrases of the from $\ulcorner$the $F\urcorner$, for example nominals ("Socrates' death"), ordinary proper names ("Socrates"), that-clauses ("that Socrates died in prison"), demonstratives ("that", "this vase"), indexical pronouns ("I", "you"), anaphoric pronouns ("it" as it occurs in, e.g., "A lone shot rang out from the hills; Tex acknowledged it with a smile"). Russell himself claimed that, from certain perspectives, ordinary proper names should be analysed in terms of definite descriptions (a handful of "logically proper names" (basically, "this" and "that") resisting analysis). The precise content of this claim and its relevance to semantics, as opposed to pragmatics, is a matter of debate[15];

[14] See, e.g., Quine (1941, 1953a, 1953b, 1953d, 1960, 1982).

[15] See Kripke (1972), Searle (1979), and Sainsbury (1993) for quite different views.

but in the light of Kripke's work it is now widely held that it is not possible to provide an adequate semantical analysis of ordinary proper names by treating them as *synonymous* with definite descriptions or as having their *references fixed* by description.

In view of what is ahead of us, it will be expedient to say something about Quine's allegiance to the Theory of Descriptions in and the connection he makes between descriptions and names. As an account of descriptive phrases Quine sees only logical and philosophical good coming from Russell's theory.[16] In addition, he suggests that proper names are "frills" that can be omitted, that they can be "trivially" reconstrued as descriptions. The basic idea is this: "Fa" is "equivalent," he says to "$(\exists x)(a = x \bullet Fx)$"; so the former can be rewritten as the latter; so the name "a" need never occur in a formula except in the context "a="; but "a=" can be rendered as a simple one-place predicate "A", uniquely true of the object a; so "Fa" can, in fact, be rendered as "$(\exists x)(Ax \bullet Fx)$", which contains no occurrence of "a"; indeed all occurrences of "a"—or any other name—are everywhere replaceable by combinations of quantifiers, variables, connectives, and predicates.

The Quinean "paraphrase" of "Fa" might be questioned on the following grounds: it is in the nature of a name that it is understood as applying to a single object; but it is not in the nature of a predicate that it is understood as satisfied by a single object; so the "paraphrase deprives us of an assurance of uniqueness that the name afforded." Quine's response to this point is straightforward: if we are worried about uniqueness we can import it explicitly in the way Russell does in his analyses of sentences containing definite descriptions. That is, "$(\exists x)(Ax \bullet Fx)$" can give way to the following:[17]

(10) $(\exists x)((\forall y)(Ay \leftrightarrow x = y) \bullet Fx)$.

Everything that can be said using names, claims Quine, can be said by way of expressions like (10) because the objects that names name are the values of variables. Names are a "convenient redundancy" that can be "restored at pleasure ... by convention of abbreviation" (1941, p. 25). A predication such as "Fa" containing the name "a" can be explained as an abbreviation of (11)—or as an abbreviation of "$(\exists x)(Ax \bullet Fx)$" if the uniqueness condition is cashed out elsewhere. "In effect," Quine adds, "this is somewhat the idea behind Russell's theory of singular descriptions" (1941, p. 25).

It is debatable whether Quine is correct to maintain that names can be eliminated in this way in all linguistic contexts. But for present purposes, we can put aside this worry; the important points are the following: (i) Quine envisions a language in which the devices of quantification, varia-

[16] See, e.g., Quine (1941, 1951, 1953a, 1953b, 1953d, 1982).

[17] This is the Russellian spelling out of "$F(\iota x)(Ax)$" i.e. of "$F(\iota x)(a = x)$".

tion, truth-functional connection, and predication do the work that we normally associate with names; and (ii) he sees this idea as essentially a refinement of, or a twist on, Russell's idea that ordinary proper names can be analysed in terms of definite descriptions.

The "theoretical advantages" of analysing names as descriptions are "overwhelming" says Quine (1953b, p. 167):

> The whole category of singular terms is thereby swept away, so far as theory is concerned; for we know how to eliminate descriptions. In dispensing with the category of singular terms we dispense with a major source of theoretical confusion, to instances of which I have called attention in ... discussions of ontological commitment.[18]

Unfortunately, Quine's own use and understanding of descriptions and of Russell's theory create considerably more confusion than they eradicate. For despite lauding the elimination of singular terms with the help of Russell's theory, Quine argues against nonextensional logics using slingshot and other substitution arguments that are stated and defended in ways that reveal either defection from or misunderstanding of Russell's theory. Of course, it is no *objection* to Quine's arguments that their statements and defences reveal either defection or misunderstanding; objections must concern points of logic upon which Quine is straight-forwardly in error. The reason I allude to confusion in Quine's thinking about descriptions is that it helps to explain *why* he makes the mistakes he does and why others have followed suit. As Russell stressed from the outset, the use of the Theory of Descriptions has interesting and far-reaching consequences for logical issues involving substitutivity. And it is clear that Quine and others have not recognised this, a fact that has a considerable bearing on the interpretation of slingshot arguments. This matter will be investigated in detail shortly. Right now, let us put together facts and descriptions.

5. *Facts and descriptions*

The syntactic and semantic similarities between "the" and the other quantificational determiners suggests using van Fraassen's notation to represent the general fact for which (4) stands as (4′):

(4) the king is mortal

(4′) $\{\langle\langle the, king\rangle, mortal\rangle\}$.

[18] If Kripke is right that it is not possible, in general, to replace every name X in every context by a definite description that is true of the referent of X, then the elimination of singular terms that Quine envisions is not viable.

We can think of this fact having as its components (i) the logical complex composed of (a) the property of being king, and (b) the *the*-relation (a relation that holds between pairs of properties $\langle P, Q\rangle$ if and only if there is exactly one thing that has P and nothing that has P but does not also have Q), and (ii) the property of being mortal.

We have in place only the barest outline of a Russellian account of facts, stripped of many of the features that were of importance to Russell himself (e.g., an account of "negative facts" and a sense-datum epistemology). But it is only a stripped down account we shall need in what follows. A Russellian account of facts is meant to be committed neither to the view that every true sentence stands for a *distinct* fact nor to the view that they all stand for the *same* fact. As Davidson (1969a, 1969b) points out, the challenge for the friend of facts is to come up with something between these poles: if all true sentences stand for the *same* fact, the notion is useless; if every true sentence stands for a *distinct* fact, then as Strawson (1950a) argues, facts can shed no light on truth as they are individuated in terms of true sentences (or statements).

It is within the spirit of a Russellian account of facts that a true sentence ϕ might be reorganized or converted into a related sentence ϕ' that stands for the same fact (in order, say, to highlight a particular expression for some purpose). Suppose (11) is true and stands for the fact given by (11′):

(11) Cicero denounced Catiline[19]

(11′) $\{\langle \textit{Cicero}, \langle \textit{denounced, Catiline}\rangle\rangle\}$.

Then the following sentences (obtained from (11) by "passivization" and "topicalization" respectively) are likely to be viewed as standing for (11′) too:

(12) Catiline was denounced by Cicero

(13) It was *Cicero* who denounced Catiline.[20]

A more interesting case involves coreferring singular terms. If the fact that a true sentence stands for is determined by, and only by, what its parts stand for (and their mode of combination), then certainly two true sentences $\Sigma[\alpha]$ and $\Sigma[\beta]$ will stand for the same fact if they differ only in that the position occupied by a singular term α in $\Sigma[\alpha]$ is occupied by a core-

[19] It is usually held that Russell's final semantics treats ordinary proper names like "Cicero" as definite descriptions to which his Theory of Descriptions applies (for doubts about this interpretation, see Sainsbury (1993)). I shall treat ordinary proper names as singular terms.

[20] To the extent that the fact theorist is happy with "conjunctive" facts, much the same point could be made using sentences of the forms "*Fa* and *Gb*", "*Gb* and *Fa*", "*Fa* but *Gb*", etc. Using van Fraassen's notation, such a theorist might say that each of these sentences stands for/is made true by $\{\langle F, a\rangle, \langle G, b\rangle\}$.

ferring singular term β in $\Sigma[\beta]$. For example, taking "Cicero" and "Tully" to be coreferring singular terms, (11) and (14) both stand for (11′):

(14) Tully denounced Catiline.

By contrast, although Cicero is the author of *De Fato*, on Russell's account

(15) The author of *De Fato* denounced Catiline

stands for a quite different fact, the *general* fact that (i) exactly one individual authored *De Fato* and (ii) every individual who authored *De Fato* also denounced Catiline, i.e. the fact given by (15′):

(15′) $\{\langle\langle \textit{the}, \langle \textit{authored}, \underline{\text{De Fato}}\rangle\rangle, \langle \textit{denounced, Catiline}\rangle\rangle\}$.

If Russell had treated "the author of *De Fato*" as a singular term that referred to Cicero, (15) would stand for (15′), just like (11)-(14). And, according to Gödel, this would have had a surprising and devastating consequence.

6. *Gödel's slingshot*

According to Gödel, there is an important connection between theories of facts and theories of descriptions: if a true sentence stands for a fact, then in order to avoid the collapse of all facts into one, one must give up either an intuitive and straightforward Fregean compositionality assumption or else the idea that definite descriptions are singular terms:

> An interesting example of Russell's analysis of the fundamental logical concepts is his treatment of the definite article "the." The problem is: what do the so-called descriptive phrases (i.e., phrases as, e.g., "the author of *Waverley*" or "the king of England") denote or signify [*footnote*: I use the term "signify" in the sequel because it corresponds to the German word "*bedeuten*" which Frege, who first treated the question under consideration, first used in this connection.] and what is the meaning of sentences in which they occur? The apparently obvious answer that, e.g., "the author of *Waverley*" signifies Walter Scott, leads to unexpected difficulties. For, if we admit the further apparently obvious axiom, that the signification of a complex expression, containing constituents which have themselves a signification, depends only on the signification of these constituents (not on the manner in which this signification is expressed), then it follows that the sentence "Scott is the author of Waverley" signifies the same thing as "Scott is Scott;" *and this again leads almost inevitably to the conclusion that all true sentences have the same signification (as well as all the false ones)* [my italics, SN] [Footnote omitted, SN]. Frege actually drew this conclusion; and he meant it in an

> almost metaphysical sense, reminding one somewhat of the Eleatic doctrine of the "One." "The True"—according to Frege's view—is analysed by us in different ways in different propositions; "the True" being the name he uses for the common signification of all true propositions (1944, pp. 128–9).

Since giving up the compositionality assumption seems impossible, Russell would appear to be fortunate, then, in having his Theory of Descriptions on hand to save his theory of facts from a collapse.

But why does Gödel think a treatment of descriptions as singular terms will precipitate such a collapse? Consider a complex description of the form

(16) $(\iota x)(x = a \bullet Fx)$

read as "the unique x such that x is identical to a and x is F"—where "a" is a singular term (for example, a name). From a formal point of view such a description is unremarkable because its matrix "$(x = a \bullet Fx)$" is a well-formed formula. So on Russell's account, (17) is simply shorthand for the well-formed formula (18):

(17) $G(\iota x)(x = a \bullet Fx)$

(18) $(\exists x)((\forall y)((Fy \bullet y = a) \leftrightarrow y = x) \bullet Gx)$.

Gödel's claim boils down to this: if an expression of the form $\ulcorner(\iota x)\phi\urcorner$ were viewed as a genuine singular term standing for the unique object satisfying ϕ, then by invoking minimal logical principles in connection with formulae containing descriptions of the form of (16) it would be possible to demonstrate that all true sentences must stand for the same fact.

In the footnote omitted from the quotation above, Gödel hints at a proof of his claim:

> The only further assumptions one would need in order to obtain a rigorous proof would be: [G1] that "ϕ(a)" and the proposition "a is the object which has the property ϕ and is identical to a" mean the same thing and [G2] that every proposition "speaks about something," i.e. can be brought to the form ϕ(a). Furthermore one would have to use the fact that for any two objects a . b. there exists a true proposition of the form ϕ(a,b) as, e.g., a ≠ b or a = a. b = b (1944, p. 129).

[G1] The first assumption is less worrying than Gödel's wording might suggest. The footnote does not reveal what he intends by saying that (Γ) and (Γ') "mean the same thing":

(Γ) Fa

(Γ') $a = (\iota x)(x = a \bullet Fx)$.

An examination of the main text (pp. 128–9) quoted above might suggest that he intends "signify the same thing." Whatever Gödel's intention, for the purposes of the argument I shall attribute to him it is both sufficient and necessary that (Γ) and (Γ') stand for the same fact.

[G2] Gödel's second assumption is that any sentence that stands for a fact can be put into predicate-argument form. Without this assumption, his slingshot will show only that all true *atomic* sentences stand for the same fact—of course, this conclusion would be every bit as devastating for the friend of facts, but Gödel thinks the more comprehensive conclusion can be proved. (Presumably Gödel would say that "Socrates snored and Plato snored" can be rendered as "Plato is *an x such that x snored and Plato snored*", and that "all men snore" can be rendered as something like "Clinton is *an x such that all men snore*" (harmlessly assuming a non-empty universe). If such conversions are found repugnant, one can still follow Gödel's argument through in connection with atomic sentences.)

[G3] A third assumption—mentioned not in the footnote but in the earlier quotation from the main text—is the compositionality assumption that "the signification of a composite expression, containing constituents which themselves have a signification, depends only on the signification of these constituents (not on the manner in which this signification is expressed." This Gödel takes to be an "apparently obvious axiom" (I shall return to its interpretation).

A proof that all true sentences stand for the same fact can now proceed as follows.[21] Assume the following three sentences are all true:

(I) Fa

(II) $a \neq b$

(III) Gb.

Then each stands for some fact or other; call the facts in question f_{I}, f_{II}, and f_{III} respectively. By (G1), since (I) stands for f_{I} so does

(IV) $a = (\iota x)(x = a \bullet Fx)$.

By the same assumption, since (II) stands for f_{II}, so does

(V) $a = (\iota x)(x = a \bullet x \neq b)$.

If a definite description $\ulcorner(\iota x)\phi\urcorner$ stands for the unique thing satisfying ϕ, then the descriptions in (IV) and (V) both stand for the same thing, viz. a. So, by (G3), sentences (IV) and (V) stand for the same fact, i.e. $f_{\mathrm{I}} = f_{\mathrm{II}}$. By (G1), since (III) stands for f_{III}, so does

(VI) $b = (\iota x)(x = b \bullet Gx)$.

And by the same assumption, since (II) stands for f_{II} so does

(VII) $b = (\iota x)(x = b \bullet x \neq a)$.

[21] I have benefited from comparing my reconstruction to those by Wedberg (1966, 1984), Wallace (1969), and Olson (1987). My reconstruction is considerably leaner and better suited to the tasks at hand than its predecessors; moreover, I believe its leanness captures Gödel's intentions precisely.

Again, on the assumption that a definite description $\ulcorner(\iota x)\phi\urcorner$ stands for the unique thing satisfying ϕ, the descriptions in (VI) and (VII) stand for the same thing, viz. *b*. So, by (G3), sentences (VI) and (VII) stand for the same fact, i.e. $f_{\mathrm{III}} = f_{\mathrm{II}}$. Thus $f_{\mathrm{I}} = f_{\mathrm{II}} = f_{\mathrm{III}}$, i.e. "*Fa*" and "*Gb*" stand for the same fact. *Mutatis mutandis* where "$a = b$" (rather than "$a \neq b$") is true. So all true sentences stand for the same fact.

If, by contrast, definite descriptions are treated in accordance with Russell's Theory of Descriptions—which has independent motivation of course—then, Gödel claims, the threatened collapse is straight-forwardly avoided. Gödel's main point here is surely that on Russell's account, since descriptions do not stand for things (they are not singular terms), neither (IV) nor (V) can be obtained from the other by the replacement of expressions that stand for the same thing, therefore it does not follow from (G3) that (IV) and (V) stand for the same fact.[22] *Mutatis mutandis* for (VI) and (VII). (Additionally, on Russell's account, there is no reason to think that even (I) and (II) stand for the same fact—similarly (VI) and (VII)—because the facts for which they stand will have different components.) In short, then, Russell avoids the "Eleatic" conclusion because he is a Russellian about definite descriptions.

