Notes on Realism, Propositions, and the Liar Paradox

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In these notes, I will develop a (propositional) realist resolution of one version of the liar paradox, and I will explain why a similar approach will *not* lead to a resolution of another version of the liar paradox. Ironically, it is the so-called "strengthened" version that *can* be resolved by the present approach, and the "ordinary" or "weaker" version of the paradox (which is typically thought to be *easier*) *cannot* be! I begin by explaining the two versions of the paradox.

Liar Paradox #1 (Ordinary). Consider the following declarative sentence:

(1) Sentence (1) is false.

That is, (1) says *of itself* that it is false. Now, consider the that-clause *that*-(1). And, assume (as a realist might) *that*-(1) denotes a proposition p_1. Now, p_1 must be either true or false, and not both (by the definition of a proposition). If p_1 is true, then p_1 is false (since, in this case, p_1 says of itself – *falsely* – that it is false). And, if p_1 is false, then p_1 is true (since, in this case, p_1 says of itself – *truly* – that it is false). Therefore, p_1 is both true and false. This contradicts our assumption that p_1 is a proposition, since propositions cannot be both true and false. So, our assumption that (1) expresses a proposition must be false. As a result, the definite description "the proposition denoted by *that*-(1)" is *empty*. This completes the set-up for liar paradox #1. Next, we set-up liar paradox #2, and then we will discuss possible resolutions of each of the paradoxes.

Liar Paradox #2 (Strengthened). Consider the following sentence:

(2) Sentence (2) is not true.

That is, (2) says *of itself* that it is not true. Now, consider the that-clause *that*-(2). And, assume (as a realist might) *that*-(2) denotes a proposition p_2. Now, p_2 must be either true or false, and not both (by the definition of a proposition). If p_2 is true, then p_2 is false (since, in this case, p_2 says of itself – *falsely* – that it is not true). And, if p_2 is false, then p_2 is true (since, in this case, p_2 says of itself – *truly* – that it is not true). Therefore, p_2 is both true and false. This contradicts our assumption that p_2 is a proposition, since propositions cannot be both true and false. So, our assumption that (2) expresses a proposition is false. And, the description "the proposition denoted by *that*-(2)" is *empty*.

A Failed Russellian Resolution of the Strengthened Liar Paradox. Since the definite description "the proposition denoted by *that*-(2)" is empty, this seems like a perfect place to apply the Russellian theory of descriptions, which was designed to handle cases involving empty names (*e.g.*, "the present King of France"). Realists about propositions tell us that sentences like "Sentence (2) is false" are really just shorthand for talk about propositions denoted by *that-clauses* like *that Sentence* (2) *is false* (or, *that*-(2), for short). Thus, we can (as realists!) safely rewrite (2), as follows:

(2′) The proposition denoted by *that*-(2′) is not true.

What can we say about (2′)? If (2′) is true, then (2′) expresses a true proposition. But, if (2′) expresses a true proposition, then the proposition (2′) expresses is *not* true. Thus, if (2′) is true, then (2′) is not true, which implies that (2′) is not true. This is not yet enough to show that (2′) is *paradoxical*, however. For that, we would also need to show that if (2′) is not true, then (2′) is true. Can we show this? Yes, but not so obviously this time. It seems that (2′) can be untrue *merely in virtue* of the fact that "the proposition denoted by *that*-(2′)" is empty. Recall, on Russell's theory, all sentences with empty subjects are untrue. So, you might think that (2′) can be untrue *without* implying that (2′) is true. But, this doesn't quite work. Russell's theory, in fact, implies something *stronger* here — that (2′) is *false*. But, if (2′) is *false*, then (2′) *must* express a false *proposition* (*only* propositions can be *false* on the realist theory!). And, if (2′) expresses a false proposition, then it expresses an untrue proposition, and therefore it expresses a true proposition (since it now says of itself truly that it expresses an untrue proposition!). So, the paradox is back, and the Russellian/Realist attempt to resolve the paradox fails. But, we were ever so close to extricating ourselves from its grip. The problem here is Russell's theory of descriptions, on which sentences the subjects of which are empty definite descriptions (or names) come out *false*. If they had come out *merely untrue*, then we would have been home free! Frege and Strawson to the rescue!

A Fregean Resolution of the Strengthened Liar Paradox. According to Frege (and later Strawson), sentences with empty subjects are *neither true nor false*. This allows us to resolve the Strengthened Liar paradox by paraphrasing (2) as (2′), above. The easy part is to see that if (2′) is true, then (2′) is not true. Hence, (2′) is not true. The trickier part is to make use of the fact that, on the Fregean reading, it *cannot* be shown that if (2′) is not true, then (2′) is true, since on the Fregean theory (2′) is untrue (and *merely* untrue) *just because* its subject is an empty description. We cannot infer that (2′) is true from this, because we are not forced to say that (2′) has *any* truth-value *whatsoever* on the Fregean account of empty names. This, it seems to me, is an advantage of the Fregean theory over the Russellian theory. On this account, (2′) comes out neither true nor false (which makes perfect sense, since *that*-(2′) doesn't denote a proposition, as we have already shown!), and the paradox is resolved, without remainder. Interestingly, this solution *only* works for the Strenthened Liar, and not for the Ordinary Liar (which, ironically, is supposed to be "less paradoxical" in some respects). I will conclude this note by explaining why this is so.

A Fregean Resolution of the Ordinary Liar Paradox. If we use our realist paraphrase on (1), we end up with the following:

(1′) The proposition denoted by *that*-(1′) is false.

The good news is that, if $(1')$ is not true, then (assuming a Fregean theory of descriptions) it does *not* follow that $(1')$ is true (for precisely the same reasons given above for $(2')$). But, the bad news is – that's not enough to avoid the paradox in this case! For, if $(1')$ is true, then $(1')$ must (as it says!) express a proposition that is false. Hence, if $(1')$ is true, then $(1')$ is false. It follows that $(1')$ is false. But, this is inconsistent with $(1')$ being neither true nor false, which is assumed by Fregean theory (and which is what allowed us to avoid the "not-$(1') \Rightarrow (1')$" direction of the paradox). Thus, on a Fregean theory, $(1')$ comes out both false and neither true nor false, which is still paradoxical. So, even a Fregean theory of descriptions can't allow the realist to escape the ordinary liar paradox (and, of course, Russell's theory of descriptions does "even worse" – if you believe in the lesser of two evils! – by making $(1')$ come out both true and false). This is ironic, since – from this point of view – the "weaker", "less paradoxical" version of the paradox is *harder to resolve* than the "stronger", "more paradoxical" version! So much for labels of philosophical problems!