At this point, we can draw a general moral from Gödel's discussion. Anyone who wishes to maintain that descriptions stand for (signify, refer to, designate, &c.) things, and at the same time hold that sentences stand for things (determined by what their parts stand for), will have to hold that (I) and (IV) stand for different things or accept that all true sentences stand for the same thing. This much is beyond dispute. Russell, as Gödel sees it, is able to avoid the conclusion that all true sentences stand for the same fact by denying that descriptions stand for things, i.e. by denying that they are singular terms. However, Gödel was not entirely convinced Russell was off the hook:

> As to the question in the logical sense, I cannot help feeling that the problem raised by Frege's puzzling conclusion has only been evaded by Russell's theory of descriptions and that there is something behind it which is not yet clearly understood (1944, p. 130).

Gödel does not specify his residual worries, but I am confident he was responding to a superficial feature of Russell's theory that generalised quantifier theory has shown to be completely dispensable. Russell's own implementation of his theory involved defining descriptions contextually, and Gödel must have thought that the effective exclusion of descriptions from the primitive notation was tantamount to a dodge, a ducking of the

[22] This corresponds to the fact that, on a Russellian account, (IV) and (V) stand for different general facts with different components: the property of being *F* is a component of the fact corresponding to (IV) but not the fact corresponding to (V).

real philosophical issue (see also Wallace (1969) and Burge (1986)). This is borne out by Gödel's next (and final) paragraph on the topic:

> There seems to be one purely formal respect in which one may give preference to Russell's theory of descriptions. By defining the meaning of sentences involving descriptions in the above manner, he avoids in his logical system any axioms about the particle "the," i.e., the analyticity of the theorems about "the" is made explicit; they can be shown to follow from the explicit definition of the meaning of sentences involving "the." Frege, on the contrary, has to assume an axiom about "the," which of course is also analytic, but only in the implicit sense that it follows from the meaning of the undefined terms. Closer examination, however, shows that this advantage of Russell's theory over Frege's subsists only as long as one interprets definitions as mere typographical abbreviations, not as introducing names for objects described by the definitions, a feature which is common to Frege and Russell (1944, pp. 130–1).

Certainly in 1943/44 it was not obvious how descriptions, if analysed in accordance with Russell's theory, should fit into a general account of natural language quantification; as Gödel's last sentence reveals, the possibilities that presented themselves at that time were just two in number: (i) descriptions are mere typographical abbreviations, or (ii) they are terms that have their references fixed quantificationally (by overlapping satisfactions). Subsequent work on generalised quantifiers and on the syntax and semantics of natural language reveals a third, and far superior, possibility: (iii) a description ⌜the F⌝ is a quantified noun phrase on an equal footing with ⌜every F⌝, ⌜some F⌝, ⌜no F⌝, etc (see § 3).[23] The fact that Russell's theory can be implemented within a theory of restricted quantification ought to dispel worries about the artificial banishment of descriptions.

In subsequent sections, I will show that Gödel's argument demonstrates all sorts of interesting facts about facts, the semantics of descriptions, and limitations on logics of purportedly nonextensional sentence connectives. In order to get everything clear and in perspective, it is necessary to step back and reflect upon some of the most basic ideas in philosophical logic.

7. *Sentence connectives*

Let us adopt some well-defined theoretical vocabulary and stipulate that terms, predicates, sentences, and sentence connectives all have *extensions*. (i) The extension of a singular term is simply its referent (for ease of expo-

[23] See the work of, e.g., Mostowski (1957), Lindström (1966), Barwise and Cooper (1981), van Benthem (1986), and Westerståhl (1989).

sition, let us agree to exile terms that fail to refer, if there are such expressions). (ii) The extension of an n-place predicate is the set of ordered n-tuples of which the predicate holds. (iii) The extension of a sentence is its truth-value. (Given (ii), (iii) is not entirely arbitrary: two n-place predicates $\Re$ and $\Re'$ have the same extension if, and only if, $\ulcorner(\forall x_1 \ldots x_n)(\Re(x_1 \ldots x_n) \leftrightarrow \Re'(x_1 \ldots x_n))\urcorner$ is true; if a sentence can be viewed as a 0-place predicate, then two sentences ϕ and ψ have the same extension if, and only if, $\ulcorner \phi \leftrightarrow \psi \urcorner$ is true; so as Carnap (1947, p. 26) points out, on such an account it seems "natural" to regard the truth-values of sentences as their extensions.) (iv) The extension of an n-place, truth-functional, sentence connective (e.g. "not" ("$\neg$"), "and" ("$\bullet$"), "or" ("$\vee$"), "if...then" ("$\rightarrow$"), and "if and only if" ("$\leftrightarrow$")) is a function from n-tuples of sentence extensions (i.e. n-tuples of truth-values) to sentence extensions (i.e. truth-values).

Let us call any expression that combines with one or more sentences to form a sentence an *S*-CONNECTIVE. ("Sentence" is to be understood as including open sentences, i.e. it is to be understood as "formula".) Thus the truth-functional connectives just mentioned and expressions such as "necessarily" ("$\Box$"), "possibly" ("$\Diamond$"), "it is causally necessary that" ("$\boxed{c}$"), "because" ("©"), "before", and "after" are all *S*-connectives (on some of their uses).

The SCOPE of an n-place *S*-connective Ω is simply the sentence (sanctioned by the syntax) that results from combining Ω with n sentences $\phi_1 \ldots \phi_k$, i.e. the smallest sentence that contains both Ω and $\phi_1 \ldots \phi_n$.[24]

EXTENSIONAL OPERATORS map *extensions* into extensions, i.e. they operate on the *extensions* of their operands. Consider an expression $\ulcorner \Omega(\phi_1 \ldots \phi_n) \urcorner$ composed of an n-place operator Ω and operands $\phi_1 \ldots \phi_n$. Ω is an extensional operator if, and only if, the extension of $\ulcorner \Omega(\phi_1 \ldots \phi_n) \urcorner$ depends only upon the *extensions* of Ω and $\phi_1 \ldots \phi_n$ (and the syntactical structure of $\ulcorner \Omega(\phi_1 \ldots \phi_n) \urcorner$). Because they take us from the extensions of expressions to the extensions of larger expressions, the class of EXTENSIONAL *S*-CONNECTIVES is a subclass of the class of extensional operators: an n-place *S*-connective Ω is extensional if, and only if, the extension of $\ulcorner \Omega(\phi_1 \ldots \phi_n) \urcorner$ is determined by the extension of Ω and the extensions of the sentences $\phi_1 \ldots \phi_n$. Thus an *S*-connective Ω is extensional if, and only if, any sentence with the same extension (i.e. truth value) as the sentence ϕ_k can be substituted for ϕ_k in $\ulcorner \Omega(\phi_1 \ldots \phi_n) \urcorner$ (where $1 \leq k \leq n$) to produce a sentence with the same extension (i.e. truth value) as $\ulcorner \Omega(\phi_1 \ldots \phi_n) \urcorner$. Since the extension of a sentence is stipulated to be a truth-value, the class of TRUTH-FUNCTIONAL *S*-CONNECTIVES is the same thing as the class of EXTENSIONAL *S*-CONNECTIVES.

[24] In the terminology of tree-geometry, the scope of an *S*-connective (or any other expression for that matter) is the first branching node properly dominating it.

A sentence ϕ is EXTENSIONAL if, and only if, its extension is determined by the extensions of its parts (and ϕ's syntactical structure). If $\phi_1 \ldots \phi_n$ are all extensional sentences and Ω is an extensional *S*-connective, then *any* component—not just an immediate component—of $\ulcorner\Omega(\phi_1 \ldots \phi_n)\urcorner$ can be replaced by a coextensional expression (of the same syntactic category) to produce a sentence that has the same extension (i.e. truth-value) as $\ulcorner\Omega(\phi_1 \ldots \phi_n)\urcorner$. Thus an extensional *S*-connective (i.e. truth-functional *S*-connective) Ω permits the substitution *salva veritate* (henceforth *s.v.*) of coextensional terms, predicates, and sentences (assuming, of course, that the term, predicate, or sentence being replaced is not within the scope of a nonextensional expression that is itself within the scope of Ω).

The issues that I want to address in the sequel require that we at least entertain the possibility of *S*-connectives that are nonextensional. In particular, they require reflection on fragments of English containing purportedly nonextensional *S*-connectives such as the expressions italicised in the following:

(19) The fire broke out *because* (*after/before*) there was a short-circuit.

(20) *Because (after/before)* there was a short-circuit, a fire broke out.

(21) *The fact that* there was a short-circuit *caused it to be the case that* there was a fire.

(22) *The statement that* there was a fire *corresponds to the fact that* there was a fire.

(23) *Necessarily (possibly/probably)* two plus three is five.

(24) *It is physically necessary that* metals expand when heated.

The idea is that, from a syntactic perspective, the italicised expressions in (19) work rather like "and", those in (20) rather like "if", those in (21) and (22) rather like "if...then", and those in (23) and (24) rather like "it is not the case that". If they are *S*-connectives, clearly these expressions are not extensional as they do not permit the substitution *s.v.* of coextensional sentences.

Quine and Davidson have used slingshot arguments to cast doubt upon the viability of nonextensional *S*-connectives. For the moment, I want to bracket such worries. All that is required right now is a grasp of the *intended* difference between extensional and (purportedly) nonextensional *S*-connectives: extensional *S*-connectives allow the substitution *s.v.* of coextensional sentences; nonextensional *S*-connectives do not.

In order to keep things as simple as possible and avoid digressions on semantical issues that are orthogonal to the issues to be discussed here, let us ignore the existence of any purportedly nonextensional operators that are not *S*-connectives (e.g. let us ignore the existence of adjectives and verbs such as "fake", "alleged", "fear", and "want"). Where *X* is a particular occurrence of an expression we can say that (i) X occupies an EXTENSIONAL POSITION, and (ii) X occurs in an EXTENSIONAL CONTEXT, if and only if X is not within the scope of any nonextensional S-connective.

8. Rules of substitution

Rules of inference are truth-preserving. Two philosophically useful rules of inference that can be employed in extensional contexts concern the substitution of coextensional sentences and coextensional singular terms.

(i) *The Principle of Substitutivity for Material Equivalents* (PSME) can be put thus:

(i) PSME:
$$\frac{\phi \leftrightarrow \psi \qquad \Sigma[\phi]}{\Sigma[\psi].}$$

This just says that if two sentences ϕ and ψ have the same truth-value and $\Sigma[\phi]$ is a sentence containing at least one occurrence of ϕ, then $\Sigma[\psi]$ and $\Sigma[\phi]$ have the same truth-value, where $\Sigma[\psi]$ is the result of replacing at least one occurrence of ϕ in $\Sigma[\phi]$ by ψ.

By definition, a context is extensional if, and only if, it permits the substitution of coextensional terms, predicates, and sentences. So it is a truism that PSME is a valid rule of inference in extensional contexts. As shorthand for this, let us say that extensional contexts are +PSME (as opposed to –PSME). And by an obvious extension of terminology, let us say that extensional S-connectives are +PSME.

(ii) *The Principle of Substitutivity for Singular Terms* (PSST) can be depicted thus:

(i) PSST:
$$\frac{\alpha = \beta \qquad \Sigma[\alpha]}{\Sigma[\beta].}$$

This just says that if two singular terms α and β have the same extension (i.e. if $\ulcorner \alpha = \beta \urcorner$ is a true identity statement) and $\Sigma[\alpha]$ is a sentence containing at least one occurrence of α, then $\Sigma[\beta]$ and $\Sigma[\alpha]$ have the same truth-value, where $\Sigma[\beta]$ is the result of replacing at least one occurrence of α in $\Sigma[\alpha]$ by β.

There is a difficulty in applying PSST: it presupposes a clear answer to the question "which singular noun phrases are singular terms?" For the sake of having a provisional answer—I admit, however, to thinking of it as rather more than provisional—let us suppose the class of singular terms to comprise the following: (i) ordinary proper names; (ii) the simple demonstratives "this" and "that"; (iii) complex demonstratives of the forms ⌜this F⌝ and ⌜that F⌝; (iv) the first- and second-person singular pronouns "I", "me", and "you"; and (v) at least some occurrences of the third-person singular pronouns "he", "him", "she", "her", and "it" (including those occurrences that (as Quine and Geach have stressed) function as variables hooked up to quantified noun phrases). In order to get things moving, I have simply stipulated that descriptions are *not* singular terms, reserving the right to redraw the boundaries of the class of singular terms if this provisional characterization proves to be lacking in any way. This decision will not prejudice our inquiries as we will explore the consequences of overturning it at all key points.

By definition, a context is extensional only if it permits the substitution of coextensional singular terms. So it is a truism that PSST is a valid rule of inference when the singular term α occurs in an extensional context. As shorthand for this, let us say that extensional contexts and connectives are +PSST (as opposed to –PSST). (Of course a context or S-connective is +PSME if, and only if, it is *extensional*. Thus any context or S-connective that is +PSME is also +PSST; but nothing on the table guarantees the converse (an argument would be needed to demonstrate it).)

We turn now to two substitution rules that are less familiar. If descriptions are treated in accordance with Russell's theory—or any other theory that does not treat descriptions as singular terms—then, as Russell stressed, substitutions involving descriptions are not licensed directly by PSST.[25] This matter merits some attention as philosophers who appeal or profess allegiance to Russell's Theory of Descriptions often fail to do justice to the point and thereby run into logical difficulties of a type that will concern us very soon.

On Russell's account, what might *look like* an identity statement involving one or two descriptive phrases is really no such thing. An identity statement has the general form ⌜$\alpha = \beta$⌝, where α and β are singular terms. The way PSST was stated, it is the truth of a statement of this form that licenses its applications. But on a Russellian analysis of descriptive phrases, the logical forms of sentences of the superficial grammatical forms ⌜$a =$ the F⌝ and ⌜the $G =$ the F⌝ are given by the following quantificational formulae:

(25) $(\exists x)((\forall y)(Fy \leftrightarrow y = x) \bullet \underline{x = a})$

[25] See in particular Russell (1905, p. 47 and pp. 51–2) and Whitehead and Russell (1927, *14).

(26) $(\exists x)\{(\forall y)(Fy \leftrightarrow y = x) \bullet (\exists u)((\forall v)(Gu \leftrightarrow v = u) \bullet \underline{u = x})\}$.[26]

And neither (25) nor (26) is an identity statement; each is a quantificational statement that *contains* an important identity statement (underlined) as a proper part.

The real force of this point emerges once we reflect on the nature of derivations in first-order logic with identity. The inference in (27) is obviously valid (on the currently standard definition of validity):

(27) [1] Cicero = Tully;
[2] Cicero snored;
―――――――――
[3] Tully snored.

In order to provide a formal derivation of the conclusion from the premises, we can use PSST, which sanctions a direct move from [1] and [2] to [3]:

1	[1]	$c = t$	premiss
2	[2]	Sc	premiss
1,2	[3]	St	1, 2, PSST.

Now consider (28), which looks like a very similar argument.

(28) [1] Cicero = the greatest Roman orator;
[2] Cicero snored;
―――――――――
[3] The greatest Roman orator snored.

Clearly this is valid. But—and this is the important point—*if* definite descriptions are Russellian, then they are not singular terms so we cannot use PSST to move *directly* from lines [1] and [2] to line [3] in the formal analogue of this argument in first-order logic with identity. Reading "Rx" as "x is greatest Roman orator" it might be tempting to set out a derivation as follows:

1	[1]	$c = (\iota x)(Rx)$	premiss
2	[2]	Sc	premiss
1,2	[3]	$S(\iota x)(Rx)$	1, 2, PSST.

But on Russell's treatment of descriptions this derivation is illegitimate because PSST can be invoked only where we have an identity statement, and an identity statement has *singular terms* on either side of the identity sign. Premise [1] is not an identity statement; on Russell's account it is merely *shorthand* for a complex quantificational statement; indeed, the

[26] Or, in restricted quantifier notation, (i) and (ii) respectively:
(i) [the x: Fx] ($\underline{x = a}$)
(ii) [the x: Fx] ([the y: Gy] ($\underline{x = y}$)).

purported derivation is just shorthand for the following illegitimate derivation:

1	[1]	$(\exists x)((\forall y)(Ry \leftrightarrow y = x) \bullet x = \mathrm{c})$	premiss
2	[2]	Sc	premiss
1,2	[3]	$(\exists x)((\forall y)(Ry \leftrightarrow y = x) \bullet Sx)$	1, 2, PSST.

To say that PSST does not sanction a direct move from line [2] to line [3] on the basis of the truth of the entry on line [1] is *not* to say that one cannot *derive* the entry on line [3] from the entries on lines [1] and [2] using standard rules of inference, which include, of course, PSST. Indeed, it is a routine exercise—the logical and philosophical relevance of which is stressed in some of the better introductory logic texts—to provide the relevant derivation:

1	[1]	$c = (\iota x)Rx$	premiss
2	[2]	Sc	premiss
1	[3]	$(\exists x)((\forall y)(Ry \leftrightarrow y = x) \bullet c = x)$	1, def. of "(ιx)"
4	[4]	$(\forall y)(Ry \leftrightarrow y = \alpha) \bullet c = \alpha$	assumption
4	[5]	$c = \alpha$	4, $\bullet$ ELIM
2,4	[6]	$S\alpha$	2, 5, PSST[27]
4	[7]	$(\forall y)(Ry \leftrightarrow y = \alpha)$	4, $\bullet$ ELIM
2,4	[8]	$(\forall y)(Ry \leftrightarrow y = \alpha) \bullet S\alpha$	6, 7, $\bullet$ INTR
2,4	[9]	$(\exists x)((\forall y)(Ry \leftrightarrow y = x) \bullet Sx)$	8, EG
1,2	[10]	$(\exists x)((\forall y)(Ry \leftrightarrow y = x) \bullet Sx)$	3, 4, 9, EI
1,2	[11]	$S(\iota x)Rx$	10, def. of "(ιx)".

Within a purely extensional system, it would be tedious to proceed in this way every time one wanted to prove something involving one or more descriptions, and it would be practical to have a fool-proof method of shortening such proofs. Whitehead and Russell reduced their workload by demonstrating that, although descriptions are not genuine singular terms (in their system), if a predicate F applies to exactly one object (i.e. if it has exactly one thing in its extension), in truth-functional (i.e. extensional) contexts the description "$(\iota x)Fx$" can be treated *as if* it were a singular term for derivational purposes. The following theorem to this effect is proved by them for truth-functional contexts:

*14.15 $\{(\iota x)\phi = \alpha\} \rightarrow \{G(\iota x)\phi \leftrightarrow G\alpha\}$.

[27] This particular application of PSST assumes that variables and temporary names function as genuine singular terms. I am fully at ease with this assumption, as, in effect, were Whitehead and Russell. It is not obvious how it might be contested, but it is an assumption nonetheless.

This says that if the individual that α stands for is the unique object satisfying a formula ϕ, then one can "verbally substitute" α for the description $\ulcorner(\iota x)\phi\urcorner$, or vice-versa (in truth-functional contexts). If descriptions are treated in accordance with Russell's theory, it is a mistake to think that when one performs a "verbal substitution" of this sort, one is simply making a direct application of PSST. *14.15 is *not* PSST; it is a *derived rule of inference* that can be used in truth-functional contexts, a rule that licenses certain substitutions when the referent of a particular singular term is identical to the unique object satisfying a particular formula. Naturally, Whitehead and Russell prove the analogue of *14.15 where both noun phrases are descriptions:

*14.16 $\{(\iota x)\phi = (\iota x)\psi\} \rightarrow \{G(\iota x)\phi \leftrightarrow G(\iota x)\psi\}$.

This says that if the unique object satisfying a formula ϕ is identical to the unique object satisfying a formula ψ, then one can "verbally substitute" the description $\ulcorner(\iota x)\phi\urcorner$ for the description $\ulcorner(\iota x)\psi\urcorner$, or vice versa.

On the basis of *14.15 and *14.16, we can add a third inference rule (actually, a triple of rules) to our collection, ι-SUBSTITUTION:

(iii) ι-SUB: $\dfrac{(\iota x)\phi = (\iota x)\psi \quad \Sigma[(\iota x)\phi]}{\Sigma[(\iota x)\psi]}$ $\qquad \dfrac{(\iota x)\phi = \alpha \quad \Sigma[(\iota x)\phi]}{\Sigma[\alpha]}$ $\qquad \dfrac{(\iota x)\phi = \alpha \quad \Sigma[\alpha]}{\Sigma[(\iota x)\phi]}$.

(Of course, if descriptions are treated as singular terms, this rule is redundant, its work already done by PSST.)

It is surely only *because* truth-functional (i.e. extensional) contexts support ι-SUB that Whitehead and Russell introduce descriptive terms into the formal language of *Principia Mathematica*: they simplify both formulae and proofs. Adding such rules to an extensional deductive system, we can now formally capture the inference from "Cicero = the greatest roman orator" and "Cicero snored" to "the greatest roman orator snored":

1	[1]	$c = (\iota x)Rx$	premiss
2	[2]	Sc	premiss
1,2	[3]	$S(\iota x)Rx$	1, 2, ι-SUB.

In harmony with shorthand introduced earlier in connection with PSST and PSME, we can note that extensional contexts and extensional S-connectives are $+\iota$-SUB.

9. Rules of conversion

As noted earlier, in natural language there are ways of reorganising a sentence (or converting it into a related sentence) without altering meaning, in a sense of the word "meaning" that the friend of facts is trying to get at with talk of true sentences standing for facts (see the discussion of examples (12)-(15) above). Reorganisations and conversions not entirely dissimilar to passivization and topicalization are sometimes employed in logic and semantics, perhaps the most common being those that involve λ-conversion.[28] For certain purposes the sentence "$Fa \bullet Ga$" might be rendered as

(29) $((\lambda x)(Fx \bullet Gx))a$

which, depending upon the one's taste, can be read as (a) "a is something that is both F and G"; (b) "the class of things that are both F and G contains a"; or (c) "the property of being both F and G is a property a has." For concreteness, let us think of λ-conversions as sanctioned by two rules of inference, λ-INTRODUCTION and λ-ELIMINATION:[29]

(v) λ-INTR: $\dfrac{\Sigma[x/\alpha]}{((\lambda x)\Sigma[x])\alpha}$ (vi) λ-ELIM: $\dfrac{(\lambda x \Sigma[x])\alpha}{\Sigma[x/\alpha].}$

($\Sigma[x]$ is any sentence containing *at least one* occurrence of a variable x, and $\Sigma[x/\alpha]$ is the result of replacing *every* occurrence of the variable x in $\Sigma[x]$ by the (closed) singular term α.) On the weakest reading of λ-expressions—viz (a) above—those who make use of such expressions will view λ-INTR and λ-ELIM as valid rules of inference in extensional contexts, just like PSST, PSME, and ι-SUB. Consonant with shorthand introduced earlier, we can say that extensional contexts and extensional S-connectives are $+\lambda$-INTR and $+\lambda$-ELIM. And when a context or connective is both $+\lambda$-INTR and $+\lambda$-ELIM, let us say that it is $+\lambda$-CONV (it permits λ-conversion *s.v.*). Similarly, in a proof we can say we are using "λ-CONV" when we are using either λ-INTR or λ-ELIM.

With Gödel's proof in mind, let us now draw up two similar inference rules involving the description-operator, rules we can call ι-INTRODUCTION and ι-ELIMINATION, where α is a singular term and x is a variable:

[28] For the origins of λ-conversion see Church (1940).

[29] Here "$((\lambda x)(Fx \bullet Gx))$" is a predicate and "$((\lambda x)(Fx \bullet Gx))a$" is a sentence. This is the usage of "λ" found in much contemporary work in semantics and differs in a harmless way from Church's usage according to which "$(\lambda x)(Fx)$" functions syntactically as a singular term admitting of contextual definition (see note 31). I follow Church in introducing λ-conversion by way of inference rules.

(vii) ι-INTR: $\dfrac{\Sigma[x/\alpha]}{\alpha = (\iota x)(x = \alpha \bullet \Sigma[x])}$ (viii) ι-ELIM: $\dfrac{\alpha = (\iota x)(x = \alpha \bullet \Sigma[x])}{\Sigma[x/\alpha].}$

Certainly ι-INTR and ι-ELIM are valid rules of inference in extensional contexts, and any theory of descriptions must be compatible with this fact (as Russell's is). Adding to our shorthand, let us say that extensional contexts and extensional *S*-connectives are $+\iota$-INTR and $+\iota$-ELIM. And when a context or connective is both $+\iota$-INTR and $+\iota$-ELIM, let us say that it is $+\iota$-CONV (it permits ι-conversion *s.v.*). Similarly, in a proof we can say we are using "ι-CONV" when we are using either ι-INTR or ι-ELIM.

As with other rules of inference, we would like to know if there are purportedly nonextensional contexts for which rules like ι-INTR and ι-ELIM are not valid, thereby putting ourselves in a better position to avoid and detect certain forms of logical error.

10. The fundamental constraint

Gödel's proof has interesting consequences for proposed treatments of purportedly nonextensional *S*-connectives. In effect, it shows conclusively that no nonextensional *S*-connective can be both $+\iota$-SUB and $+\iota$-CONV, i.e. it shows that no *S*-connective can have the following combination of features:

(30) $+\iota$-CONV $+\iota$-SUBS $-$PSME.

This is most readily seen by converting Gödel's proof into the following proof within a system of first-order logic with identity, augmented with the *iota*-operator and a purportedly nonextensional *S*-connective ❽ that has the features given in (30):

1	[1]	Fa	premiss
2	[2]	$a \neq b$	premiss
3	[3]	Gb	premiss
1	[4]	$a = (\iota x)(x = a \bullet Fx)$	1, ι-INTR
2	[5]	$a = (\iota x)(x = a \bullet x \neq b)$	2, ι-INTR
2	[6]	$b = (\iota x)(x = b \bullet x \neq a)$	2, ι-INTR
3	[7]	$b = (\iota x)(x = b \bullet Gx)$	3, ι-INTR
1,2	[8]	$(\iota x)(x = a \bullet Fx) = (\iota x)(x = a \bullet x \neq b)$	4,5, ι-SUB
2,3	[9]	$(\iota x)(x = b \bullet Gx) = (\iota x)(x = b \bullet x \neq a)$	6,7, ι-SUB
10	[10]	❽(Fa)	premiss
10	[11]	❽$(a = (\iota x)(x = a \bullet Fx))$	10, ι-INTR
1,2,10	[12]	❽$(a = (\iota x)(x = a \bullet x \neq b))$	11,8, ι-SUB

1,2,10	[13]	❽$(a \neq b)$	12, ι-ELIM
1,2,10	[14]	❽$(b = (\iota x)(x = b \bullet x \neq a))$	13, ι-INTR
1,2,3,10	[15]	❽$(b = (\iota x)(x = b \bullet Gx))$	14,9, ι-SUB
1,2,3,10	[16]	❽(Gb)	15, ι-ELIM.

Mutatis mutandis where premise [2], "$a \neq b$", is replaced by [2′], "$a = b$".

What this version of the proof shows is that if an *S*-connective is +ι-SUB, +ι-CONV, then it is also +PSME—i.e. it permits the substitution *s.v.* of mere material equivalents such as "*Fa*" and "*Gb*". In short, it shows that ❽ is, after all, an *extensional S*-connective, contrary to initial hypothesis. (Again, this assumes, with Gödel, that every sentence can be brought into subject-predicate form. If this is problematic, the argument demonstrates the technically weaker—but equally important—conclusion that if an *S*-connective is +ι-SUB, +ι-CONV, then it also permits the substitution *s.v.* of materially equivalent *atomic* sentences.)

The precise relation between this proof and Gödel's original proof comes clearly into focus if "❽( )" is interpreted as "the fact that *Fa* = the fact that ( )" or as "the sentence '*Fa*' corresponds to the fact that ( )".

How worrying is this fundamental constraint for friends of facts, nonextensional logics, and purportedly nonextensional *S*-connectives in natural language? The first thing to note is that the proof certainly does *not* show that an *S*-connective is extensional if it is both +PSST and +ι-CONV, i.e. it does *not* directly demonstrate that the following combination of features is inconsistent:

(31) +ι-CONV +PSST —PSME.

But the inconsistency of (31) *would* be shown if it were possible to prove that any *S*-connective that is +PSST is also +ι-SUB, or prove that any *S*-connective with the three features in (31) must also be +ι-SUB. I know of no attempts to construct such proofs; indeed, if descriptions are Russellian such proofs cannot be constructed. So anyone hoping to use Gödel's proof as part of an argument against the consistency of (31) needs to (i) provide a viable treatment of descriptions according to which they are (a) singular terms (and hence subject to PSST), and (b) still the plausible inputs and outputs of ι-CONV, and then (ii) construct a proof exactly like the one above except that it appeals to PSST (rather than ι-SUB) at lines [8], [9], [12], and [15]. I shall return to this matter in §12.

11. The Church-Quine-Davidson slingshot

The proof implicit in Gödel's (1944) paper is certain to call to mind a better known proof that appears explicitly in Church's (1943) review of Carnap's (1942) book *Introduction to Semantics*. Carnap had broken with Frege by taking sentences to designate *propositions*—which he took to be something like *states-of-affairs*—rather than truth-values. Church's slingshot was meant to show that, in Carnap's system, sentences could not designate propositions on pain of entailing that all true sentences designate the same proposition.[30]

Versions of Church's proof have been deployed by Quine and Davidson to various philosophical ends, and discussed widely in the literature. By contrast, the literature contains relatively few discussions of Gödel's proof, which is interesting because Gödel's premises are weaker.[31] We are now in a position to articulate the difference clearly and explore its ramifications.

The basic difference is that Church, Quine, and Davidson draw upon purported *logical equivalences*, such as those between (Δ) and (Δ'), or (Δ) and (Δ''):

(Δ) ϕ

(Δ') $a = (\iota x)(x = a \bullet \phi)$

(Δ'') $(\iota x)(x = a) = (\iota x)(x = a \bullet \phi)$.

The net effect of this is that the Church-Quine-Davidson slingshot makes use of a more contentious substitution principle.

Following common practice, let us use "$\phi \vDash\dashv \psi$" as shorthand for "ϕ and ψ are logically equivalent". And following Tarski, and common practice, let us say that $\phi \vDash\dashv \psi$ if, and only if, ϕ and ψ have the same truth-value in every model. We can now state one last rule of inference:

[30] It seems likely that Carnap (1947) accepted Church's argument, taking the referents of sentences to be truth-values rather than propositions. Church sees his argument as a "reproduction in more exact form by means of Carnap's semantical terminology" of Frege's argument in support of the view that "a *sentence* (Behauptungssatz) *expresses a proposition* (drückt aus einen Gedanken) but denotes or *designates a truth-value* (bedeutet einen Wahrheitswerth)" (p. 301). It is not easy to interpret Frege as advancing an inexact form of Church's argument (see note 6). Probably, Church was reading his own thoughts into Frege here (something we do easily when expounding the work of those who have influenced us so profoundly).

A superficial difference between the arguments of Church and Gödel is that Church uses the abstraction operator "(λx)"—where "$(\lambda x)\phi$" is read as "the class

(ix) PSLE:
$$\frac{\begin{array}{c}\phi \models\dashv \psi \\ \Sigma[\phi]\end{array}}{\Sigma[\psi].}$$

The *Principle of Substitutivity for Logical Equivalents* (PSLE) says that if sentences ϕ and ψ are logically equivalent and $\ulcorner\Sigma[\phi]\urcorner$ is a sentence con-

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of all x such that ϕ"—while Gödel (implicitly) uses the definite description operator "(ιx)". This difference should not obscure the fact that Gödel and Church are in complete harmony on the matter of contextual definitions of their respective term-forming devices. Gödel points out that if "(ιx)" does not belong to the primitive symbols but is provided with a Russellian contextual definition, then it will not be possible to use his slingshot to demonstrate that if true sentences stand for facts, all true sentences stand for the same fact. Similarly Church (1943a, pp. 302–3) points out that if "(λx)" does not belong to the primitive symbols but is provided with a contextual definition such as the following,

(i) $(\lambda x)(Fx) = (\lambda x)(Gx) =_{\text{df}} (\forall x)(Fx \leftrightarrow Gx)$

then it will not be possible to use his slingshot to demonstrate that if sentences designate propositions, all true sentences designate the same proposition. An alternative to the contextual definition in (i) would be to view "$(\lambda x)Fx$" as a definite description ("the set of things that are F") that can be analysed in accordance with Russell's theory, as suggested by Smullyan (1948) and Quine (1941), respectively (ii) and (iii):

(ii) $[(\lambda x)Fx]G(\lambda x)Fx =_{\text{df}} (\exists\alpha)((\forall x)(Fx \leftrightarrow x \in \alpha) \bullet G\alpha)$

(iii) $(\lambda x)Fx =_{\text{df}} (\iota\alpha)((\forall x)(Fx \leftrightarrow x \in \alpha))$.

(In Smullyan's (ii), α is a class variable and "$[(\lambda x)Fx)]$" is a scope marker just like Whitehead and Russell's "$[(\iota x)Fx]$".) Church is fully aware that it makes no difference whether descriptions or class abstracts are used in setting up the slingshot, as are Quine and Davidson. For the sake of continuity, I will stick to statements that contain descriptions.

[31] Gödel's proof is discussed explicitly by Wedberg (1966, 1984) Wallace (1969), Burge (1986), Olson (1987), and Parsons (1991), and mentioned in passing by Morton (1969), Widerker (1983), and Davidson (forthcoming). That Gödel's proof does not overtly invoke the notion of logical equivalence—the notion central to the Church-Quine-Davidson slingshot—is recognized by Wedberg, Wallace, Olson, and Parsons. Burge and Davidson note that Gödel's argument is different in form from Church's, but they do not discuss the nature of the difference. The possibility of a modification of the Church-style proof that differs from the original in the crucial respect in which Gödel's does is mentioned by Dale (1978), Taylor (1976, 1985), and Widerker (1978, 1985)—though only Widerker actually mentions Gödel in this connection. In these articles, there appears to be no recognition of the fact that an argument of the modified form can be stated in such a way that it relies on strictly weaker premises. Dale remarks, correctly, that claims about logical equivalence are easier to justify within the context of the modified proof. In his discussion of Quine's (1960) slingshot, Sharvy (1969) notes that the argument could be reformulated using a weaker premise, i.e. by appealing to a tighter notion than the purported logical equivalence of ϕ and "the number x such that ((x = 1) and ϕ) or ((x = 0) and not ϕ)", the purported equivalence that does the work in Quine's slingshot. Føllesdal (1983) notes in passing that not all versions of the slingshot involve the interchange of logical equivalents.

taining at least one occurrence of ϕ, then $\ulcorner\Sigma[\psi]\urcorner$ and $\ulcorner\Sigma[\phi]\urcorner$ have the same truth-value, where $\ulcorner\Sigma[\psi]\urcorner$ is the result of replacing at least one occurrence of ϕ in $\ulcorner\Sigma[\phi]\urcorner$ by ψ. PSLE is of course valid in extensional contexts (i.e. contexts that are +PSME). As shorthand for this, let us say that extensional contexts and extensional *S*-connectives are +PSLE (as opposed to –PSLE). (The modal *S*-connectives "$\Box$" and "$\Diamond$" are standardly taken to be +PSLE.)

On the assumption that talk by Church, Quine, and Davidson of "logical equivalence" is talk of the Tarskian notion used to state PSLE—an alternative characterization will be examined soon enough—we can now set out the Church-Quine-Davidson slingshot in *S*-connective format. Let ❽ be an arbitrary nonextensional *S*-connective that is +PSLE and +ι-SUB. The central component of the Church-Quine-Davidson slingshot usually takes the following form:

1	[1]	$\phi \leftrightarrow \psi$	premiss
2	[2]	❽(ϕ)	premiss
2	[3]	❽$((\iota x)(x = a) = (\iota x)(x = a \bullet \phi))$	2, PSLE
1	[4]	$(\iota x)(x = a \bullet \phi) = (\iota x)(x = a \bullet \psi)$	1, def. of "(ιx)"
1,2	[5]	❽$((\iota x)(x = a) = (\iota x)(x = a \bullet \psi))$	3, 4, ι-SUB
1,2	[6]	❽(ψ)	5, PSLE.

Since $\{(\phi \leftrightarrow \psi)$, ❽$(\phi)\} \models$ ❽(ψ), contrary to initial assumption "❽" is actually an extensional *S*-connective since it has been shown to be +PSME ("❽(ϕ)" differs from "❽(ψ)" only in the substitution of the mere material equivalents ϕ and ψ). The philosophical consequences of this argument are then drawn by interpreting "❽" as, e.g., "necessarily ( )" or "the statement that ϕ corresponds to the fact that ( )".[32]

It will not do to object to this argument on the grounds that "$(\iota x)(x = a \bullet \phi)$" is not well-formed or not interpretable unless ϕ contains an occurrence of x that "(ιx)" can bind. Even if it would normally be odd to use anything like the analogue of such a description in ordinary or theoretical talk, there is no more formal difficulty involved in making sense of such a description than there is in making sense of those that Gödel uses in his slingshot.

An important but overlooked fact about this argument is that, unlike Gödel's, if it is to be of any interest whatsoever it must be supplemented

[32] Related arguments have been used by Quine (1960) and Føllesdal (1965, 1966, 1969, 1983) with a view to demonstrating that modal distinctions collapse (i.e. that "$p \leftrightarrow \Box p$" is valid) in systems that attempt to combine devices of modality, quantification, and description (or abstraction). Nothing of formal interest emerges from examining these arguments that does not emerge more readily from an examination of the proof just presented. This undoubtedly accounts for the fact that they have attracted far less attention. What discussion there is tends to be overly reliant on, and influenced by, Carnap's idiosyncratic account of definite descriptions, and consequently ride roughshod over delicate matters of substitutivity, scope, and counterfactual truth-conditions.

with a precise semantics for definite descriptions. The reason is simple: (i) the notion of *logical equivalence* is invoked in getting from line [2] to line [3], and from line [5] to line [6]; (ii) lines [3] and [5] both contain definite descriptions; (iii) on some treatments of descriptions the logical equivalences obtain, on others they do not; (iv) the treatment of descriptions assumed by the argument will determine whether it is PSST or ι-SUB that is invoked in getting from lines [3] and [4] to line [5].[33]

If descriptions are given a Russellian analysis, then the proof is valid, for on such an analysis (32) is just shorthand for the first-order sentence (33):

(32) $(\iota x)(x = a) = (\iota x)(x = a \bullet \phi)$

(33) $(\exists x)\{(\forall y)(x = a \leftrightarrow y = x) \bullet$
$(\exists u)((\forall w)(w = a \bullet \phi \leftrightarrow w = u) \bullet u = x)\}$.

And (33) agrees in truth-value with ϕ in every model. So the argument is valid and the proof perfectly legitimate: if descriptions are Russellian and if ❽ is +PSLE and +ι-SUB, then ❽ is also +PSME, contrary to initial assumption. Thus it has been shown that no connective can have the following combination of features:

(34) +PSLE +ι-SUB −PSME.

But *nothing* follows about *S*-connectives that are claimed to have the following combination of features:

(35) +PSLE + PSST −PSME.

So although the argument is valid, it does not lend any support to the view that there cannot be nonextensional *S*-connectives: any connective with the features in (35) would be such a connective.

A philosopher still determined to cause trouble for nonextensional *S*-connectives could choose to treat descriptions as singular terms and then restate line [5] of the proof just given as

1,2 [5′] ❽$((\iota x)(x = a) = (\iota x)(x = a \bullet \psi))$ 3, 4, PSST.

But such a philosopher would then be saddled with the task of providing a referential semantics for descriptions that justifies (a) the move from line [2] to line [3], (b) the entry at line [4], and (c) the move from line [5] to line [6]. On the Russellian analysis, all is clear and automatic; but on a referential treatment, difficult choices must be made about the truth-theoretic contributions of improper descriptions, choices that bear crucially on claims of logical equivalence.

Of course, an explicit appeal to a referential treatment of descriptions would be an odd move for *Quine* to make: as noted earlier, it is his view

[33] As Church's remarks about contextual definitions of class abstracts reveal (see footnote 30), if the argument is restated using class abstracts, exactly analogous questions must be answered concerning logical equivalence and the precise semantics for class abstracts.

that a Russellian Theory of Descriptions can be used to eliminate names and other singular terms in favour of devices of quantification, predication, and truth-functional connection. But there is no guarantee that Quine is right about this, so in the interests of a comprehensive examination of the slingshot we must explore the consequences of referential treatments of descriptions.

12. Descriptions as singular terms

There appear to be extremely good reasons for thinking that descriptions should be analysed in accordance with Russell's theory, as devices of quantification rather than reference.[34] However, there are a number of apparently attractive referential treatments of descriptions on the market, some of which give rise to interesting questions in connection with slingshot arguments. Those virtuous souls who, like Quine, are thoroughly content with Russell's Theory of Descriptions can comfortably skip this section and head on to the main business in §13.

If the intuitive meaning of the description operator is to be honoured, one definite condition must be satisfied by any referential treatment: if a formula $\Sigma[x]$ containing at least one occurrence of the variable x (and no free occurrence of any other variable) is uniquely satisfied by A, then the description $\ulcorner(\iota x)\Sigma[x]\urcorner$ must refer to A. The wording of this condition brings out questions that any referential treatment must answer, questions that can be sharpened by reflecting on the logical simplicity afforded by Russell's treatment.

As noted earlier, Russell's treatment provides straightforward accounts of sentences containing descriptions whose matrices are not uniquely satisfied, so-called "improper" descriptions. Refining our terminology, let us say that a description $\ulcorner(\iota x)\Sigma[x]\urcorner$ is *proper* (according to some model M) if, and only if, its matrix $\Sigma[x]$ is (on M's interpretation) true of exactly one item in the domain over which the variables of quantification range, and *improper* otherwise.[35] There is no worry about proper descriptions: a model

[34] See (e.g.) Mates (1973), Neale (1990, 1993), and Quine (1941, 1953a, 1953b, 1953d, 1960).

[35] Terminology aside, I here follow Taylor (1985). For the sake of simplicity I propose (again with Taylor) to ignore the irrelevant complexities raised by relativized descriptions such as 'the woman sitting next to him' where 'him' is bound by a higher quantifier. Nothing of any bearing upon the point at hand turns on the existence or interpretation of such descriptions. Russell's theory both predicts the existence of and provides an automatic and successful interpretation of such descriptions without any additional machinery. With some work, presumably some referential accounts of descriptions can also supply what is necessary here, hence I propose to ignore any potential problems that relativization creates for the non-Russellian.

M interprets a proper description $\ulcorner(\iota x)\Sigma[x]\urcorner$ as referring to the unique object satisfying $\Sigma[x]$. But a number of competing approaches to improper descriptions can be found in the literature, or else constructed on the basis of existing informal suggestions (a good deal of this work has been done by Carnap (1947) and Taylor (1985), from whom I shall draw liberally).

In the system of Hilbert & Bernays (1934), a description $\ulcorner(\iota x)\Sigma[x]\urcorner$ can be used only after it has been proved proper, i.e. only after (36) has been proved:

(36) $(\exists x)((\forall y)(\Sigma[x/y] \leftrightarrow y = x)$.

Although this treatment may be useful for certain mathematical purposes, as Quine (1941), Carnap (1947), Scott (1967) and others have pointed out, there are insurmountable problems involved in viewing it as a treatment of descriptions in any interesting fragment of natural language. Firstly, as far as natural language is concerned the class of well-formed formulae will not be recursive on this account, the question of whether a string of symbols containing the substring "(ιx)" is a formula depending upon rather more than a set of syntactical rules, for example matters of logic and the "contingency of facts". And secondly, utterances of many sentences of natural language containing improper descriptions (or descriptions not known to be proper) are straightforwardly true or false (e.g. "last night I dined with the king of France") or straightforwardly used to conjecture. There would appear to be no prospect, then, of using Hilbert & Bernays' treatment in connection with descriptions belonging to any interesting fragment of natural language.

Furthermore, since the use of a description $\ulcorner(\iota x)(x = a \bullet \Sigma)\urcorner$ is permitted on Hilbert & Bernays' treatment only if either Σ or $\ulcorner\Sigma[x/a]\urcorner$ is provable, the adoption of this treatment will render neither of the slingshots we have examined a valid proof that there can be no S-connective with the combinations of features given in (31) and (35):

(31)	+ι-CONV	+PSST	–PSME.
(35)	+PSLE	+PSST	–PSME.

So the two slingshots would show only that ❽ permits the substitution *s.v.* of *logical truths*.[36] (Carried back over to the discussion of facts, both would show only that all *logical truths* stand for the same fact.) Thus Gödel's slingshot would demonstrate something of mild interest: if ❽ is +PSST and +ι-CONV then it also permits the substitution *s.v.* of logical truths. There is a temptation to think that if ❽ permits the substitution of logical truths *s.v.*, it must also be +PSLE. I am not interested in investigat-

[36] Following Tarski (1956) and common practice, let us say that a sentence ϕ of first-order extensional logic is *logically true* if, and only if, it is true in every model for first-order extensional logic.

ing this matter, but even if it is correct the conclusion of the Gödel slingshot is still meagre. (Carried over to facts it would show only that logically equivalent sentences stand for the same fact). And the Church-Quine-Davidson slingshot would demonstrate, at most, only the truth of one of its own premisses, viz that ❽ is +PSLE (and thereby that all logically equivalent sentences stand for the same fact).

To sum up, on the assumption—which may be correct—that any S-connective that permits the substitution *s.v.* of logical truths is +PSLE, all that can be demonstrated by adopting Hilbert & Bernays' treatment of descriptions is that no *S*-connective can have the following combination of features:

(37) $+\iota$-CONV +PSST −PSLE.

Rather different approaches to improper descriptions have been inspired by Frege, who thought it an imperfection of natural and mathematical languages that they contain apparent singular terms that fail to refer. Frege (1892) suggests that a description is a "compound proper name" and as such

> ... must actually always be assured a reference, by means of a special stipulation, e.g. by the convention that that it shall count as referring to 0 when the concept applies to no object or to more than one (p. 71).

Elsewhere, Frege (1893) suggests an alternative treatment according to which an improper description refers to *the class of entities satisfying its matrix* (thus all empty descriptions refer to the empty class). Within the context of Frege's overall theory of reference, this is certainly an improvement as far as compositionality and extensionality are concerned. On Frege's account, the matrix of a description is a concept expression and, as such, it is paired with a class of entities, its extension. As required by the meaning of the definite article, where the extension of a matrix F is one-membered, the member qualifies as the referent of the resulting description ⌜the F⌝; if the class in question is anything other than one-membered, *the set itself* serves as the reference. So, on Frege's (1893) account, there is a straightforward extensionality constraint governing *all* definite descriptions: if ⌜$\Sigma[x]$⌝ and ⌜$\Sigma'[x]$⌝ are satisfied by the same elements, then ⌜$(\iota x)\Sigma[x]$⌝ and ⌜$(\iota x)\Sigma'[x]$⌝ have the same reference.[37]

Frege's suggestions have been developed in a number of ways, most notably by Carnap (1947), Scott (1967), and Grandy (1972). Carnap's

[37] I am here indebted to Mark Sainsbury and Barry Smith. Once the suggestion has been made that empty descriptions refer to the empty class, it would be misleading to say, with Quine (1940, p. 149) that there is something "arbitrary" about Frege's suggestion that those "uninteresting" descriptions whose matrices are satisfied by more that one entity refer to the class of things satisfying the matrix: for this is *exactly* the suggestion for empty descriptions.

position on improper descriptions might be summarized model-theoretically as follows: in each model M, some arbitrary element $*_M$ in the domain (over which the variables of quantification range) serves as the referent (in M) of all descriptions that are improper with respect to M.[38] On this treatment of descriptions, about the only thing that Gödel's slingshot demonstrates is just how bad a treatment it is. (It was hardly a plausible treatment of descriptions in natural language anyway.) Consider a model M in which "Fa" is false and the singular term "a" refers to $*_M$. (The existence of such a model presupposes that which element of the domain is functioning as the referent of improper descriptions is one feature (i.e. assignment) that is used in individuating models. This is surely the way to make sense of the idea that an "arbitrarily" chosen entity in the domain serves as the referent of improper descriptions). In M, (Γ) is false while (Γ') is true:

(Γ) Fa

(Γ') $a = (\iota x)(x = a \bullet Fx)$.

So on this model-theoretic treatment of descriptions, neither $+\iota$-INTR nor $+\iota$-ELIM is truth-preserving in truth-functional contexts. I am inclined to think this finishes off the treatment once and for all; but even if it doesn't, the fact that (Γ) and (Γ') can differ in truth-value on this treatment means it cannot be used in conjunction with Gödel's slingshot to demonstrate anything interesting about nonextensional S-connectives.

The problem just raised concerning $+\iota$-CONV could be eradicated, of course, by a special stipulation to the effect that only those singular terms that are also descriptions can be assigned $*_M$ as their reference in M. If the resulting treatment of descriptions turns out to be the correct one—remember descriptions are singular terms on this treatment—then Gödel's slingshot demonstrates that no S-connective can have the combination of features given in (31), a fact that would be devastating for any theory of facts requiring "the fact that Fa = the fact that ( )" to have this set of features:

(31) $+\iota$-CONV +PSST −PSME.

[38] It is unclear whether it makes much sense to attribute to Carnap, as part of his overall account of descriptions—which, as he points out (1947, p. 8), "deviates deliberately from the meaning of descriptions in the ordinary language"—even the informal analogue of this model-theoretic account of improper descriptions. Moreover, in the light of the work of Smullyan (1948) and Kripke (1972), it is not easy to make sense of Carnap's account of descriptions within modal systems as it rides roughshod over issues of scope, substitutivity, and singular vs. general counterfactual truth-conditions. No doubt this explains the mess people get into when examining modal collpases in Carnap's S_2 and other systems that claim to incorporate Carnap's overall account of descriptions. As Donald Davidson has pointed out to me, most of these problems for Carnap's overall account of descriptions do not arise in purely extensional systems.

The situation is slightly different when it comes to the Church-Quine-Davidson slingshot. On the Carnapian treatment, (Δ) and (Δ') are not logically equivalent, as pointed out by Taylor (1985):

(Δ) ϕ

(Δ') $a = (\iota x)(x = a \bullet \phi)$.

Consider a model M in which ϕ is false, and the singular term "a" refers to $*_M$. (Δ') will be true in M; and since (Δ) and (Δ') have different truth values in M, they do not have the same truth value in all models, hence they are not logically equivalent. Thus the Church-Quine-Davidson slingshot appears to collapse if descriptions are treated in the Carnapian way. Again the logical equivalence could be regained by stipulating that only descriptions can refer, in M, to $*_M$, in which case the Church-Quine-Davidson slingshot does produce a result on a modified Carnapian treatment, viz. that no S-connective can have the combination of features given in (35):

(35) +PSLE +PSST —PSME.

The failure of the desired logical equivalence on the original Carnapian treatment of descriptions suggests to Taylor a modified slingshot. The idea, put into S-connective format, is to tack "$a \neq (\iota x)(x \neq x)$" onto ϕ and derive "$\circledS(\psi \bullet a \neq (\iota x)(x \neq x))$" from "$\circledS(\phi \bullet a \neq (\iota x)(x \neq x))$" and "$\phi \leftrightarrow \psi$" in exactly the same way as "$\circledS(\psi)$" is meant to be derived from "$\circledS(\phi)$" and "$\phi \leftrightarrow \psi$" using the original Church-Quine-Davidson slingshot. The beauty of Taylor's slingshot is that it avoids any special stipulation concerning which terms can refer to which entities *and* guarantees the logical equivalence of (Δ) and (Δ'). The conclusion could be viewed as just as damaging to theories of facts and states-of-affairs as the original Church-Quine-Davidson version as it demonstrates the truth of such statements as "the fact that (Davidson teaches at Berkeley and Davidson is not identical to the non-self-identical entity) = the fact that (Wiggins teaches at Oxford and Davidson is not identical to the non-self-identical entity)".

Taylor's strategy for dealing with the Church-Quine-Davidson slingshot (and his own modification) is to define a notion of "tight" logical equivalence, and then maintain that tight logical equivalents stand for the same state-of-affairs whereas mere logical equivalents need not. This involves defining a class of expressions that might be called the "tight" logical constants, a class that includes the quantifiers and truth-functional connectives but not the description operator or the identity sign. Whilst I have sympathy with Taylor's view that the standard notion of logical equivalence is somewhat murky, I am not convinced that he improves matters by bringing tight equivalence into the picture. More importantly, Taylor's manoeuvres do not allow him to avoid *Gödel's* slingshot, which makes no appeal to logical equivalence whether standard or tight.

Fregean treatments of descriptions have also been proposed by Scott (1967) and Grandy (1972). On these treatments, bound variables range over a domain D, but the values of singular terms and free variables may lie in a so-called "pseudo-domain" D°, stipulated to be disjoint from D and non-empty. An improper description is given a value in D°, "thereby emphasising its impropriety" as Scott says.[39] The situation with respect to Gödel's slingshot is much as before. Consider a model M in which "Fa" is false and the singular term "a" refers to $*_M{}^\circ$, the pseudo-object selected from D° to be the referent of descriptions that are improper with respect to M. In such a model, (Γ) is false while (Γ') is true. So, again, we have treatments of descriptions according to which truth-functional contexts are neither $+\iota$-ELIM nor $+\iota$-INTR. So while these treatments seem to solve some of the problems that Scott and Grandy are grappling with, the fact that they permit (Γ) and (Γ') to differ in truth-value ensures that they cannot be used in conjunction with Gödel's slingshot to show anything about nonextensional S-connectives and also suggests strongly that they are inadequate as treatments of descriptions in natural language. The Church-Quine-Davidson slingshot fares no better. As Taylor notes, again (Δ) and (Δ') are not logically equivalent. Scott's treatment declares (Δ) false and (Δ') true in any model M in which ϕ is false and "a" refers to $*_M{}^\circ$; Grandy's declares (Δ) false and (Δ') true in any model M in which ϕ is false and "a" refers to the referent of descriptions whose matrix has the *intension* of "$(x = a \bullet \phi)$". Thus the Church-Quine-Davidson slingshot fails if descriptions are treated in the ways Scott and Grandy suggest.

Again, tinkering with the class of expressions that can take $*_M{}^\circ$ (or anything else in D°) as a value would alter things, but such a move would constitute a clear departure from the theories of Scott and Grandy. Such tinkering may be what Olson has in mind when he suggests that (Δ) and (Δ') are logically equivalent upon a "Fregean" theory of descriptions according to which an improper description refers to "some object outside the universe" (1987, p. 84, footnote 9). Assuming that Olson has not simply overlooked models in which 'a' refers to $*_M{}^\circ$, he must have in mind a semantics quite different from those envisaged by Scott and Grandy. It is a feature of the Scott-Grandy systems that the values of singular terms (but not bound variables) may lie in D°, and it is this feature that legitimizes the selection of an element in D° to serve as the value of a description—a singular term on this proposal—whose matrix is not uniquely satisfied by something in D. So if Olson has is mind a referential seman-

[39] According to Grandy, "Not all objects in the pseudo-domain are possible objects for one of them will be the denotation of $(\iota x)(x \neq x)$" (1972, p. 175).

tics according to which (Δ) and (Δ′) are logically equivalent, then he must postulate two distinct classes of singular terms, those that can take values in D° and those that cannot, and he must put descriptions into the former class and proper names into the latter. Treating definite descriptions so differently from other singular terms would certainly make the resulting theory less attractive than the Scott-Grandy theories, and it might also lead to results that their systems are carefully designed to avoid.

The final referential treatment of descriptions I want to consider is the one Taylor (1985) produces (but does not endorse) by recasting some of Strawson's (1950b) views in model-theoretic terms.[40] The key features of this account are (a) the rejection of bivalence: a sentence containing a description that is improper with respect to a model *M* will lack a truth-value in *M*; and (b) a refinement of the notion of logical equivalence to take into account cases in which sentences lack truth-values: two sentences are *logically equivalent* if and only if they have the same truth-value *in every model in which they both have a truth-value*. (Of course it would be very odd for *Quine* to pursue such an approach to descriptions as he has consistently opposed truth-value gaps and praised Russell's theory for eliminating them where descriptions are concerned.) On this account, (Δ) and (Δ′) are logically equivalent: in any model in which (Δ) is true, so is (Δ′); in any model in which (Δ) is either false or lacks a truth-value, the description "$(\iota x)(x = a \bullet \phi)$" is improper and so (Δ′) lacks a truth value; so every model in which (Δ) and (Δ′) both have a truth-value is a model in which they are both true; thus they are ("Strawsonian") logical equivalents.

On this treatment, the Church-Quine-Davidson slingshot appears to successfully demonstrate that no *S*-connective can have the following combination of features:

(38) +PSST +PSLE –PSME.

However, the rejection of bivalence and subsequent refining of logical equivalence bring up important questions. Firstly, logical equivalence is standardly taken to be tightly, if not definitionally, connected to other notions, for example *logical consequence*, *logical truth*, and *material equivalence*. On the proposed refinement, is there pressure to redefine the notion of logical truth (from the standard (i) "$\models \phi$ if, and only if, ϕ is true in all models," to (ii) "$\models \phi$ if, and only if, ϕ is true in all models in which ϕ

[40] Taylor is well aware that his reformulation cannot capture Strawson's own intentions and that these intentions are not important for the purposes at hand. On Strawson's account it is *speakers* rather than singular terms that refer; and his assault on Russell's Theory of Descriptions is part of a general campaign against the ideas that terms refer and sentences are true or false; thus some distortion of Strawson's views is inevitable in any attempt to recast them model-theoretically; important choices where Strawson is unclear are also necessary.

has a truth-value")? And given that standardly $\phi \vDash\Dashv \psi$ if, and only if, $\phi \vDash \psi$ and $\psi \vDash \phi$, is there pressure to redefine the notion of logical consequence? And given that standardly if $\phi \vDash\Dashv \psi$ then $\vDash \phi \leftrightarrow \psi$, should the truth-table for "$\leftrightarrow$" be the one given by Halldén (1949) and Körner (1960) for certain logics in which bivalence is rejected—"$\phi \leftrightarrow \psi$ is true if, and only if, ϕ and ψ are either both true or both false; without (standard) truth-value otherwise"—or should it differ in some way? And what of the truth-table for negation? I do not mean to be insisting that all of these (and related) questions cannot be answered together to produce a consistent and attractive package; I simply want to point out that such questions need to be answered by anyone who wants to give up bivalence and refine logical consequence in the way Taylor suggests.

Secondly, and more importantly, even if there is no formal problem with the account, it appears to be inadequate to the task of providing an account of descriptions in natural language. Put bluntly, there are just too many (utterances of) sentences of natural language that seem to have clear truth-values despite containing improper descriptions. I have discussed such cases at length elsewhere (see Neale, 1990), so I will be brief. Utterances of (39), (40) and (41) made today would surely be true, false, and false respectively, precisely because there is no king of France:

(39) The king of France does not exist

(40) Bill Clinton is the King of France

(41) The king of France is not bald since there is no king of France.

Perhaps clever theories of negation, existence, predication and identity could help the Strawsonian here, but they could not help with (42):

(42) Last night Clinton dined with the king of France.

And appeals to a semantically relevant asymmetry between singular terms in subject position and those that form part of a predicate phrase will not help with the following:

(43) The king of France stole my car last night.

(44) The king of France shot himself last night.

Descriptions occurring in nonextensional contexts create similar problems. I may say something true or false by uttering (45) or (46):

(45) The first person to land on Mars in 1990 might have been Australian

(46) Bill thinks the largest prime lies between 10^{27} and 10^{31}.

At the very least, then, we must reject the view that the use of an empty description *always* results in an utterance without a (standard) truth-value. Strawson (1964, 1972, 1986) came to realise this; and in an attempt to reduce the number of incorrect predictions made by his earlier theory, he suggests that sometimes the presence of an improper description renders the proposition expressed false and at other times it prevents a proposition from being expressed at all.[41] Since nothing appears to turn on structural or logical facts about the sentence used, Strawson suggests restricting the "truth-value gap" result by appealing to the *topic of discourse*. Once the Strawsonian model-theorist makes this concession, even if a workable semantics can be salvaged, the logical equivalence that is being sought must surely drift away.

In summary, the status of slingshot arguments is a very complex matter if definite descriptions are treated as singular terms. On Hilbert and Bernays' treatment, Gödel's slingshot demonstrates something of mild interest, but the Church-Quine-Davidson slingshot proves only one of its own premisses. On Fregean treatments, according to which improper descriptions refer, by stipulation, to some entity in the domain D of quantification, or some entity in a disjoint "pseudo-domain" $D°$, both slingshots demonstrate something of significance only if descriptions are treated differently from other singular terms, a move which robs the placement of descriptions into the class of singular terms of some of its appeal and has formal consequences that still need to be explored. (But as we saw, a modified slingshot, due to Taylor, appears to hit its target without such a contortion.) The full range of consequences of the model-theoretic Strawsonian treatment (which abandons bivalence) also needs to be explored. On the assumption that the treatment is coherent, both slingshots hit their targets, but the treatment itself, even if coherent, does not come at all close to succeeding as an account of descriptions in natural language.

At this juncture it is worth reminding ourselves of the force of the slingshot arguments on a standard Russellian analysis of descriptions. The Church-Quine-Davidson slingshot succeeds in showing only that any *S*-connective that is +PSLE and +ι-SUB is also +PSME. Gödel's, by contrast, shows something more worrying: any *S*-connective that is +ι-CONV and +ι-SUB is also +PSME. This is more worrying on the obvious assumption that every "Gödelian equivalence", as given by ι-CONV, is also a logical equivalence, but not vice versa. This fact will be of interest if we find *S*-connectives or contexts that are –PSLE, +ι-SUB, and +ι-CONV, because defenders of such connectives will have no recourse to the most common rejoinder to slingshot arguments: denying that the *S*-connective in question is +PSLE, a rejoinder made explicitly by defenders of facts and situations such as Barwise and Perry (1981, 1983), Bennett (1990), Searle

[41] Difficulties for this proposal are pointed out by Gale (1970), Kempson (1975), Donnellan (1981), and Lycan (1984).

(1995), and Taylor (1985). Let us now turn to some S-connectives that bear on philosophical and ordinary talk of necessity, causation, and facts.

13. Intensional connectives

A good deal of contemporary philosophy involves manoeuvring within linguistic contexts governed by modal, causal, deontic and other purportedly nonextensional operators. If this sort of manoeuvring is to be effective it must respect the logical and other semantical properties of the contexts within which it takes place. It is an unfortunate fact about much of today's technical philosophy that the relevant logico-semantical groundwork is not properly done (if it is done at all), and this is one reason that so much contemporary work in metaphysics, ethics, the philosophy of language, and the philosophy of mind reduces to utter nonsense. Great progress has been made in the last forty years in understanding the logic, structure, and use of language; technical philosophy in the absence of logical grammar in this age barely deserves the name "philosophy."

If Ω is an n-place extensional S-connective, then the extension of $\ulcorner$❽$(\phi_1 \ldots \phi_n)\urcorner$ is determined by the extension of ❽ and the extensions of $\phi_1 \ldots \phi_n$. But suppose ❽ is a *non*extensional S-connective; what properties of ❽ and $\phi_1 \ldots \phi_n$ determine the extension of $\ulcorner$❽$(\phi_1 \ldots \phi_n)\urcorner$? Inspired by Frege's distinction between *sense* and *reference*, many philosophers have attempted to answer this question, for particular values of ❽, by postulating a second level of "semantic value" to supplement extensions. For example, Carnap (1947) and those he has influenced have suggested that each expression has an *intension* as well as an extension, and that for certain interesting nonextensional S-connectives ❽, the extension of ❽ together with the *intensions* of $\phi_1 \ldots \phi_n$ determines the extension of $\ulcorner$❽$(\phi_1 \ldots \phi_n)\urcorner$.

Consider the modal S-connectives "$\Box$" and "$\Diamond$" (where $\ulcorner \Diamond \phi \urcorner$ is understood as $\ulcorner \neg\Box\neg\phi \urcorner$). Allowing ourselves talk of so-called "possible worlds" for a moment—we can, and will, avoid such talk soon enough—one common idea is to regard the intension of an expression as *a function from possible worlds to extensions*. On such an account, (1) the intension of a singular term is a (possibly partial) function from possible worlds to *objects*;[42] (2) the intension of an n-place predicate is a function from possible worlds to *sets of ordered n-tuple of objects*; (3) the intension of a sen-

[42] If all singular terms are "rigid designators" in Kripke's (1972) sense, then on such an account the extension of a singular term will be a *constant* (but, again, possibly partial) function.

tence is a function from possible worlds to *truth-values*; and (4) the intensions of "□" and "◇" are functions from functions from possible worlds to truth-values to *functions from possible worlds to truth-values*.

As far as "□" and "◇" are concerned, the answer to our original question—"if ❽ is a nonextensional *S*-connective, what properties of ❽ and $\phi_1 \ldots \phi_n$ determine the extension of ⌜❽$(\phi_1 \ldots \phi_n)$⌝?"—is now within sight. The extension of, say, ⌜$\Box\phi$⌝ is determined, in part, by the extension of "◇". And the extension of "□" must be a function from something X to the potential extensions of ⌜$\Box\phi$⌝, i.e. a function from X to truth-values. Clearly X cannot be the potential extensions (i.e. truth-values) of ϕ, for otherwise "□" would be an extensional *S*-connective. But if X is the potential *intensions* of ϕ, everything fits together perfectly. The extension of "□" is simply a function from intensions to truth-values, i.e. a function *from* functions from possible worlds to truth-values *to* truth-values. Thus the extension of ⌜$\Box\phi$⌝ is determined by (i) the *extension* of "□", and (ii) the *intension* of ϕ. We say that "□" and "◇" are INTENSIONAL *S*-CONNECTIVES because they operate on the *intensions* of their operands.

Where X is a particular occurrence of an expression we can say that (i) X occupies an INTENSIONAL POSITION, and (ii) X occurs in an INTENSIONAL CONTEXT, if and only if (a) X is within the scope of an intensional *S*-connective, and (b) any *S*-connective within whose scope X lies is either intensional or extensional.[43]

At this point we do well to recall how possible worlds and intensions have been related to several other notions in the literature. (1) Whereas

[43] Carnap (1947) was very clear in his use of "extensional" and "intensional", pointing out that the class of nonextensional contexts does *not* collapse into the class of intensional contexts. There is an unfortunate tendency for philosophers and linguists to use the words "intensional" and "intensionality" in ways that are much looser than the precise ways in which logicians use them, and this encourages talk of the "intensionality of propositional attitude reports" and of attitude constructions involving "intensional operators." Such talk is to be deplored as it muddies already cloudy waters and can lead to philosophical mistakes engendered by running together modal and attitude contexts. There is no *a priori* reason to think of the modalities and the attitudes as sharing a logic—they *don't*—nor is there any reason to think that talk of "possible worlds" is of any help in thinking about the semantics of propositional attitude reports—it *isn't*. If we need a technical word to use in connection with attitude reports and constructions we should settle for something like Cresswell's "hyperintensional". There is only disaster in store for those who would too easily lump together constructions involving the attitudes and those involving metaphysical modality. Contrary to claims made by some linguists, the use of Cresswell's "hyperintensional" leads to no confusion and is uniformly and unambiguously applied to contexts, constructions, and operators (including *S*-connectives). Almost certainly, those who do not see this are in the grip of Quine's sloppy talk of "intensionality", "opacity", "substitutivity of identicals", and "indiscernibility of identity", and his careless use of the *iota*-operator. To the extent that we want to group nonextensional contexts, constructions, and operators together, there is a perfectly good word: "nonextensional".

some philosophers have been inclined to view possible worlds as primitive, others have been tempted to view them as sets of (consistent) states-of-affairs. Still others, such as Fine (1982), have been tempted to see them as "very large facts." (2) A common idea is to equate the intension of a sentence with the *proposition* it expresses and characterise the notion of a proposition in terms of *possible worlds*. The basic idea is this: if the intension of a sentence is a function from possible worlds to truth-values, then (assuming an extensional characterization of functions) the intension of a sentence can be viewed as a set of possible worlds, viz. those in which the sentence is true. And this set of worlds can be called a "proposition." This notion of a proposition corresponds to the common philosophical notion of the *truth-condition* of a sentence, i.e. the set of conditions under which it is true. Thus we reach the familiar positions that (i) the intension of a sentence is its truth-condition, and (ii) the truth-value of an intensional sentence $\ulcorner \Box\phi \urcorner$ depends upon ϕ's *truth condition* (whereas the truth-value of the extensional sentence $\ulcorner \neg\phi \urcorner$ depends only upon ϕ's *truth-value*). Obviously this will suggest to some—for example, those who view the notion of the truth-condition of a sentence as more basic than the notion of a possible world—that we can talk perfectly well about the intensions of expressions without talking about possible worlds.

Returning to matters logical, it should be clear that intensional *S*-connectives are understood to be –PSME and +PSST. That they are –PSME is self-evident; that they are +PSST is readily demonstrated: If α and β are both singular terms that refer to X, and Φ is a one-place extensional predicate, then $\ulcorner \Phi\alpha \urcorner$ and $\ulcorner \Phi\beta \urcorner$ have the same truth-condition: $\ulcorner \Phi\alpha \urcorner$ and $\ulcorner \Phi\beta \urcorner$ are both true if, and only if, X is Φ. If ❸ is an intensional operator, then by definition the truth-value of $\ulcorner$❸$(\Phi\alpha)\urcorner$ depends only upon the truth condition of $\ulcorner \Phi\alpha \urcorner$; and the truth-value of $\ulcorner$❸$(\Phi\beta)\urcorner$ depends only upon the truth-condition of $\ulcorner \Phi\beta \urcorner$. But $\ulcorner \Phi\alpha \urcorner$ and $\ulcorner \Phi\beta \urcorner$ have the same truth-condition (they are both true if and only if X is Φ). Hence $\ulcorner$❸$(\Phi\alpha)\urcorner$ is true if and only if $\ulcorner$❸$(\Phi\beta)\urcorner$ is true. Hence ❸ is +PSST.

14. Modality and Quine's slingshot

According to Quine, talk of possible worlds, necessity, and intensions is futile. He has deployed three main formal arguments against the logical modalities, and there is some confusion in the literature as to the precise relationship between them. The first is a straightforward slingshot—an instantiation of the one discussed in §11—and it must be understood as an argument against the possibility of modal *S*-connectives occurring in a language that contains (i) two-place truth-functional *S*-connectives and (ii) definite descriptions (or class abstracts)—n.b. if descriptions (or abstracts) are treated as unanalysed singular terms, the language need not contain quantifiers. Quine's second argument also turns upon substitution inferences involving descriptions (or corresponding abstracts) of the form $\ulcorner(\iota x)(x = a \bullet \phi)\urcorner$ within the scope of modal *S*-connectives and to logical validities of standard modal systems, and for this reason it is sometimes viewed as a type of slingshot argument (see Føllesdal (1983) and Marti (1994) for discussion). This argument must be understood as an argument against the possibility of modal *S*-connectives occurring in a language that contains (i) two-place truth-functional *S*-connectives, (ii) descriptions (or abstracts), and (iii) quantifiers. So strictly speaking, the second argument demonstrates a weaker conclusion than the first. The third argument involves the substitution of a simple description within the scope of a modal *S*-connective and a thesis about the interpretation of bound variables. It must be understood as an argument against the possibility of modal *S*-connectives occurring in a language that contains (i) descriptions and (ii) quantifiers. It is the first of these arguments that I want to address here.[44]

On their standard construals, the modal *S*-connectives "$\Box$" and "$\Diamond$" are meant to be +PSLE and –PSME. With these connectives in mind, Quine proposes to show that any *S*-connective that is +PSLE and also permits what he calls "the substitutivity of identicals" is, in fact, an extensional *S*-connective. He begins his argument with an abbreviatory convention (1960, p. 148):

> Where "p" represents a sentence, let us write "δp" (following Kronecker) as short for the description: the number x such that ((x = 1) and p) or ((x = 0) and not p).

Following the discussion in §11, the central part of Quine's argument can be set out as the following derivation:

1	[1]	$p \leftrightarrow q$	premiss
2	[2]	$\Box p$	premiss
2	[3]	$\Box(\delta p = 1)$	2, PSLE (assuming "$\delta p = 1$" $\vDash\dashv$ "p")
1	[4]	$\delta p = \delta q$	1, def. of "δ"
1,2	[5]	$\Box(\delta q = 1)$	3, 4, "substitutivity of identicals"
1,2	[6]	$\Box q$	5, PSLE (assuming "$\delta q = 1$" $\vDash\dashv$ "q").

[44] I will make a few brief remarks about the others in passing. Quine's second argument is not worth addressing: (a) there is no question of getting to grips with it without a thorough understanding of the first argument, (b) it inherits any real import it has from that of the first argument, and (c) it involves tangling with thorny but irrelevant complexities. (It is not surprising that the argument has attracted so little attention.) I have addressed Quine's third argument elsewhere (Neale, 1990), and I stand by everything I said earlier, though the present essay forces some tidying up. The central problem with the third argument is obvious once the limitations of the first argument are grasped.

Gloss: on the assumptions that (i) "p" and "q" have the same truth-value, (ii) "$\Box p$" is true, (iii) "$\Box$" is +PSLE, and (iv) "$\Box$" allows the "substitutivity of identicals", it appears to be provable that "$\Box q$" is true. And since "$\Box q$" differs from "$\Box p$" just in the substitution of the mere material equivalents "p" and "q", it would seem that "$\Box$" is, contrary to hypothesis, +PSME, i.e. it would seem that "$\Box$"—and any other purportedly nonextensional S-connective that allows the substitutions given in (iii) and (iv)—is actually an extensional S-connective after all.

The validity of the derivational component of this argument turns on two related matters: (i) the interpretation of the expression "substitutivity of identicals," and (ii) the semantics ascribed to Kronecker's δ-operator. Quine uses the definite description "the number x such that (($x = 1$) and p) or (($x = 0$) and not p)" when rendering the expression "δp" in English; so the question naturally arises whether he is assuming the description to have a Russellian or a referential semantics. And as we have already seen, the power of any collapsing argument involving the substitution of descriptions whose matrices are satisfied by the same object depends upon the precise semantics assumed, all the more so when the notion of logical equivalence is invoked in connection with sentences containing descriptions. Quine's use of Kronecker's notation to produce a description "δp"—rather than, say, a use of the Peano-Russell notation to produce a description "$(\iota x)((x = 1 \bullet p) \vee (x = 0 \bullet \neg p))$"—does not excuse him from saying something about the semantics of such an expression.

If Quine is assuming that Kronecker descriptions are treated in accordance with Russell's theory, then it is easy enough to justify the logical equivalences that his proof invokes. On a Russellian account, "$\delta p = 1$" is simply an abbreviation for the first-order sentence, (47), which is logically equivalent to (48):

(47) $(\exists x)((\forall y)((y = 1 \bullet p) \vee (y = 0 \bullet \neg p) \leftrightarrow y = x) \bullet x = 1)$.

(48) p.

(Perhaps it was the logical equivalence of (47) and (48) that prompted Quine to claim that "$\delta p = 1$" and p are logically equivalent.) But now a problem arises. If Kronecker descriptions are Russellian, the entry on line [4] of Quine's proof is not a genuine identity statement; it is just shorthand for (49):

(49) $(\exists x)((\forall y)((y = 1 \bullet p) \vee (y = 0 \bullet \neg p) \leftrightarrow y = x) \bullet$
$(\exists z)((\forall w)((w = 1 \bullet q) \vee (w = 0 \bullet \neg q) \leftrightarrow w = z) \bullet x = z))$.

So on the reading that interests Quine, the central lines of the derivation given above will have the following logical forms:

[3] $\Box((\exists x)((\forall y)((y = 1 \bullet p) \vee (y = 0 \bullet \neg p) \leftrightarrow y = x)\, x = 1))$ 2, PSLE

[4] $(\exists x)((\forall y)((y = 1 \bullet p) \vee (y = 0 \bullet \neg p) \leftrightarrow y = x) \bullet (\exists z)((\forall w)((w = 1 \bullet q) \vee (w = 0 \bullet \neg q) \leftrightarrow w = z) \bullet x = z))$ 1, def of "δ"

[5] $\Box((\exists z)((\forall w)((w = 1 \bullet q) \vee (w = 0 \bullet \neg q) \leftrightarrow w = z) \bullet z = 1))$ 3,4,"subst".

But what is this inference rule "subst" ("substitutivity of identicals") that licenses the move from lines [3] and [4] to line [5]? On a Russellian treatment of descriptions, clearly "subst" is not PSST, which is a rule of inference governing the substitution of coreferring singular terms; it must be ι-SUB. So on a Russellian treatment of descriptions Quine's proof shows decisively that "□" cannot have the following combination of features:

(50) +PSLE −PSME +ι-SUB.

But this result, incontrovertible as it is, will not worry most modal logicians. First, it is the combination given in (51) that most modal logicians want to ascribe to "□" and "◇":

(51) +PSLE −PSME +PSST.

And Quine's proof has absolutely no bearing on the viability of this combination *if descriptions are provided with a Russellian treatment*. Second, modal logicians are antecedently predisposed to think that "□" and "◇" are *not* +ι-SUB, *largely because of examples brought up by Quine*. Consider the following argument, where the description "the number of planets in our solar system" is substituted for the description "the square of three" within the scope of "□":

(52) [1] □(nine exceeds seven);
[2] nine = the number of planets in our solar system;
____
[3] □(the number of planets in our solar system exceeds seven).

That is, consider the argument rendered thus in a system of restricted quantification:

(52′) [1] □(nine exceeds seven)
[2] [the x: x numbers the planets in our solar system](x= nine);
____
[3] □[the x: x numbers the planets in our solar system] (x exceeds seven).

The fact that (52) is invalid when read as (52′) shows that "□" is not +ι-SUB. The upshot of all this, then, is that if Quine treats Kronecker descriptions as Russellian, (a) the logical equivalences he needs for his slingshot are guaranteed, but (b) the conclusion of the argument is one that modal logicians will endorse, viz., that "□" and "◇" do not possess the set of features given in (50). None of this has any bearing whatsoever on whether or not "□" and "◇" are +PSST, which many modal logicians think they are. So no argument against the possibility of treating "□" and "◇" as nonextensional *S*-connectives emerges.

The appearance of (52) and (52′) is certain to bring to mind an argument that Quine uses against the possibility of combining "□" and "◇" with devices of quantification. He has argued that a variable occurring within the scope of a quantifier *Q* may not be intelligibly understood as bound by *Q* if there is an intervening modal operator. More precisely, in a formula of the following form

(53) $(Q\ldots(\Box\ldots(\ldots v\ldots)))$

—where *Q* is a quantifier, "□" is a modal operator within the scope of *Q*, and *v* is a variable within the scope of "□" (*a fortiori* within the scope of *Q*)—Quine claims there is no intelligible interpretation of the formula upon which *v* is bound by *Q*. The conclusion of (52) is one of two readings that Russell's theory ascribes to the string

(54) □(the number of planets in our solar system exceeds seven).

The other is (55):

(55) [the *x*: *x* numbers the planets in our solar system]□($x > 7$).

But (55) is of the form of (53), so on Quine's account it is unintelligible. If Quine were right about this, the Russellian would be restricted to just one reading, viz. the conclusion of (52′). But, importantly, this would not bear on the issue of whether or not there are nonextensional *S*-connectives; it would show only that one cannot quantify into the scope of the modal *S*-connectives "□" and "◇". While such a result would certainly curtail the use of the modal *S*-connectives, it would not render all modal sentences unintelligible nor would it cast aspersions on the very idea of modal or other nonextensional *S*-connectives.[45]

If one wants to treat descriptions as singular terms, getting (47) and (48) to come out logically equivalent involves nontrivial commitments. Referential treatments of descriptions were examined in section §12. Of these, Taylor's Strawsonian theory and those chosen object theories embodying stipulations to the effect that, in a model M, only descriptions can refer to the object serving as the referent of improper descriptions in *M*, are of interest to us here as they rule (47) and (48) logically equivalent. But once again we must not overlook the implausibility of such theories as accounts of descriptions occurring in natural language or modalized

[45] As my wording makes clear, I think there is no reason to think that Quine's argument against quantified modal logic demonstrates anything of significance. My reasons are essentially those given by Smullyan (1947, 1948), expounded and developed in detail by Neale (1990). On this matter, see also Marcus (1948) and Kripke (1972).

formal languages. Certainly when it comes to ambiguities and to improper descriptions Russell's theory is superior.

The modal *S*-connectives "$\Box$" and "$\Diamond$", appear to be $+\iota$-INTR and $+\iota$-ELIM, (just as they seem to be $+\lambda$-INTR and $+\lambda$-ELIM). The following inferences are unproblematic (the rules applying *within* the scope of "$\Box$" of course):

(56) $\dfrac{\Box Fa}{\Box\{a = (\iota x)(x = a \bullet Fx)\}}$ $\qquad\qquad$ $\dfrac{\Box\{a = (\iota x)(x = a \bullet Fx)\}}{\Box Fa.}$

One task for the modal logician, then, is to provide a semantics for descriptions that captures these facts. Again, Russell's theory does the job perfectly.

15. Causal statements

Some odd claims have been made by people who take themselves to be addressing the question "are causal contexts extensional?"[46] If we are to inject clarity into this question, we need to distinguish at least two brands of purportedly causal locution. Consider the following:

(57) The short-circuit *caused* the fire at the Ritz

(58) There was a fire at the Ritz *because* there was a short-circuit.

In (57) the causal expression is a transitive verb whose arguments are noun phrases (definite descriptions of events). In (58), by contrast, the causal expression looks more like a two-place *S*-connective whose arguments are sentences rather than noun phrases. Davidson has argued that this difference is superficial and that an analysis of the logical form of (58) reveals an occurrence of the transitive verb "cause" and no occurrence of any causal *S*-connective. Indeed, he uses a slingshot argument against the viability of non-extensional causal *S*-connectives.[47]

Putting aside the correct analysis of the logical form of (57) for a moment, consider (58). On the assumption that nouns such as "short-circuit" and "fire" apply to events (rather than objects), and on the plausible assumption that event descriptions, like object descriptions, are best

[46] The question is addressed by, e.g., Achinstein (1975, 1979), Anscombe (1969), Chisholm (1965), Davidson (1967c), Føllesdal (1965), Gottlieb and Davis (1975), Kim (1993), Levin (1976), Lombard (1979), Lycan (1974), Mellor (1995), Pap (1958), Rosenberg and Martin (1979), Stern (1978), and Vendler (1967a)

[47] Analyses of various causal expressions as nonextensional *S*-connectives have been provided by, e.g., Burks (1951), Mackie (1965, 1974), Pap (1958), and Needham (1994).

treated as Russellian—an assumption that looks more attractive than ever in the light of the referential theories examined earlier—the logical form of (57) is given by (57′):

(57) The short-circuit *caused* the fire at the Ritz

(57′) [the x: short-circuit(x)] [the y: fire(y) • at-the-Ritz(y)] (x caused y).

So (57) has the same logical structure as a sentence like (59), whose logical form is given by (59′):

(59) the king kissed the queen

(59′) [the x: king(x)] [the y: queen(y)] (x kissed y).

The important point here concerns *scope*. On a Russellian analysis the descriptions in (57′) and (59′), unlike the variables they bind, are not within the scopes of "caused" or "kissed". More generally, since the descriptions in (57′) and (59′) do not occur within the scopes of any non-extensional operators, they occur in extensional contexts.

Since coextensional predicate substitution in extensional contexts will not affect truth-value, obviously any predicate inside any of the descriptions in (57) and (59) can be replaced *s.v.* by any coextensional predicate. Consequently, a description of an object or event may be replaced by any other description of the same object or event (this, of course, is rolled up into Whitehead and Russell's derived rule of inference *14.16). So from (59) and (60),

(59) the king kissed the queen

(60) the queen = the most beautiful woman in the kingdom

we can validly infer (61):

(61) the king kissed the most beautiful woman in the kingdom.

Similarly, from (57) and (62),

(62) the fire at the Ritz = the fire at the most expensive hotel in town

we can validly infer (63):

(63) The short-circuit *caused* the fire at the most expensive hotel in town.

But it would be a mistake to package this into the unthinking claim that causal contexts are extensional: we haven't been talking about causal contexts because we haven't been talking about substitutions *within* the scopes of causal expressions, we've been talking about *external* substitutions, i.e. substitutions in extensional contexts.

By contrast, when it comes to (58), interesting questions about causal contexts and extensionality do emerge. In particular, if the occurrence of "because" is treated as a two-place *S*-connective, we can sensibly ask whether or not it is extensional, and then proceed to examine what happens when coextensional expressions are substituted within its scope. And it is clear that the purported connective is *not* extensional: it does not permit the substitution of coextensional sentences *s.v.* Suppose (58) is true; then the contained sentences "there was a fire at the Ritz" and "there was a short-circuit" have the same extension (they are both true); but this is not enough to guarantee the truth of (64), which is obtained by switching the contained sentences:

(64) there was a short-circuit *because* there was a fire at the Ritz.

Thus the purported connective is not +PSME.

But two further questions concerning extensions emerge at this point: (i) Is the connective +PSST? (ii) Is it +ι-SUB?

(i) Let us use "©" as shorthand for "because". It is clear that "©" is +PSST. The following is surely a valid inference:

(65) [1] Catiline fell © Cicero denounced him;
[2] Cicero = Tully;

---

[3] Catiline fell © Tully denounced him.

(ii) The *S*-connective "©" also seems to be +ι-SUB, witness the following:

(66) [1] Catiline fell © the greatest Roman orator denounced him;
[2] the greatest Roman orator = the author of *De Fato*;

---

[3] Catiline fell © the author of *De Fato* denounced him.

But on a Russellian analysis of descriptions, there is more here than meets the eye. The situation is reminiscent of Russell's (1905) discussion of George IV's wondering whether Scott was the author of *Waverley* and Smullyan's (1948) discussion of the number of planets. In (66), perhaps the entries on lines [1] and [3] are ambiguous according as the descriptions are given large or small scope. If so, then it might be possible to claim that the purported validity of (66) provides no reason to think that "©" is +ι-SUB. The idea would be that (66) is valid on the following interpretation

(66′) [1] the x: greatest Roman orator(x)] (Catiline fell © x denounced Catiline;
[2] the greatest Roman orator = the author of *De Fato*;

---

[3] [the x: author of *De Fato*(x)] (Catiline fell © x denounced Catiline).

But since there is no ι-SUBSTITUTION within the scope of "©" here, the intuitive validity of (66) gives us precious little information about the logic of "©". The relevant information must reside in the validity or invalidity of the following reading of (66):

(66″) [1] Catiline fell © [the x: greatest Roman orator(x)] (x denounced Catiline);

[2] the greatest Roman orator = the author of *De Fato*;

---

[3] Catiline fell © [the x: author of *De Fato*(x)] (x denounced Catiline).

There certainly is a reading of (66) according to which it is valid. But is that simply because (66) can be understood as (66′)? Or does (66″) give the sole logical form of (66) and thereby demonstrate that "©" is +ι-SUB? I suspect that lines [1] and [3] of (66′) are not genuine readings of lines [1] and [3] of (66), for reasons that Evans (1977) highlighted in his discussion of anaphora.[48]

Suppose "©" is +ι-SUB; there is no good reason to think it is also +PSLE. If "©" is +PSLE, then (67), (68) and (69) must all entail one another:

(67) Catiline fell © Tully denounced him

(68) (Catiline fell $\bullet$ ($\phi \vee \neg\phi$)) © Tully denounced him

(69) Catiline fell © (Tully denounced him $\bullet$ ($\phi \vee \neg\phi$)).

And since there is no compelling reason to think this is the case, the Church-Quine-Davidson slingshot poses no threat to understanding "©" as a nonextensional *S*-connective.[49]

Gödel's slingshot is more worrying, however. Again, suppose © is +ι-SUB. Is it also +ι-CONV? Do (67) and (70), or (67) and (71) entail one another?

(70) Catiline = (ιx)(x = Catiline $\bullet$ x fell) © Tully denounced him

(71) Catiline fell © Tully = (ιx)(x = Catiline $\bullet$ x denounced him).

If so, then Gödel's result has definite consequences for the causal logician: it is imperative to deny that "©" is +ι-SUB. The way for the causal logician to do this is to endorse Russell's Theory of Descriptions and argue that the argument in (66) is being read as (66′) by people who maintain that the inference is valid.

[48] Sentence (i) cannot be understood as either (ii) or (iii):

(i) * Every boy liked it$_1$ because [some car]$_1$ was red.

(ii) [some y: car y] [every x: boy x] ((x liked y) because (y was red))

(iii) [every x: boy x] [some y: car y] ((x liked y) because (y was red)).

Does this signal that it is not, in general, possible for a quantified noun phrase occurring within one of the "conjuncts" of "because" to be understood with larger scope than "because"? I suspect not, and that it shows only that the constraints on quantifier scope form a proper subset of the constraints on quantifier-variable anaphora.

[49] Neale (1993) suggests that "©" is +PSLE. This suggestion, which does not impinge upon the main points of that work, is mistaken.

16. Facts revisited

Gödel's proof gives us an elegant tool for examining philosophical theses. Let's return to where we started, to facts. We can put statements that purport to make reference to facts into *S*-connective format; for example, the following can be treated as one-place *S*-connectives:

(72) the statement that (ϕ) corresponds to the fact that ( )

(73) the fact that (ϕ) caused it to be the case that ( )

(74) the fact that (ϕ) = the fact that ( ).

If the occurrences of ϕ are removed, we can view the results as two-place *S*-connectives. Let us use "*FIC*" (for "Fact Identity Connective") for the two-place *S*-connective

(75) the fact that ( ) = the fact that ( )

Obviously "*FIC*(ϕ, ϕ)" is true.

We can begin to examine *any* theory of facts by looking at the inferential properties of *FIC*. Obviously if *FIC* is +PSME, then there is at most one fact. But Gödel's proof provides us with an additional piece of information: any fact theorist who maintains that *FIC* is +ι-SUB and +ι-CONV is committed to *FIC*'s being +PSME, and so to the existence of at most one fact. *There is no way out of this*. So the task for the fact theorist is now easy to state: articulate a theory of facts according to which *FIC* lacks one of these properties and at the same time provide a semantics for descriptions that is consistent with the logic ascribed to *FIC* and viable as an account of descriptions in natural language. (As far as Russell's theory of facts is concerned, Gödel's point is that Russell manages to do all of this by (i) individuating facts by reference to their components, i.e. objects and properties (construed nonextensionally); (ii) denying that *FIC* is +ι-SUB; and (iii)—surprise—using Russell's Theory of Descriptions.)

I take it that *no* fact theorist wants to deny that *FIC* is +PSST. So the fact theorist who wants to maintain that descriptions are singular terms has some work to do to avoid Gödel's slingshot. It cannot be avoided by agnosticism as to the semantics of descriptions and denying that *FIC* is +PSLE (the basic strategy of Barwise and Perry (1981, 1983), Bennett (1988), Searle (1995), Taylor (1985), and others). For Gödel's slingshot does not make use of PSLE, it makes use of ι-CONV. And denying that *FIC* is +ι-CONV means taking a definite position on the semantics of descriptions.

The fact theorist who is a Russellian about descriptions has an easier task, for he or she has the option of denying that *FIC* is +ι-SUB. This strategy means saying something about the following inference (analogous to (66) above):

(76) [1] the fact that the greatest Roman orator snored = the fact that the greatest Roman orator snored;
[2] the greatest Roman orator = the author of *De Fato*;

---

[3] the fact that the greatest Roman orator snored = the fact that the author of *De Fato* snored.

If this is valid, then the fact theorist who is a Russellian about descriptions will have to say that it does not involve an ι-substitution within the scope of *FIC*, but a substitution in the truth-functional context outside its scope, the argument being understood as valid only when the descriptions have large scope, e.g. when the argument is understood as (76′):

(76′) [1] [the x: greatest Roman orator(x)] ([the y: greatest Roman orator(y)] (the fact that x snored = the fact that y snored));
[2] the greatest Roman orator = the author of *De Fato*;

---

[3] [the x: greatest Roman orator(x)] ([the y: author of *De Fato*(y)] (the fact that x snored = the fact that y snored)).

So, again, we are faced with the question of determining whether or not the constraints on quantifier scope and variable-binding delivered by the best theory of syntax license (76′) as reading of (76).

Naturally enough, exactly the same considerations apply when we bring together talk of facts and talk of causes—in the spirit of those fact theorists who would maintain that facts (as well as, or instead of, events) are causal relata—as in the following inference:

(77) [1] the fact that there was a malfunction in the new sprinkler system caused the water not to flow;
[2] the new sprinkler system = the cheap sprinkler system that Bill installed;

---

[3] the fact that there was a malfunction in the cheap sprinkler system that Bill installed caused the water not to flow.

There is no knock-down argument against facts in this; but it is now abundantly clear that unless a theory of facts is presented with an accompanying theory of descriptions and an accompanying logic of *FIC*, there is good reason to treat it with caution. The task for the friend of facts is to put together a theory according to which facts are not so fine-grained that they are sentence-like and not so coarse-grained that they collapse into one.

Finally, it should be noted that analogous tests can be produced for accounts of truth, statements, and propositions that do not appeal to facts. This is easily seen by viewing the following as *S*-connectives:

(78) the sentence *S* is true iff ( )

(79) the statement that ( ) is true iff ( )

(79) the proposition that ( ) is true iff ( )

(80) the proposition that ( ) = the proposition that ( ).

We have learned several things. First, logical equivalence is not the most important issue when it comes to the force of slingshot arguments. Second, the power of Gödel's slingshot resides in the fact that it forces philosophers to say something about the semantics of definite descriptions as soon as they step outside the realm of extensional logic and language fragments, and as soon as they posit entities to which sentences are meant to correspond. Third, although no nonextensional *S*-connective can be $+\iota$-SUB and $+\iota$-CONV, this need not spell trouble for advocates of nonextensional logics and *S*-connectives who endorse Russell's Theory of Descriptions. Fourth, Gödel's argument yields an elegant test for examining the logics of purportedly nonextensional contexts. Finally, referential treatments of descriptions have unpleasant consequences that are highlighted by Gödel's slingshot and, to some extent, those due to Church, Quine, and Davidson. We have extracted much philosophy from Gödel's discussion and found a tool that should help us to avoid many philosophical mistakes.[50]

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REFERENCES

Achinstein, P. 1975: "Causation, Transparency, and Emphasis". *Canadian Journal of Philosophy* 5, pp. 1–23.

——1979: "The Causal Relation". *Midwest Studies in Philosophy IV: Studies in Metaphysics*. Minneapolis: University of Minnesota Press, pp. 369–86.

Anscombe, G. E. 1969: "Causality and Extensionality". *Journal of Philosophy* 66, pp. 152–9.

Altman, A., Bradie, M. and Miller, D. Jr. 1979: "On Doing Without Events". *Philosophical Studies* 36, pp. 301–6.

Anderson, A. and Belnap, N. 1962: "Tautological Entailment". *Philosophical Studies* 13, pp. 9–24.

Armstrong, D. 1993: "A World of States of Affairs", in J. Tomberlin, ed., *Philosophical Perspectives 7,* pp. 429–40.

Barwise, J. and Cooper, R. 1981: "Generalized Quantifiers and Natural Language". *Linguistics and Philosophy* 4, pp. 159–219.

Barwise, J. and Etchemendy, J. 1989: *The Liar: An Essay on Truth and Circularity*. Oxford: Clarendon Press.

Barwise, J. and Perry, J. 1981: "Semantic Innocence and Uncompromising Situations". *Midwest Studies in Philosophy* VI, pp. 387–403.

——1983: *Situations and Attitudes*. Cambridge: MIT Press.

Bennett, J. 1988: *Events and Their Names*. New York: Hackett.

Burge, T. 1974: "Truth and Singular Terms". *Nous* 8, pp. 309–25.

——1986: "Frege on Truth", in L. Haaparanta and J. Hintikka (eds.), *Frege Synthesized*, Dordrecht: Reidel, pp. 97–154.

Burks, A. 1951: "The Logic of Causal Propositions". *Mind* 60, pp. 363–82.

Carnap, R. 1942: *Introduction to Semantics*. Cambridge: Harvard University Press.

——1947: *Meaning and Necessity.* Chicago: University of Chicago.

Chisholm, R. 1965: "Query on Substitutivity". *Boston Studies in the Philosophy of Science*, Vol II, Dordrecht: Reidel, pp. 275–8.

Church, A. 1940: "A Formulation of the Simple Theory of Types". *Journal of Symbolic Logic* 5, pp. 56–68.

——1943a: Review of Carnap's *Introduction to Semantics*. *Philosophical Review* 52, pp. 298–304.

——1943b: Review of Quine's "Notes on Existence and Necessity". *Journal of Symbolic Logic* 8, pp. 45–7.

——1956: *Introduction to Mathematical Logic*. Princeton: Princeton University Press.

Clark, R. 1975: "Facts, Fact-correlates, and Fact-surrogates", in P. Welsh (ed.), *Fact, Value, and Perception: Essays in Honor of Charles A. Baylis*. Durham: Duke University Press.

Cummins, R. and Gottlieb, D. 1972: "On an Argument for Truth-functionality". *American Philosophical Quarterly* 9, pp. 265–9.

Dale, A. J. 1978: "Reference, Truth-functionality, and Causal Connectives". *Analysis* 38, pp. 99–106.

Davidson, D. 1967a: "Truth and Meaning". *Synthese* 7, pp. 304–23. Reprinted in Davidson 1984a, pp. 17–36.

——1967b: "The Logical Form of Action Sentences", in N. Rescher (ed.), 1967: *The Logic of Decision and Action*. Pittsburgh: University of Pittsburgh Press, pp. 81–95. Reprinted in Davidson 1980, pp. 105–48.

——1967c: "Causal Relations". *Journal of Philosophy* 64, pp. 691–703. Reprinted in Davidson 1980, pp. 105–21.

——1969a:"True to the Facts". *Journal of Philosophy* 66, pp. 748–64. Reprinted in Davidson 1984a, pp. 37–54.

——1969b: "Events and Event Descriptions" in J. Margolis (ed.), *Fact and Existence*. Oxford; Blackwell, 1969, pp. 74–84.

——1980: *Essays on Actions and Events*. Oxford: Clarendon Press.

——1984a: *Inquiries into Truth and Interpretation*. Oxford: Clarendon Press.

——1984b: "On the Very idea of a Conceptual Scheme", in Davidson 1984a, pp. 183–98.

——1989: "The Myth of the Subjective", in M. Krausz, (ed.), *Relativism: Interpretation and Confrontation*. Notre Dame: University of Notre Dame Press, pp. 159–72.

——1990: "The Structure and Content of Truth". *Journal of Philosophy* 87, pp. 279–328.

——forthcoming: "The Folly of Trying to Define Truth".

Davies, M. 1978: "Weak Necessity and Truth Theories". *Journal of Philosophical Logic* 7, pp. 15–439.

——1981: *Meaning, Quantification, Necessity*. London: Routledge.

Donnellan, K. 1981: "Presupposition and Intuitions", in P. Cole (ed.), *Radical Pragmatics*. New York: Academic Press, pp. 129–42.

Evans, G. 1977: "Pronouns, Quantifiers and Relative Clauses (I)". *Canadian Journal of Philosophy* 7, pp. 467–536.

Evans, G. 1984: *The Varieties of Reference*. Oxford: Clarendon Press.

——1985: "Does Tense Logic Rest on a Mistake?" in his *Collected Papers*. Oxford: Clarendon Press, pp. 343–63.

Evnine, S. 1991: *Donald Davidson*. Oxford: Blackwell.

Fine, K. 1982: "First-order Modal Theories III—Facts". *Synthese* 53, pp. 43–122.

Føllesdal, D. 1964: "Knowledge, Identity, and Existence". *Theoria* 33, pp. 1–27.

——1965: "Quantification into Causal Contexts". *Boston Studies in the Philosophy of Science*, Vol II, Dordrecht: Reidel, pp. 263–74.

——1966: "Referential Opacity and Modal Logic". *Filosofiske Problemer,* vol 23, Oslo: Universitetforslaget.

——1969: "Quine on Modality", in D. Davidson and J. Hintikka (eds.), *Words and Objections*. Dordrecht: Reidel, pp. 175–85.

——1983: "Situation Semantics and the 'Slingshot' Argument". *Erkenntnis* 19, pp. 91–8.

Frege, G. 1892: "Über Sinn und Bedeutung". *Zeitschrift fur Philosophie und Philosophische Kritik* 100, pp. 25–50. Translated as "On Sense and Reference" in P. Geach and M. Black (eds.): *Translations from the Philosophical Writings of Gottlob Frege*. Oxford: Blackwell, pp. 56–78.

——1893: *Die Grundgesetze der Arithmetik, vol. I.* Jena: Verlag Hermann Pohle. Reprinted at Hildesheim: Georg Olms Verlagsbuchhandlung, 1962. Translated as *The Basic Laws of Arithmetic*, by M. Furth, Berkeley and Los Angeles: University of California Press, 1964.

Gale R. 1970: "Strawson's Restricted Theory of Referring". *Philosophical Quarterly* 20, pp. 162–5.

Geach, P. T. 1972: *Logic Matters*. Berkeley: University of California Press.

Gödel, K. 1944: "Russell's Mathematical Logic", P. A. in Schillp (ed.), *The Philosophy of Bertrand Russell.* Evanston and Chicago: Northwestern University Press, pp. 125–53.

Gottlieb, D. and Davis, L. 1974: "Extensionality and Singular Causal Sentences". *Philosophical Studies* 25, pp. 69–72.

Grandy, R. 1972: "A Definition of Truth for Theories with Intensional Definite Description Operators". *Journal of Philosophical Logic* 1, pp. 137–55.

Halldén, S. 1949: *The Logic of Nonsense*. Uppsala: Uppsala Universitets Arsskrift.

Harré, R and Madden, E. H. 1975: *Causal Powers: A Theory of Natural Necessity.* Oxford: Blackwell.

Hilbert, D. and Bernays, P. 1934: *Grundlagen der Mathematik,* vol. I. Berlin.

Hochberg, H. 1978: "Descriptions, Substitution, and Intentional Contexts", in his *Thought, Fact, and Reference: Origins of Ontology and Logical Atomism*. Minneapolis: University of Minnesota Press.

——1984: "Facts and Truth", in his *Logic, Ontology, and Truth*. Philosophia Verlag, pp. 279–95.

Horgan, T. 1978: "The Case Against Events". *Philosophical Review* 87, pp. 28–47.

——1982: "Substitutivity and the Causal Connective". *Philosophical Studies* 42, pp. 47–52.

Kalish, D., Montague, R., and Marr, D. 1980: *Logic: Techniques of Formal Reasoning*. 2nd ed. New York: Harcourt, Brace, Jovanovich.

Kaplan, D. 1978: "Dthat", in P. Cole (ed.), *Syntax and Semantics, vol. 9: Pragmatics*. New York: Academic Press, pp. 221–43.

Kim, J. 1993: *Supervenience and Mind*. Cambridge: Cambridge University Press.

Körner, S. 1960: *The Philosophy of Mathematics*. London: Hutchison.

Kripke, S. 1972: "Naming and Necessity", in D. Davidson and G. Harman (eds.),*Semantics of Natural Language*, Dordrecht: Reidel, pp. 253–355. Reprinted with a preface Oxford: Blackwell Publishers, 1980.

Lambert, K. 1991: "A Theory of Definite Descriptions", in K. Lambert, (ed.), *Philosophical Applications of Free Logic*. Oxford: Clarendon Press, pp. 17–27.

Levin, M. 1976: "The Extensionality of Causation and Causal Explanatory Contexts". *Philosophy of Science* 43, pp. 266–77.

Lewis, C. I. 1943: "Modes of Meaning". *Philosophy and Phenomenological Research* IV, 2 pp. 236–49.

Lindström, P. 1966: "First-order Predicate Logic with Generalized Quantifiers". *Theoria* 32, pp. 186–95.

Lindström, S. 1991: "Critical Study of Jon Barwise and John Perry's *Situations and Attitudes*". *Nous* 25, pp. 743–70.

Lombard, B. 1979: "The Extensionality of Causal Contexts: Comments on Rosenberg and Martin". *Midwest Studies in Philosophy IV: Studies in Metaphysics*. Minneapolis: University of Minnesota Press, pp. 409–15.

Lycan, W. 1974: "The Extensionality of Cause, Space, and Time". *Mind* 83, pp. 498–511

Lycan, W. 1984: *Logical Form in Natural Language*. Cambridge, Mass.: MIT Press.

McGinn, C. 1976: "A Note on the Frege Argument". *Mind* 85, pp. 422–3.

Mackie, J. L. 1965: "Causes and Conditions". *American Philosophical Quarterly* 2, 245–64.

——1974: *The Cement of the Universe*. Oxford: Clarendon Press.

Marcus, R. 1948: Review of Smullyan's "Modality and Description". *Journal of Symbolic Logic* 13.

Marti, G. 1994: "Do Modal Distinctions Collapse in Carnap's System?". *Journal of Philosophical Logic*, 23, pp. 573–93.

Mates, B. 1973: "Descriptions and Reference". *Foundations of Language* 10, 409–18.

Mellor, D. H. 1991: *Matters of Metaphysics*. Cambridge: Cambridge University Press.

——1995: *The Facts of Causation*. London: Routledge.

Morton, A. 1969: "Extensional and Non-truth-functional Contexts". *Journal of Philosophy* 66, pp. 159–64.

Mostowski, A. 1957: "On a Generalization of Quantifiers". *Fundamenta Mathematica* 44, pp. 12–36.

Neale, S. 1990: *Descriptions*. Cambridge: MIT Press.

——1993a: "Grammatical Form, Logical Form, and Incomplete Symbols", in A. D. Irvine and G. A. Wedeking (eds.), *Russell and Analytic Philosophy.* Toronto: University of Toronto Press, pp. 97–139.

——1993b: "Term Limits", in J. Tomberlin (ed.), *Philosophical Perspectives* 7: *The Philosophy of Language and Logic*, pp. 89–124.

Needham, P. 1994: "The Causal Connective", in J. Faye, U. Scheffler, and M. Urchs (eds.), *Logic and Causal Reasoning*. Akademie Verlag, pp. 67–89.

Olson, K. 1987: *An Essay on Facts*. Lecture Notes No. 6, Stanford: CSLI Publications.

Pap, A. 1958: "Dispositions, Concepts, and Extensional Logic", in H. Feigl *et al*, (eds.), *Minnesota Studies in the Philosophy of Science*, vol. 2, pp. 196–224.

Parsons, C. 1990: "Introductory Note to [Gödel] 1944", in S. Feferman *et al* (eds.), *Kurt Gödel, Collected Works,* Vol. II Oxford: Clarendon Press, pp. 102–18.

Pollock, J. 1984: *The Foundations of Philosophical Semantics*. Princeton: Princeton University Press.

Prior, A. 1967: "The Correspondence Theory of Truth". *Encylopaedia of Philosophy*, P. Edwards (ed.). New York: Macmillan.

Quine, W. V. 1940: *Mathematical Logic*. Cambridge, Mass.: Harvard University Press, revised edition 1951.

——1941: "Whitehead and the Rise of Modern Logic", in P. A. Schillp (ed.), *The Philosophy of Alfred North Whitehead*. Evanston, Illinois: Northwestern University Press, pp. 127–63.

——1943: "Notes on Existence and Necessity". *Journal of Philosophy* 40, pp. 113–27.

——1947: "On the Problem of Interpreting Modal Logic". *Journal of Symbolic Logic* 12, pp. 43–8.

——1953a: "On What There is", in his *From a Logical Point of View*. Cambridge: Harvard University Press, revised 1961 and 1980, pp. 1–19.

——1953b: "New Foundations for Mathematical Logic", in his *From a Logical Point of View*, pp. 80–101.

——1953c: "Reference and Modality", in his *From a Logical Point of View*, pp. 139–59.

——1953d: "Meaning and Inference", in his *From a Logical Point of View*, pp. 160–7.

——1953e: "Three Grades of Modal Involvement". *Proceedings of the XIth International Congress of Philosophy* (Brussels 1953), vol. 14, pp. 65–81.

——1960: *Word and Object*. Cambridge, Mass.: MIT Press.

——1982: *Methods of Logic*. Cambridge Mass.: Harvard University Press, fourth edition.

Rorty, R. 1992a: *Objectivity, Relativism, and Truth*. Cambridge: Cambridge University Press.

——1992b: "Twenty-Five Years After", in R. Rorty (ed.), *The Linguistic Turn,* second edition. London: Hackett.

Rosenberg, A. and Martin, R. 1979: "The Extensionality of causal Contexts". *Midwest Studies in Philosophy IV: Studies in Metaphysics*. Minneapolis: University of Minnesota Press, pp. 401–8.

Russell, B. 1905: "On Denoting". *Mind* 14 1905: pp. 479–93.

——1918: "The Philosophy of Logical Atomism", in R. C. Marsh (ed.), *Logic and Knowledge*. London: George Allen and Unwin, 1956.

Sainsbury, M. 1990: *Logical Forms: An Introduction to Philosophical Logic.* Oxford: Blackwell.

——1993: "Russell on Names and Communication", in A. Irvine and G.A. Wedeking (eds.), *Russell and Analytic Philosophy*. Toronto: University of Toronto Press, pp. 3–21.

Scott, D. 1967: "Existence and Description in Formal Logic", in R. Schoenman (ed.), *Bertrand Russell: Philosopher of the Century*. London: Allen & Unwin, pp. 181–200.

Searle, J. 1969: *Speech Acts*. Cambridge: Cambridge University Press.

——1995: *The Construction of Social Reality.* London: Free Press.

Sharvy, R, 1970: "Truth Functionality and Referential Opacity". *Philosophical Studies* 21, pp. 5–9.

——1972: "Three Types of Referential Opacity". *Philosophy of Science* 39, pp. 153–61.

——1983: "Reply to Wideker". *Philosophia* 3/4, pp. 453–5.

Skyrms, B. 1981: "Tractarian Nominalism". *Philosophical Studies* 40, pp. 199–206.

Sleigh, R. C. 1966: "Note on an Argument of Quine's". *Philosophical Studies* 17, pp. 91–3.

Smullyan, A. F. 1947: Review of Quine's "The Problem of Interpreting Modal Logic". *Journal of Symbolic Logic* 12, pp. 139–41.

——"Modality and Description", *Journal of Symbolic Logic* 13, pp. 31–7.

Stern, C. 1978: "The Alleged Extensionality of 'Causal Explanatory Concepts'". *Philosophy* of *Science* 45, pp. 614–25.

Strawson, P. F. 1950a: "Truth". *Proceedings of the Aristotelian Society*, supp. vol. 24, pp. 129–56.

——1950b: "On Referring", *Mind* 59, pp. 320–44.

——1954: "Reply to Mr Sellars", *Philosophical Review* 63, pp. 216–31.

——1964: "Identifying Reference and Truth-Values", *Theoria* 30, pp. 96–118.

——1972: *Subject and Predicate in Logic and Grammar*. London: Methuen.

——1986: "Direct Singular Reference: Intended Reference and Actual Reference", in L. Nagl and R. Heinrich (eds.), *Wo Steht die Analytische Philosophie Heute*. Vienna: Oldenbourg Verlag, pp. 75–81.

Tarski, A. 1956: *Logic, Semantics, Metamathematics*. Oxford: Clarendon Press.

Taylor, B. 1976: "States of Affairs", in G. Evans and J. McDowell (eds.), *Truth and Meaning*. Oxford: Clarendon, pp. 263–84.

——1985: *Modes of Occurrence: Verbs, Adverbs, and Events*. Oxford: Blackwell.

Thomason, R. 1969: "Modal Logic and Metaphysics", in K. Lambert (ed.), *The Logical Way of Doing Things*. New Haven: Yale University Press, pp. 119–46.

Trenholme, R. 1975: "Causation and Necessity". *Journal of Philosophy* 72, pp. 444–65

van Benthem, J. 1986: *Essays in Logical Semantics*. Dordrecht: Reidel.

van Fraassen, B. 1969: "Facts and Tautological Entailments". *Journal of Philosophy*, 67, 15, pp. 477–87.

Vendler, Z. 1967a: "Causal Relations". *Journal of Philosophy* 64, pp. 704–13.

——1967b: *Linguistics in Philosophy*. Ithaca, NY: Cornell University Press.

Wagner, S. 1986: "California Semantics Meets the Great Fact". *Notre Dame Journal of Formal Logic* 27, pp. 430–55.

Wallace, J. 1969: "Propositional Attitudes and Identity". *Journal of Philosophy,* 66, pp. 145–52.

Wedberg, A. 1966: *Filosofins Historia: Fran Bolzano till Wittgenstein.* Stockholm: Bonniers.

——1984: *A History of Philosophy. Volume 3: From Bolzano to Wittgenstein.* Oxford: Clarendon.

Westerståhl, D. 1989: "Quantifiers in Formal and Natural Languages", in D. Gabbay and F. Guenther (eds.), *Handbook of Philosophical Logic,* Vol IV. Dordrecht: Reidel, pp. 1–131.

Whitehead, A. N. and Russell, B. 1927: *Principia Mathematica*, vol. I, 2nd ed. Cambridge: Cambridge University Press.

Wideker, D. 1983: "The Extensionality Argument". *Nous* 17, pp. 457–68.

——1985: "Davidson on Singular Causal Sentences". *Erkenntnis* 23, pp. 223–42.

Williamson J. 1976: "Facts and Truth". *Philosophical Quarterly* 26, pp. 203–16.

Wilson, N. L. 1959a: "Substances without Substrata". *Review of Metaphysics* 12, pp. 521–39.

——1959b: *The Concept of Language*. Toronto: University of Toronto Press.

——1974: "Facts, Events, and their Identity Conditions". *Philosophical Studies* 25, pp. 303–21.

Wittgenstein, L. 1921: *Tractatus Logico-Philosophicus*, English tr. by D. Pears and B. McGuinness. London: Routledge and Kegan Paul, 1961.